Gravitational wave sources in Dynamical Chern-Simons gravity

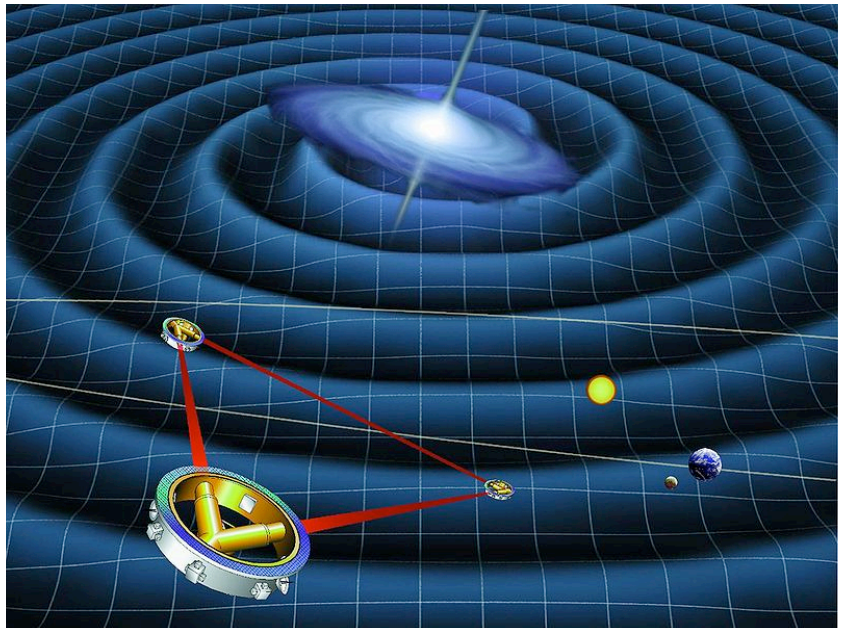
Leonardo Gualtieri, "Sapienza" University of Rome

(in collaboration with V. Cardoso, P. Pani, C. Molina)

Motivation

Testing the strong field limit of gravity is one of the main objectives of current gravitational experiments, both ground-based (Adv. LIGO/Virgo) and space-based (LISA or its smaller version by ESA).

These detectors are expected not only to directly observe gravitational waves, but to study their features, opening up a new window to the universe ("gravitational wave astronomy") and enabling us to test General Relativity (GR), in its strong field, fully non-linear regime, against other theories.



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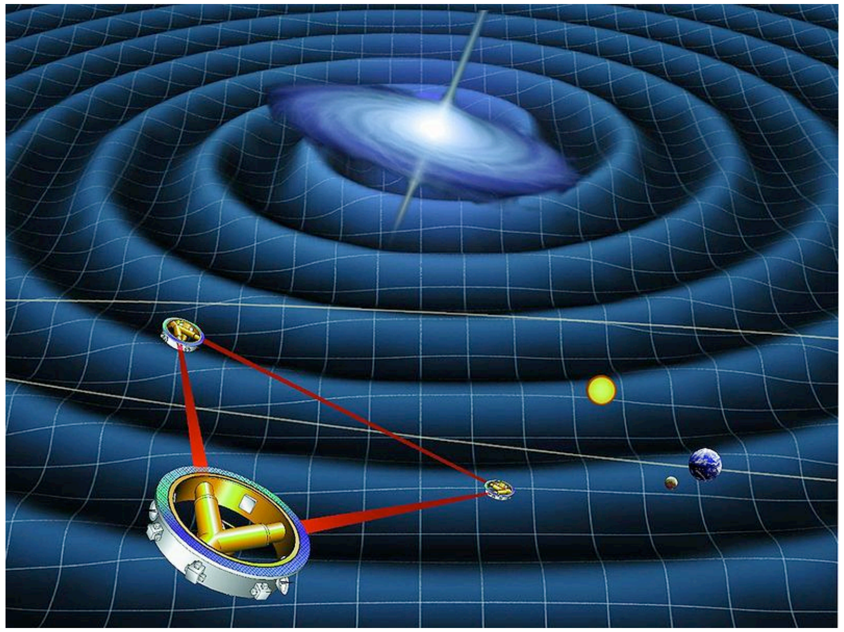

Indeed, even though General Relativity has passed many tests in the weak field regime, open problems from both theory and observations suggest that it should not be considered as the ultimate theory of gravity, but as the low energy limit of a more fundamental theory. A signature of new physics in gravitational experiments could be a sort of "message in a bottle" coming from a theory standing at energies far beyond our reach.

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Chern-Simons modified gravity (*see S.Alexander & N.Yunes '09 and references therein*)
is one of the most interesting extensions of General Relativity.
The gravitational field is coupled with a scalar field through a parity-violating Chern-Simons term:

$$S = \kappa \int d^4x\sqrt{-g}R + \frac{\alpha}{4}\int d^4x\sqrt{-g}\vartheta\ {}^*RR$$

where ϑ is a scalar field and ${}^*RR = -\frac{1}{2}R_{abcd}\epsilon^{abef}R^{cd}{}_{ef}$

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This theory has first been introduced in a *non-dynamical version,*
in which the scalar field is given *a priori*, like an external field.
Later, a *dynamical* version of the theory has been proposed, in which the scalar field is treated as a *dynamical* field:

$$-\frac{\beta}{2} \int d^4x \sqrt{-g} \left[g^{ab} \nabla_a \vartheta \nabla_b \vartheta + V(\vartheta) \right]$$

I will speak about Dynamical Chern-Simons (DCS) gravity and its possible gravitational signatures.

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We will discuss two possible observational signature of DCS gravity:

- Black hole oscillations
- Extreme mass-ratio inspirals

Black hole oscillations

A perturbed black hole oscillates with damped pulsations
at characteristic frequencies and damping times, the so-called quasi-normal modes (QNMs),
which are expected to be detected by gravitational-wave experiments.
For instance, in the coalsescence of two black holes
(which is one of the most promising sources for both ground-based and space-based detectors),
the last stages of the signal (the so-called *ringdown)* will be characterized by
the quasi-normal oscillations of the final black hole.
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Recently (*V. Cardoso & L.G. '09; C. Molina, P. Pani, V. Cardoso, L.G. '10*) we have determined the QNMs of spherically symmetric black holes in DCS gravity, finding how they differ from those obtained in GR.
Detection of a black hole ringdown could allow to discriminate between GR and DCS gravity.

Black hole oscillations

Spherically symmetric BH in DCS gravity are described by the Schwarzschild metric.

In previous work on this subject (*N. Yunes & C. Sopuerta, '08*) is was found that in presence of a background scalar field,

$$g_{\mu\nu} = g_{\mu\nu}^{(Schw)} + h_{\mu\nu} \qquad \vartheta = \vartheta^{(0)} + \delta\vartheta$$

perturbations of the spacetime metric with polar parity and axial parity are mixed, and their equations are very involved; their explicit expression has been worked out only recently (*H. Motohashi & T. Suyama, '11*).

We have found that if $\vartheta^{(0)} = 0$, polar and axial metric perturbations decouple.

Only axial perturbations are coupled with the scalar field; polar perturbations are not affected by the scalar field, and satisfy the equations of General Relativity (the Zerilli equation).

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If we expand the scalar field in scalar spherical harmonics: $\vartheta = \frac{\Theta}{r} Y^{lm} e^{-\mathrm{i}\omega t}$

and we expand the metric perturbations in tensor spherical harmonics, defining the Regge-Wheeler master function $\psi(r)$, which describes axial perturbations, as in GR, we find a system of two coupled differential equations (we set $\alpha=1$ using the freedom of scalar field normalization):

Black hole oscillations

$$\frac{d^2}{dr_*^2}\Psi + \left\{\omega^2 - f\left[\frac{l(l+1)}{r^2} - \frac{6M}{r^3}\right]\right\}\Psi = \frac{96\pi M f}{r^5}\Theta$$

$$\frac{d^2}{dr_*^2}\Theta + \left\{\omega^2 - f\left[\frac{l(l+1)}{r^2}\left(1 + \frac{576\pi M^2}{r^6\beta}\right) + \frac{2M}{r^3}\right]\right\}\Theta = f\frac{(l+2)!}{(l-2)!}\frac{6M}{r^5\beta}\Psi$$

The solutions behaving as purely ingoing waves at the horizon and outgoing waves at infinity are the black hole proper oscillation modes.

The system admits such solutions for a discrete set of values of the complex frequency $\omega_n=\sigma_n+i/\tau_n$: *the QNMs of the black hole.*

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Solving numerically this system is not an easy task: the well-known numerical problems related to the divergence of stable solutions far away from the black hole (which also plague perturbations of black holes in GR) are much more severe in the case of coupled equations, since the solution through the continued fraction methond can not be applied.

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At the end, we overcame these technical problems, and found the QNM solutions using *two* independent numerical approaches: time evolution and a formulation of the frequency domain approach which had never been used before to study black hole perturbations.
The results of the two independent methods agree within 0.1%, validating each other.
The solutions crucially depend on the coupling parameter β (or the dimensionless quantity βM^4).

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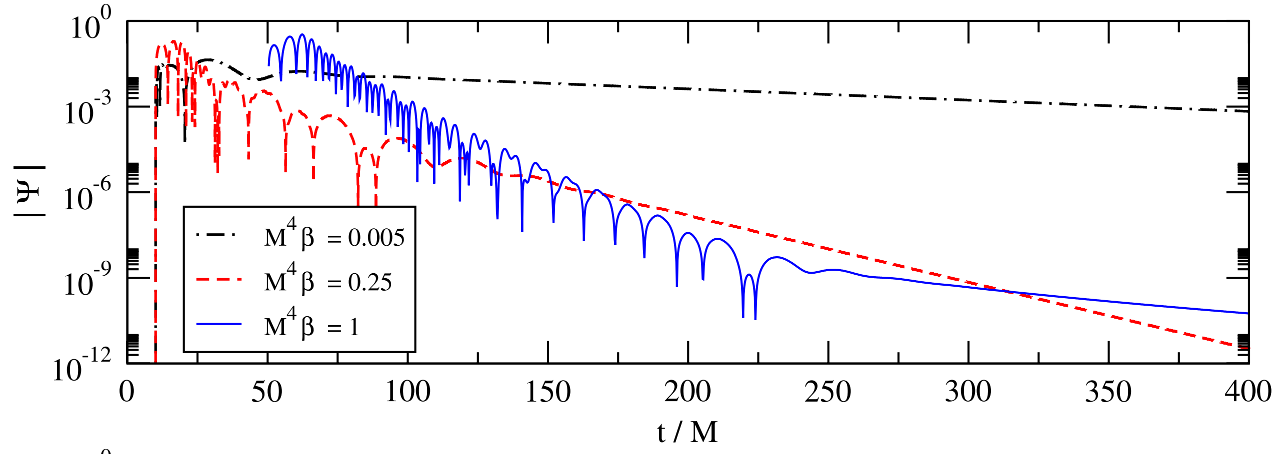

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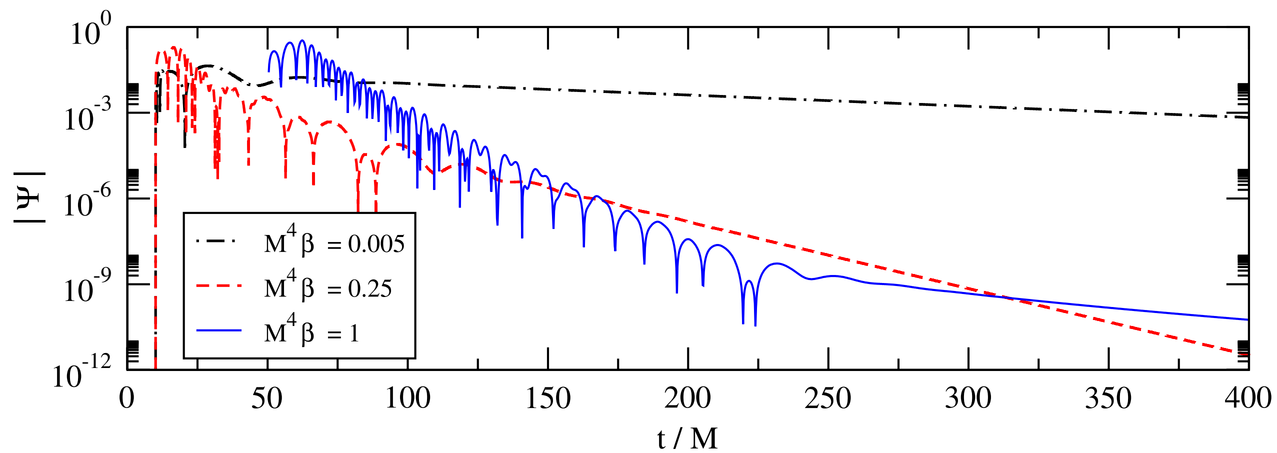
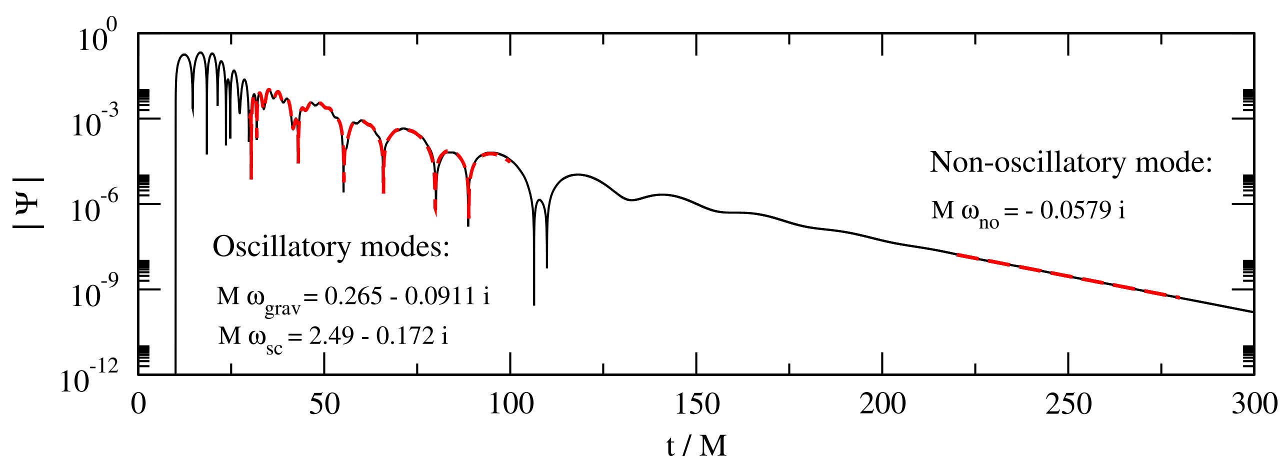


- For intermediate values of the coupling constant, there is oscillatory behaviour, with two families of modes, which we call "*scalar*" modes and "*gravitational*" modes. These names are related to their $\beta \rightarrow \infty$ limit, but both perturbations (Ψ and Θ) oscillate with (different superpositions of) both families.



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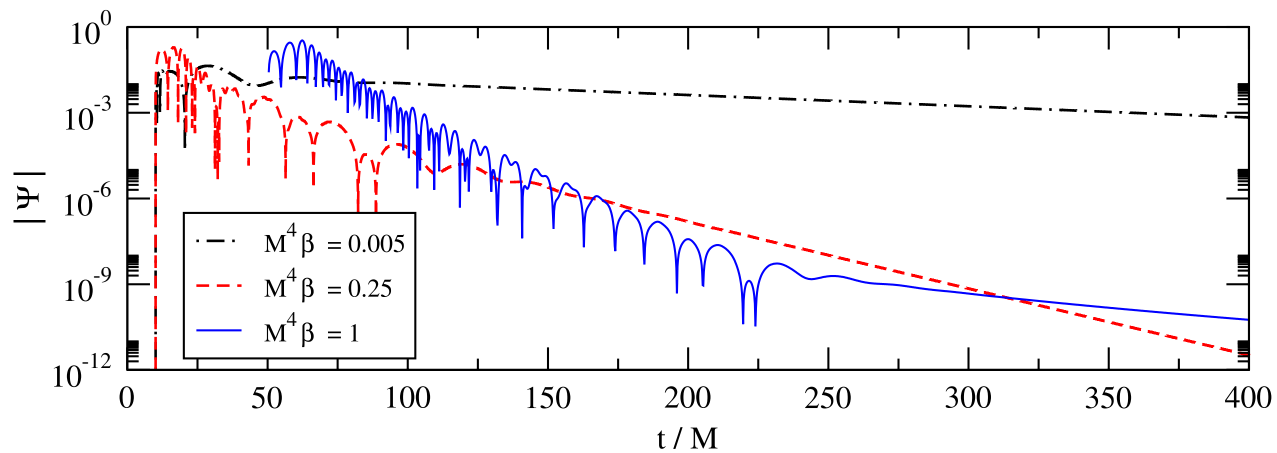

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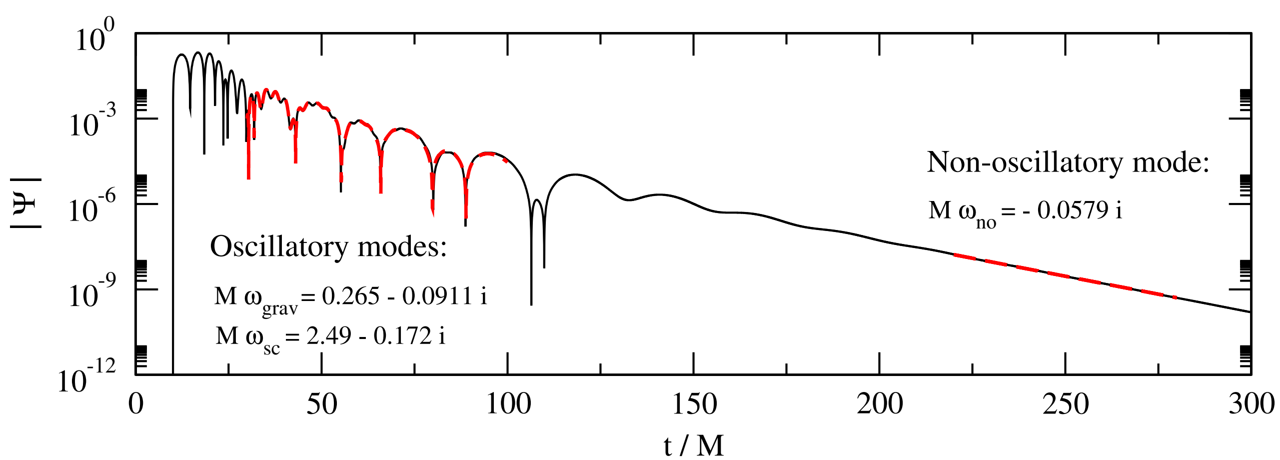

- For $\beta \to \infty$, the "gravitational" and "scalar" branches tend to the QNMs of gravitational and scalar perturbations in GR. ψ oscillates with a combination of the two, Θ oscillates with the scalar mode only.

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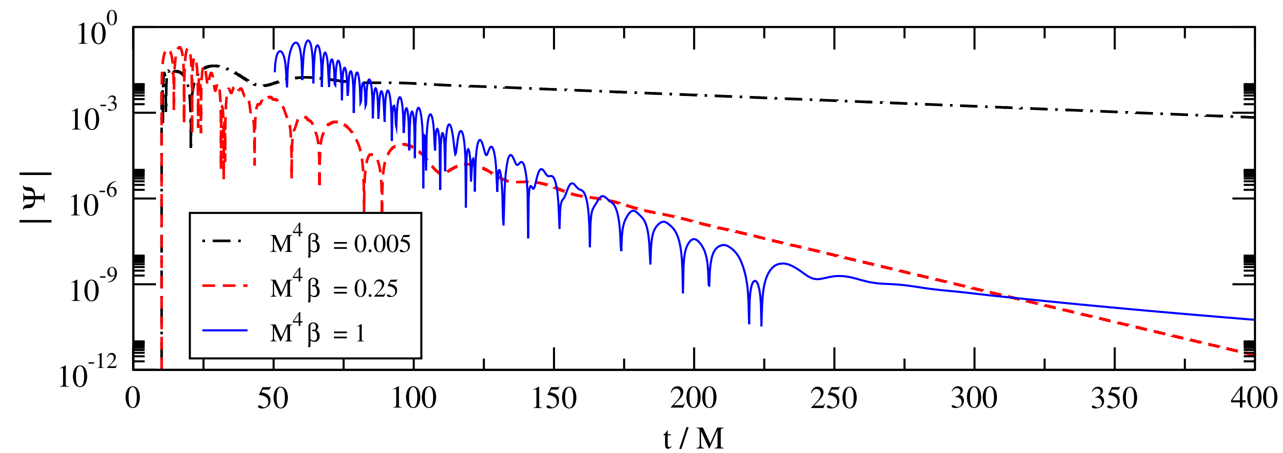

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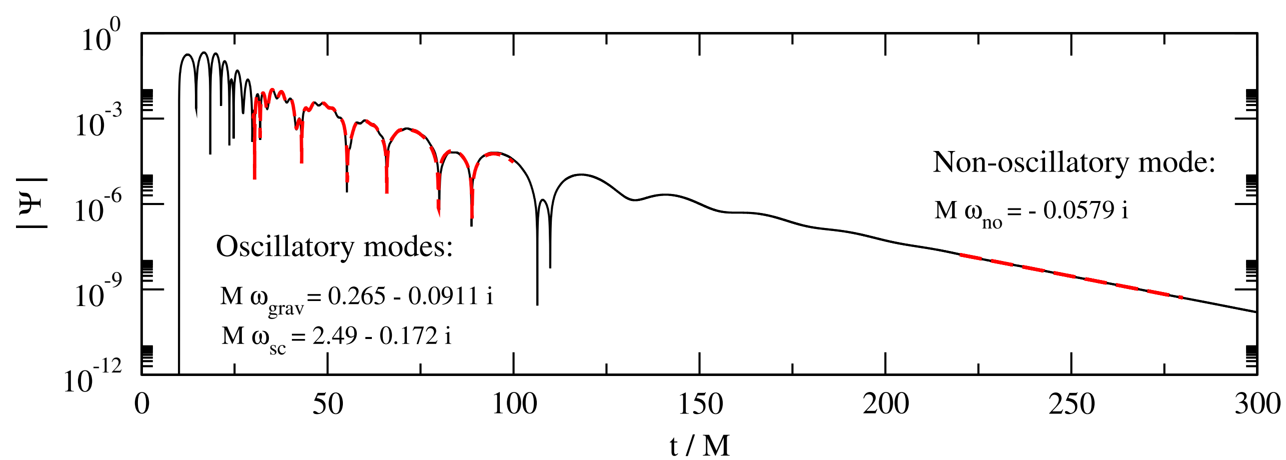

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- At late times, the field decays with a power-law tail, depending neither by β nor by M.

Black hole oscillations

What kind of information one can extract from the observation of black hole QNMs?

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Let us imagine that a ringdown is detected,
and that it is a superposition with the same amplitude of the two lowest lying QNMs.

If GR is the theory of gravity, these modes would probably be the fundamental l=2 and l=3 modes:

$M\omega^{l=2} = 0.37367 - i\, 0.8896$ $\quad$ $M\omega^{l=3} = 0.59944 - i\, 0.09270.$

If, instead, DCS is the theory of gravity, these modes would probably be the fundamental l=2 modes of the gravitational and scalar branches: for a large value of β

$M\omega_{grav} = 0.37367 - i\, 0.8896$ $\quad$ $M\omega_{scal} = 0.4839 - i\, 0.09671.$

A standard Fisher matrix computation shows that with a signal-to-noise ratio greater than 6
(which is expected to be well reached by Advanced LIGO/Virgo detectors)
it would be possible to discriminate between the two scenarios.

$$SNR_{crit} = \frac{\max(\rho\sigma_{f_1}, \rho\sigma_{f_2})}{|f_1 - f_2|} \qquad \text{with} \qquad \rho\sigma_f \sim \frac{0.1}{M}$$

For smaller values of β, the frequency of the l=2 “scalar” mode would be larger,
and a larger signal-to-noise ratio would be required.

Black hole oscillations

Black hole oscillations

This is in a sense an optimistic scenario, but it shows that it could be possible, with gravitational wave observation expected in the next few years, to discriminate between General Relativity and Dynamical Chern-Simons gravity.

This study also gives strong evidence that spherically symmetric black holes are stable (at least, when the background scalar field vanishes - see *H. Motohashi & T. Suyama, '11)*.

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It would be extremely important to generalize these studies to rotating black holes, or to other spacetimes with a nonvanishing background scalar field.

Extreme mass-ratio inspirals

Inspirals of stellar mass compact objects into supermassive black holes at galactic centers.

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(even in its ESA-led, reduced configuration).
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- The signal is emitted when the stellar-mass object ("particle") is close to the horizon of the supermassive black hole, thus encoding the features of the strong-field black hole spacetime and of the strong-field regime of GR.

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- EMRIs are expected to be relatively clean systems, with negligible perturbation from surrounding matter.

In (*P. Pani, V. Cardoso, L. G. '11*) we have studied how EMRI signals in DCS gravity
would differ from the corresponding signals in GR,
and which are the prospects of discriminating between these two theories
(or setting limits on DCS parameters) through detection of gravitational waves from EMRIs.

Extreme mass-ratio inspirals

The stellar mass object can be treated as a perturbation of the supermassive black hole spacetime.
We have neglected rotation, and assumed vanishing background scalar field,
thus the perturbation equations are those discussed above,
describing the scalar field and the gravitational perturbation with axial parity,
together with the equation for gravitational perturbations with polar parity
(which is identical to that of GR, i.e., the Zerilli equation).
All these equations have a source describing a particle (the stellar mass object).

$$\left[\frac{d^2}{dr_*^2}+\omega^2-V_{RW}(r)\right]Q^{\ell m}(r)=T_{RW}(r)\Theta^{\ell m}(r)+S^{\ell m}_{RW}(r)$$

$$\left[\frac{d^2}{dr_*^2}+\omega^2-V_S(r)\right]\Theta^{\ell m}(r)=T_S(r)Q^{\ell m}(r)+S^{\ell m}_S(r)$$

$$\left[\frac{d^2}{dr_*^2}+\omega^2-V_Z(r)\right]Z^{\ell m}(r)=S^{\ell m}_Z(r)$$

$$V_{RW}(r)=f\left(\frac{\ell(\ell+1)}{r^2}-\frac{6M}{r^3}\right)$$

$$T_{RW}(r)=f\,\frac{96i\pi M\omega\alpha}{r^5}$$

$$V_S(r)=f\left(\frac{\ell(\ell+1)}{r^2}\left[1+\frac{576\pi M^2\alpha^2}{r^6\beta}\right]+\frac{2M}{r^3}\right)$$

$$T_S(r)=-f\,\frac{(\ell+2)!}{(\ell-2)!}\,\frac{6Mi\alpha}{r^5\beta\omega}$$

$$V_Z(r)=\frac{f}{r^2\Lambda^2}\left[2\lambda^2\left(\lambda+1+\frac{3M}{r}\right)+\frac{18M^2}{r^2}\left(\lambda+\frac{M}{r}\right)\right]$$

$$((\lambda=(\ell+2)(\ell-1)/2\,;\ \ \Lambda=\lambda+3M/r)$$

S_{RW}, S_Z are standard “particle” sources in GR
(sources of Regge-Wheeler and Zerilli equations).
S_S is the corresponding term
for the scalar field equation.
These terms are proportional to a delta function
on the particle worldline.

Extreme mass-ratio inspirals

We normalize the field such that $\alpha=1$ and express the results in terms of the parameter $\xi=16\pi/\beta$, or in terms of the dimensionless parameter $\zeta=16\pi/\beta M^4$.

Current constraints from astrophysical observations (*N. Yunes & F. Pretorius, '09*) imply

$$\xi \leq 10^{16}\ \mathrm{km}^4 ,$$

which, for supermassive black holes with $M \sim 10^4$-$10^7\ M_{sun}$, corresponds to values for ζ which can be as large as 1-10.

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We have solved the perturbation equations with source using an improved version of the Green function approach, finding the amplitude of the gravitational and scalar perturbations both at infinity and at the horizon. Then, we have computed the gravitational and scalar energy flux (at infinity and at the horizon):

$$\dot{E}^{\pm}_{grav} = \frac{1}{64\pi}\frac{(\ell+2)!}{(\ell-2)!}\sum_{\ell m}\left[(m\omega_K)^2\left|\bar{\mathcal{Z}}^{\ell m}_{\pm}\right|^2 + 4\left|\bar{\mathcal{Q}}^{\ell m}_{\pm}\right|^2\right]$$

$$\dot{E}^{\pm}_{scal} = \sum_{\ell m}(m\omega_K)^2\beta\left|\bar{\Theta}^{\ell m}_{\pm}\right|^2 .$$

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We compare the total energy flux in DCS gravity with that in GR:

$$\dot{E}_{DCS} = \dot{E}^{H}_{grav} + \dot{E}^{\infty}_{grav} + \dot{E}^{H}_{scal} + \dot{E}^{\infty}_{scal}$$

$$\dot{E}_{GR} = +\dot{E}^{\infty}_{grav}(\zeta = 0)$$

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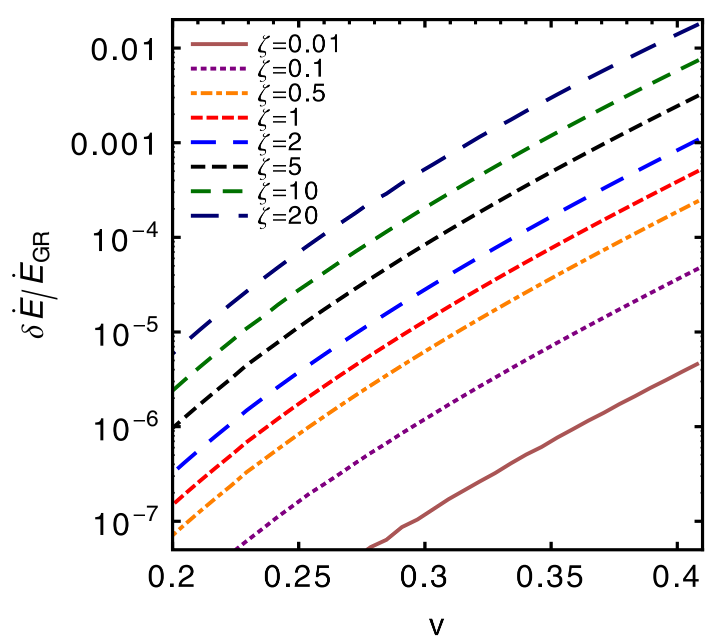
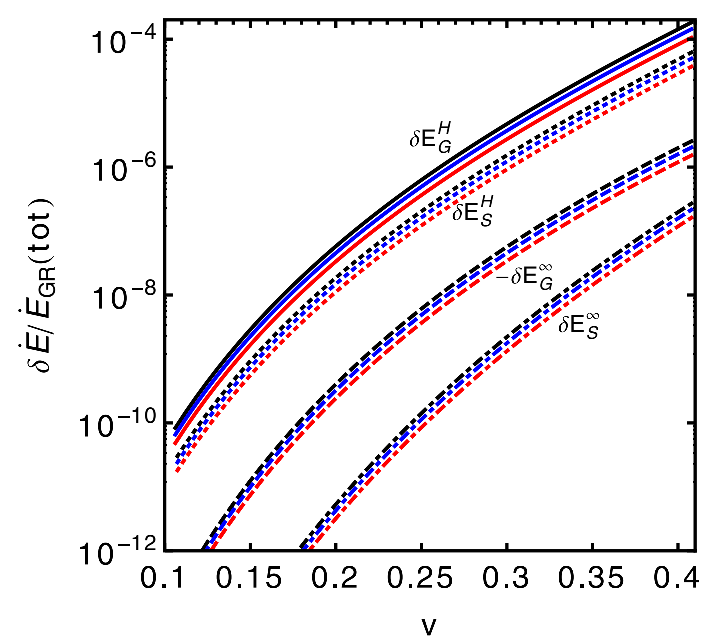
$$\frac{\delta \dot{E}}{\dot{E}_{GR}} = \frac{\dot{E}_{DCS} - \dot{E}_{GR}}{\dot{E}_{GR}}$$

- In DCS gravity, the total energy flux is larger than in GR (as expected: extra dissipation channel).
- The modification is negligible far from the ISCO.
- The main contribution comes from the horizon radiation (which is, instead, almost negligible in GR); the correction is positive for the horizon radiation, and negative for the radiation at infinity.
- Only axial radiation is affected. Close to the ISCO, the axial flux can be as large as twice than in GR.
- However, the axial flux is a subleading contribution, so that the total flux increases at most by a few percent.





$$v = (M\omega_K)^{1/3} = p^{-1/2}$$

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Although the difference in the energy emission from EMRIs between DCS gravity and GR is small, stellar mass objects can have up to ~10^5 cycles (many of which are near the ISCO) while emitting radiation in the bandwidth of LISA or of similar detectors. Small deviations accumulate in the phase, and can yield detectable effects.

The number of gravitational wave cycles accumulated can be computed as:

$$\mathcal{N} = \int_{f_i}^{f_f} \frac{f}{\dot{f}} df$$

with

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We compute $\dot{f}$ assuming the *adiabatic approximation*: the particle is in nearly geodesic motion, thus we compute, at each time, the energy flux assuming a geodesic orbit; then, we update the orbital constant of motion E_{orb} using the flux balance equation:

$$\dot{E}_{orb} + \dot{E}_{grav} + \dot{E}_{scal} = 0$$

Extreme mass-ratio inspirals

Although the difference in the energy emission from EMRIs between DCS gravity and GR is small, stellar mass objects can have up to ~10^5 cycles (many of which are near the ISCO) while emitting radiation in the bandwidth of LISA or of similar detectors. Small deviations accumulate in the phase, and can yield detectable effects.

The number of gravitational wave cycles accumulated can be computed as:

$$\mathcal{N} = \int_{f_i}^{f_f} \frac{f}{\dot{f}} df \qquad \text{with} \qquad \begin{array}{rcl} f_i & = & \max(f_{low}, f_{1\,yr}) \\ f_f & = & \min(f_{ISCO}, f_{up}) \end{array}$$

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This approximation neglects the so-called "conservative part of the self-force", but this contribution (for non-spinning black holes) should be less than a cycle in the entire process (few cycles at most for spinning black holes). It has to be taken into account in the data analysis of the process, but it can be neglected in assessing the relevance of DCS corrections to the EMRI signal.

Extreme mass-ratio inspirals

Since $$\dot{f} = -\frac{3}{2}\frac{f}{r}\frac{dr}{dE_{orb}}\dot{E}_{orb}$$ we have that $$\frac{\delta\dot{f}}{\dot{f}} = \frac{\delta\dot{E}}{\dot{E}_{GR}}$$

and we can compute the correction in the number of cycles

$$\frac{\delta\mathcal{N}}{\mathcal{N}} = -\frac{\int_{f_i}^{f_f}\frac{\dot{f}}{f}\frac{\delta\dot{E}}{\dot{E}_{GR}}df}{\int_{f_i}^{f_f}\frac{\dot{f}}{f}df}$$

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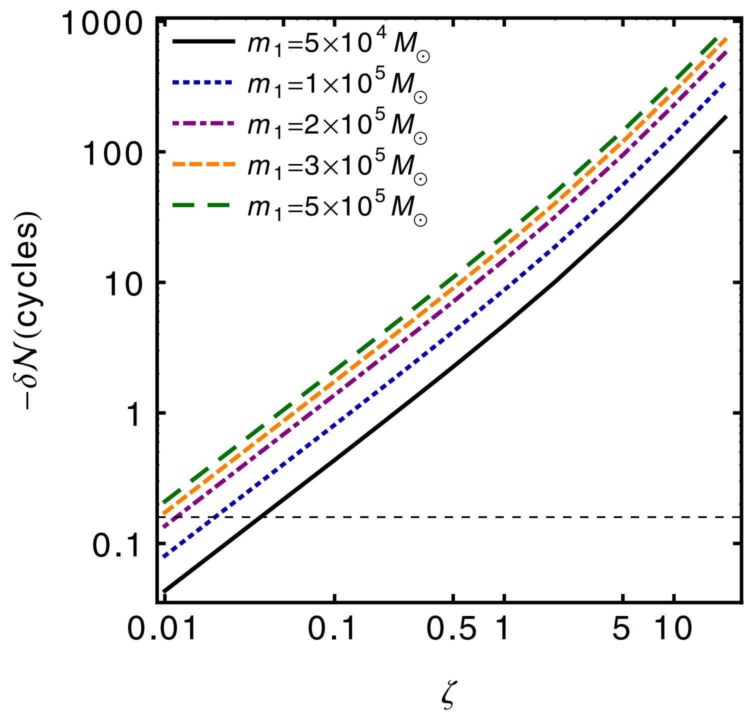

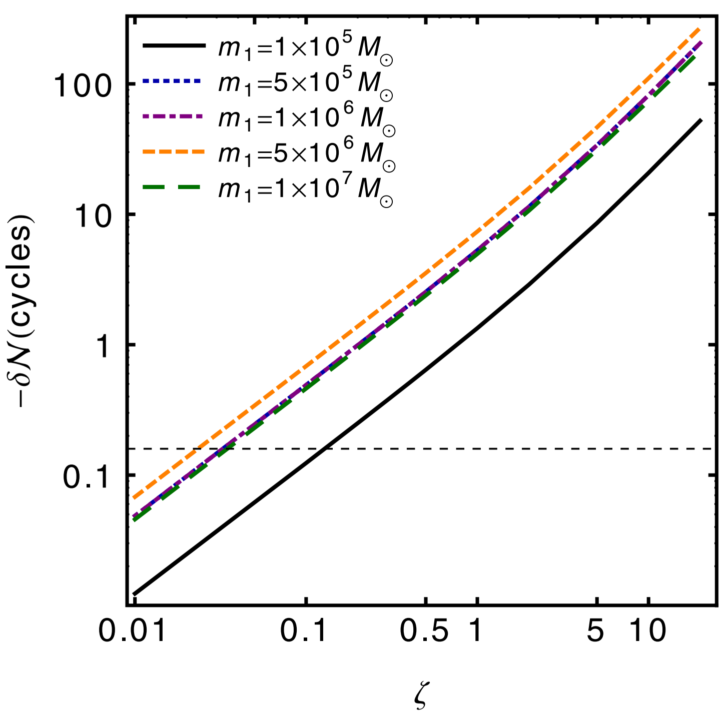

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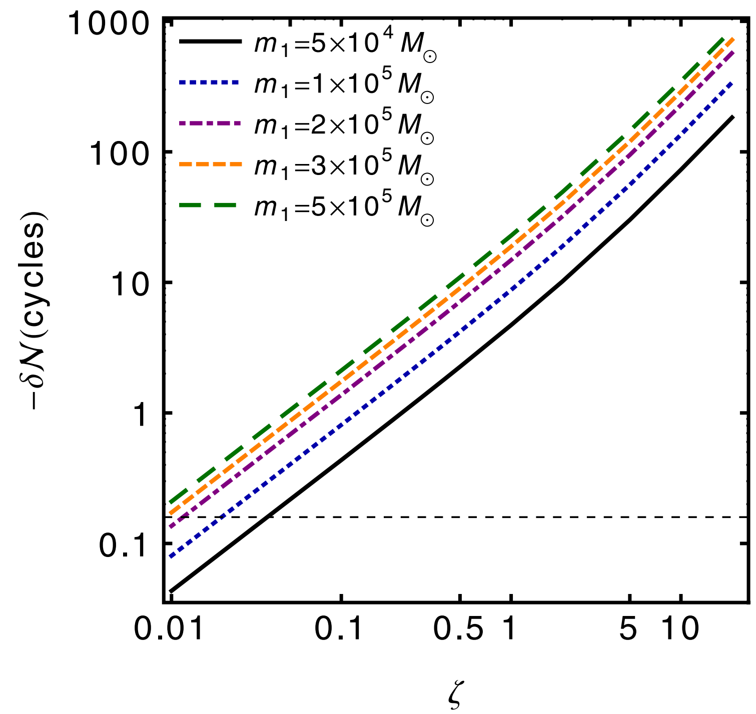

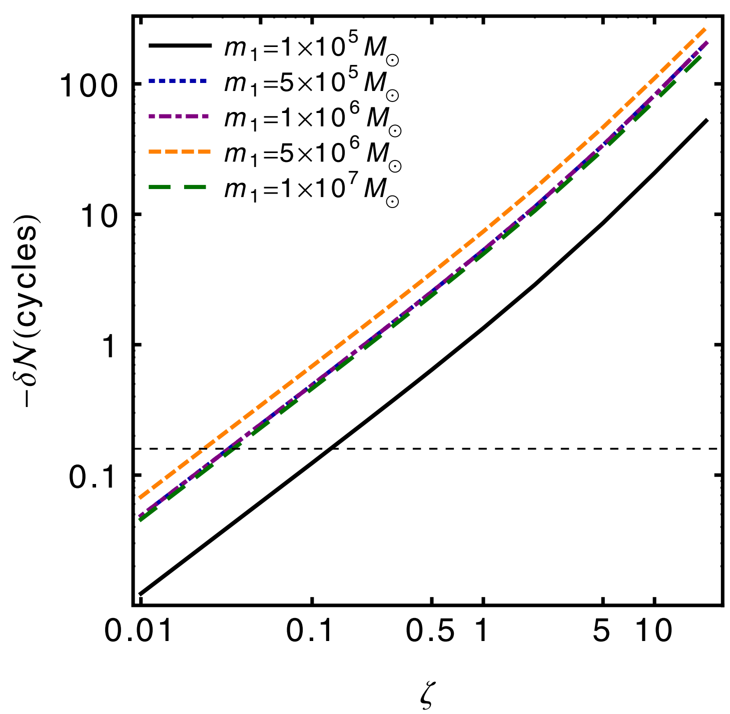

Corrections of GR are generally considered significant if they exceed 1 rad over the observation time.

Corrections from DCS gravity could be of several cycles, and this could be detected very well by LISA or a similar space-based gravitational detector!

Conclusions

- Detection of gravitational waves will enable us to study the strong field limit of gravity, where possible deviations from GR may show up.

- DCS gravity is a very promising theory, since it has a characteristic observational signature, and it is grounded on (possible) more fundamental theories.

- Studies of processes involving black holes show that upcoming gravitational experiments will be able to discriminate GR from DCS gravity (in a large part of its parameter space).

- Quasi-normal modes of black holes, which should be detected in the last phase of black hole binary coalescence, have the imprint of the strong field gravitational theory, and thus are a promising tool to discriminate between theories of gravity. In DCS gravity, a detection with a signal-to-noise ratio as low 6 could be sufficient to this aim.

- Extreme mass-ratio inspirals, which should be detected by LISA or a similar space-based detector, are very sensitive to DCS gravity, since the stellar mass object spends many cycles in the detector bandwidth, and the small effect of the DCS coupling "piles up", giving rise to effects which could be measurable.

- It would be very important to extend these studies to (fastly) rotating black holes and to other spacetimes with non-vanishing background scalar fields.