

Astrophysical signatures of theories beyond GR

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<http://blackholes.ist.utl.pt>



Outline

Testing alternative theories of gravity

- Motivation
- Modified gravity
- Strategies
- Neutron stars as strong-curvature probes

Part I

Coupling to matter

- Non-relativistic phenomenology
- Eddington-inspired gravity

Part II

Coupling to scalars

- Quadratic gravity
- Spontaneous scalarization

GR is NOT well tested in the strong-field regime!

$$\frac{\Phi_{\text{Newton}}}{c^2} = \frac{GM_{\odot}}{c^2 1\text{AU}} \sim 10^{-8}$$

Solar system tests

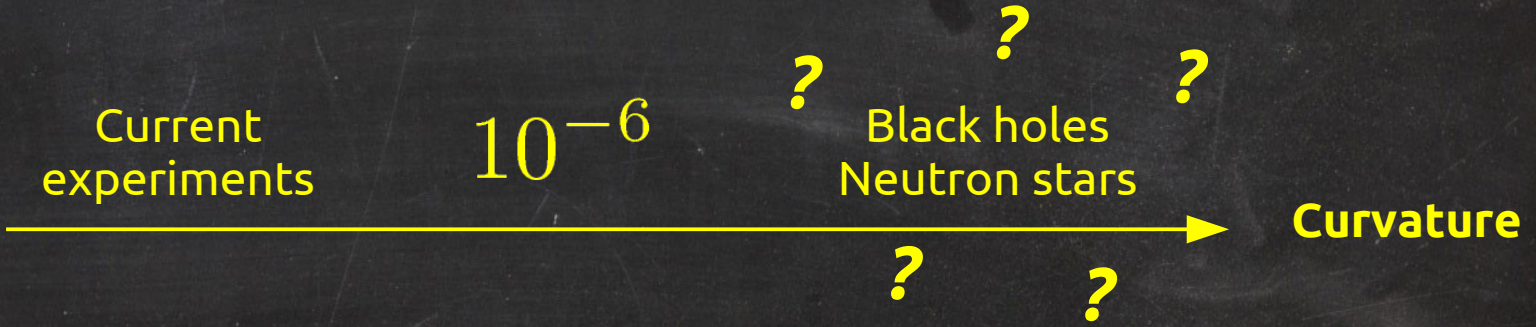
$$\frac{\Phi_{\text{Newton}}}{c^2} \sim \frac{GM_{\odot}}{c^2 r_{\text{periastron}}^{\text{Hulse-Taylor}}} \sim 10^{-6}$$

Millisecond binary pulsar

$$\frac{\Phi_{\text{Newton}}}{c^2} \sim \frac{GM_{\text{BH}}}{c^2 \mathcal{O}(r_{\text{H}})} \sim 10^{-1} - 1$$

Binary black holes

Gravity:

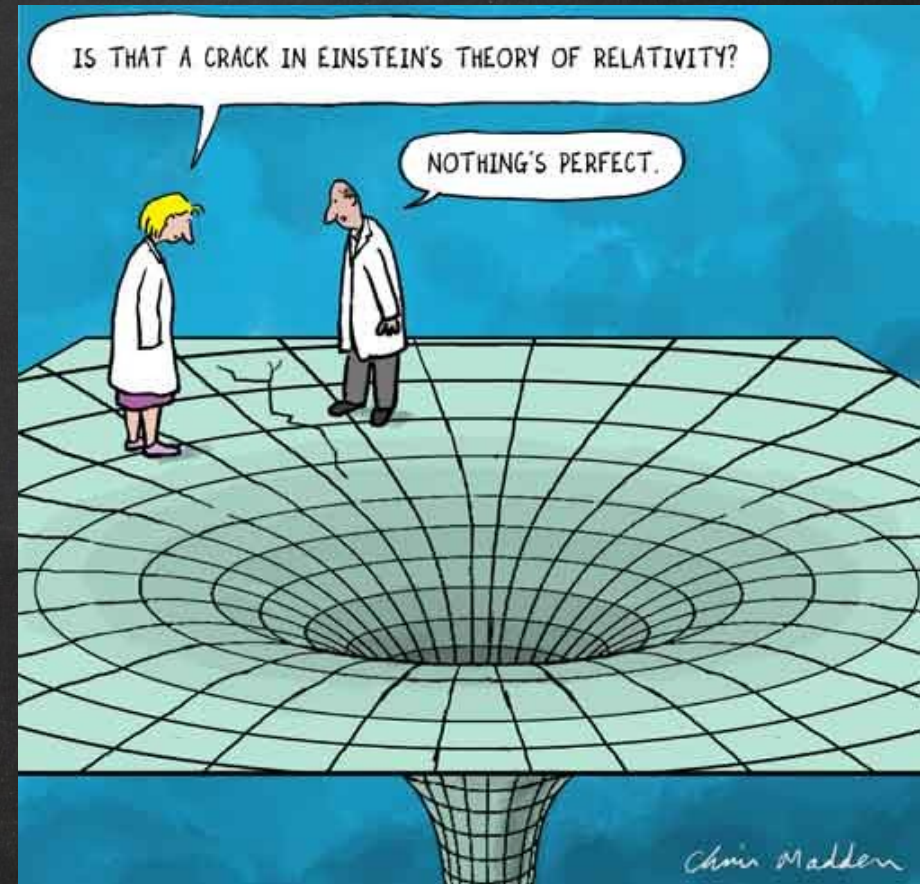


Particle physics:



Modifying GR: motivation

- **Phenomenologically:** elusive sectors of Einstein theory
 - Strong-field regime
 - **Coupling with matter**
- **Experimentally:**
 - Dark matter and dark energy
 - Gravitational waves
- **Theoretically:** many issues
 - **Singularities ?**
 - Gravity and QFT
 - Hints of UV completion



Modifying GR: strategies

- Imagination beats reality: **plethora** of alternative theories
- Unified approach **VS** case-by-case analysis:

1) Select a specific theory/effect and look for “**smoking guns**”

- **Floating orbits** in scalar-tensor theories [Cardoso's previous talk]
- **GWs birefringence** in parity-violating theories
- **Singularity avoidance** in cosmology and stellar collapse
- **Spontaneous scalarization** in scalar-tensor neutron stars

2) Parametrize a general action / field equations

- Quadratic couplings
- Scalar-tensor theories / $f(R)$ theories
- Coupling with matter

3) Look for accumulated effects

- **Two-body inspiral** [Gualtieri's next talk]

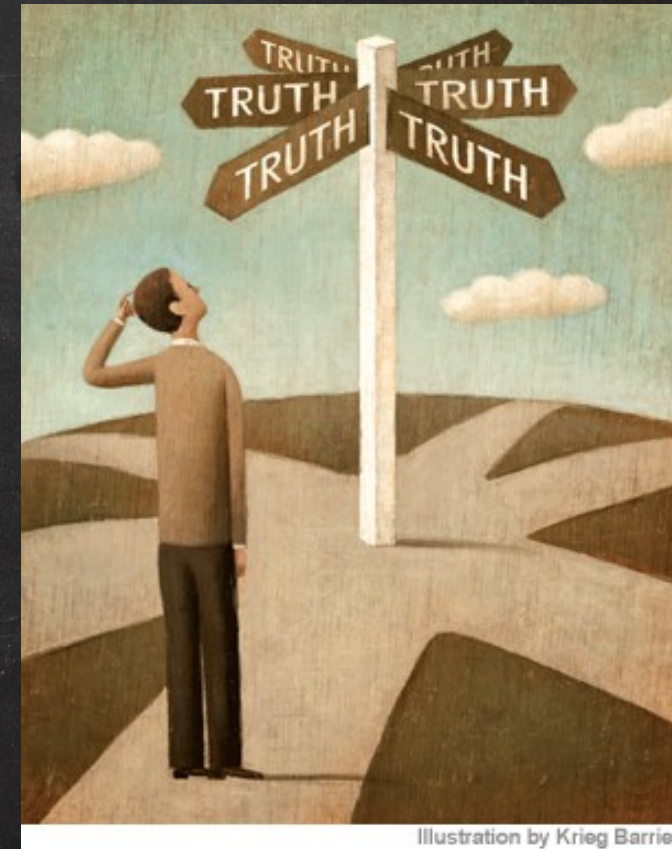


Illustration by Krieg Barrie

Part I

Coupling to matter

Based on:

P. Pani, V. Cardoso, T. Delsate

J. Casanellas, P. Pani, I. Lopes, V. Cardoso

T. Delsate and J. Steinhoff

Phys. Rev. Lett. 107 031101 (2011) & Work in Progress

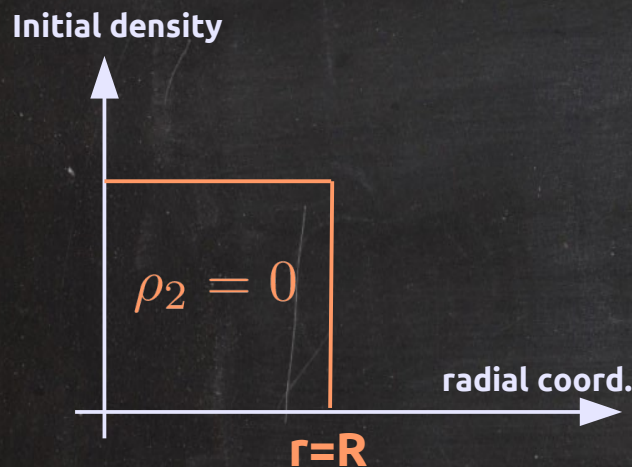
ApJ (in press) astro-ph.SR/1109.0249

Work in progress

Singularities in GR

[Joshi and Malafarina 2000-2011,
Joshi, Dadhich, Maartens 2002]

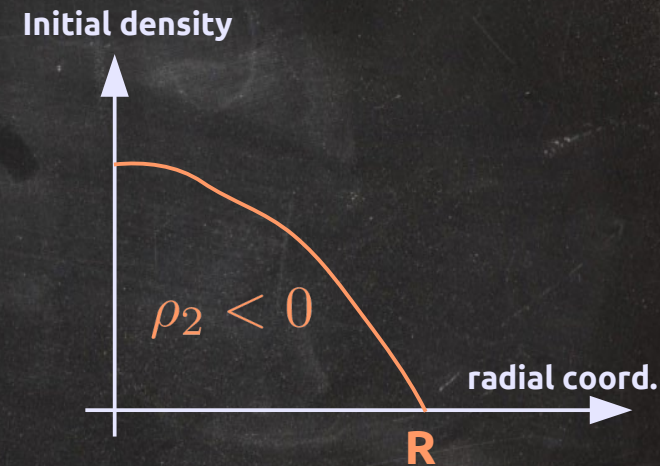
- Singularity are common in GR (Big Bang, black holes...)
 - Can be produced in dynamical processes (e.g. stellar collapse)
 - **Cosmic Censorship**
 - Naked singularities in **realistic scenarios** ?



$$\rho(r) \sim \rho_c + \rho_2 r^2 + \dots$$

$$t_{\text{ah}} \approx t_s - \rho_c r^3 - \rho_2 r^2$$

Time of formation of an apparent horizon



Density gradient produces a **shear** that postpones the **apparent horizon** formation

Locally naked singularities may be **globally naked!** [Joshi, Dwivedi, 2002]

In the whole process it is **crucial how gravity is coupled to matter**

Coupling to matter beyond GR

- In vacuum $\rightarrow R_{\mu\nu} = 0 \quad \nabla_{\mu} T^{\mu\nu} = 0$
- Matter sector is **extremely difficult to probe** \rightarrow caution (e.g. **extra dims**)
- Crucial to describe stars and cosmology

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi T_{\mu\nu} + \underbrace{S_{\mu\nu}[g, T, \partial g, \partial T]}$$

Can become dominant
at high density or high gradients

- **No extra** fundamental fields
- **Vanishing in vacuum** [Bertolami et al., 2007]
- Geodesic equation, **minimal coupling** $\rightarrow \nabla_{\mu} T^{\mu\nu} = 0$
- Quadratic in T? **Matter derivatives?** Action principle?

Born-Infeld-Eddington gravity is a prototype of this kind of corrections

Have we tested Newtonian gravity enough?

- Parametrized Post-Poissonian approach:

Most general Poisson eq. which is covariant, perturbative to 2nd derivatives and reduces to Laplace eq. in vacuum:

$$\underbrace{\nabla^2 \Phi = 4\pi G \rho}_{\text{standard}} + \underbrace{\frac{\kappa_g}{4} \nabla^2 \rho + \alpha_g \epsilon^{ij} \nabla_i \Phi \nabla_j \rho + \eta \rho^2 + \gamma \nabla \rho \cdot \nabla \rho + \epsilon_1 \nabla \Phi \cdot \nabla \rho + \epsilon_2 \Phi \nabla^2 \rho + \epsilon_3 \rho \nabla^2 \Phi + \dots}_{\text{Linear corrections}} + \underbrace{\dots}_{\text{Quadratic corrections}}$$

- Tests of the **equivalence principle** constrain many terms
- Linear corrections are compatible with all observations so far

$$\nabla^2 \Phi = 4\pi G \rho + \frac{\kappa}{4} \nabla^2 \rho$$

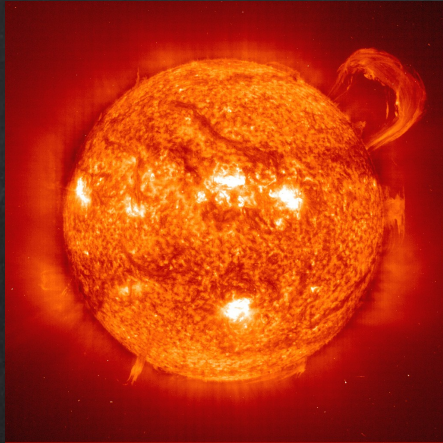
- Precise measurements of **solar neutrinos** and **helioseismology**

[Casanellas, Pani, Lopes, Cardoso, ApJ (in press) astro-ph.SR/1109.0249]

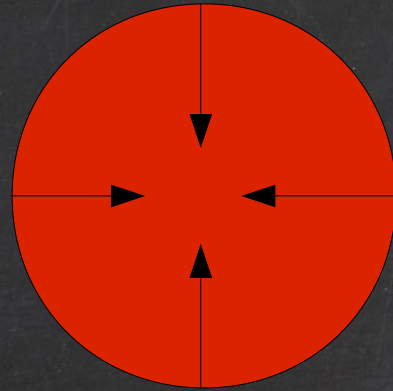
Collapse in modified Newtonian gravity

[Pani, Cardoso, Delsate, PRL 107, 2011]

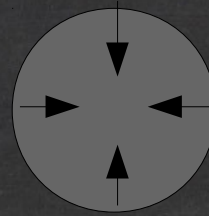
$$\nabla^2\Phi = 4\pi G\rho$$



Massive star



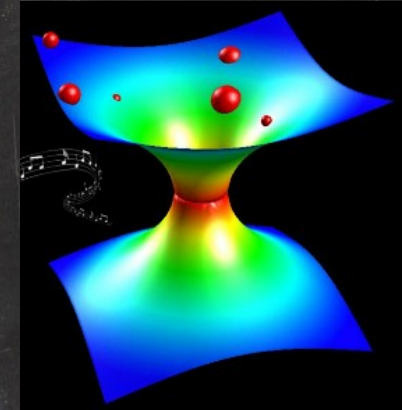
End of nuclear reactions



Collapse



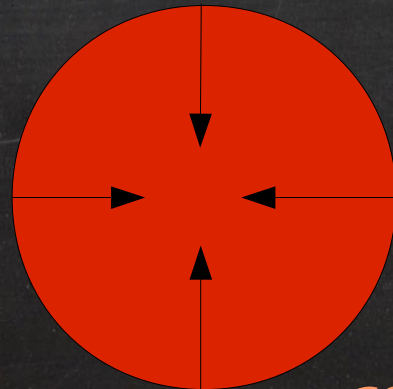
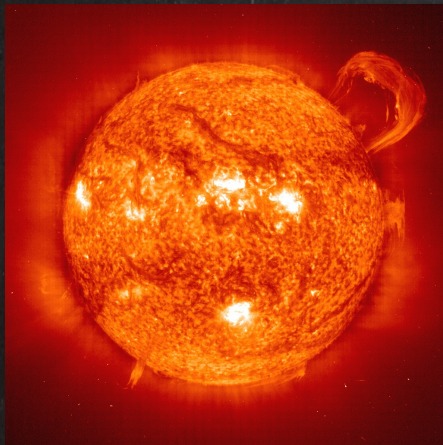
Singularity



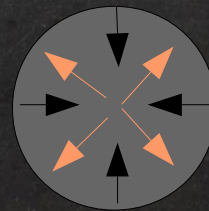
Black hole (?)

time

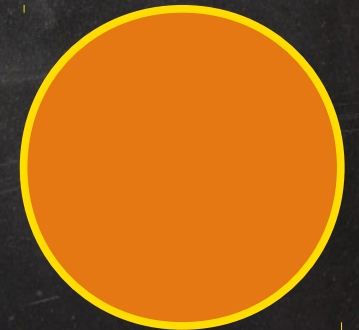
$$\nabla^2\Phi = 4\pi G\rho + \frac{\kappa}{4}\nabla^2\rho$$



New repulsive gravity



NO Singularity!



Regular "star"

Collapse in modified Newtonian gravity

[Pani, Cardoso, Delsate, PRL 107, 2011]

- In GR, the Newtonian collapse of non-interacting particles reproduces the **Oppenheimer-Snyder collapse quantitatively** [Florides 1977]

$$\frac{\partial u(t, r)}{\partial t} + u(t, r) \frac{\partial u(t, r)}{\partial r} = - \frac{GM(t, r)}{r^2} - \frac{\kappa}{4} \frac{\partial \rho(t, r)}{\partial r}$$

Modified Euler equation

$$\frac{\partial \rho(t, r)}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} [\rho(t, r) r^2 u(t, r)] \quad \frac{\partial M(t, r)}{\partial r} = 4\pi r^2 \rho(t, r)$$

Continuity equation

Mass function

- In practice:
 - **Lagrangian formulation** (less time consuming, “comoving coords.”)
 - **Artificial viscosity** (to smear out fluid shocks)

Stellar collapse: *1+1 evolution of non-interacting particles*



Stars in modified Newtonian gravity

$$\nabla^2 \Phi = 4\pi G \rho + \frac{\kappa}{4} \nabla^2 \rho$$

- **Modified hydrostatic equilibrium:**

$$\frac{dP}{dr} = -\frac{Gm(r)\rho}{r^2} - \frac{\kappa}{4} \rho \rho'$$

- **Admits “dark matter stars” (P=0 and $\kappa > 0$)**

$$\rho(r) = \rho_c \frac{\sin \varpi r}{\varpi r} \quad \varpi = 4 \sqrt{\frac{\pi G}{\kappa}}$$

- **Linearly stable:**

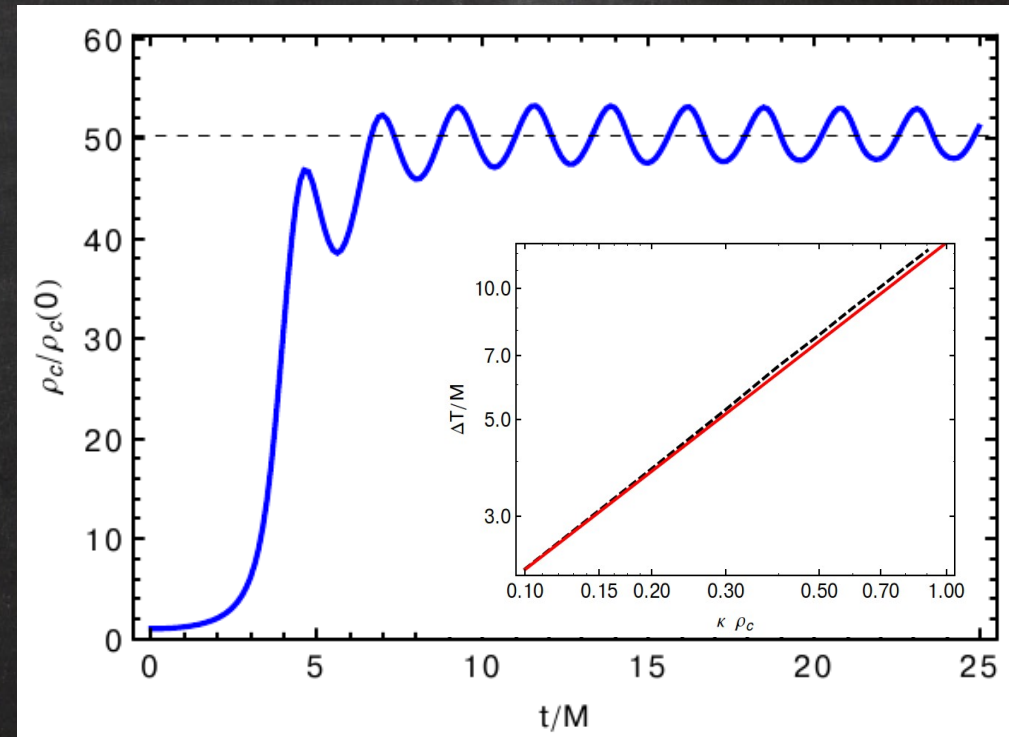
$$\frac{\Delta T}{M} \approx \frac{\pi^{5/4}}{4.4} \left(\frac{\kappa}{M^2} \right)^{3/4}$$

- **No dissipation in the collapse**

Equivalent to standard gravity

with a polytropic EOS:

$$P(\rho) = \frac{\kappa}{8} \rho^2$$



Modified Chandrasekhar model

- Ultra-relativistic matter $P=K \rho^{4/3}$

- Energy:

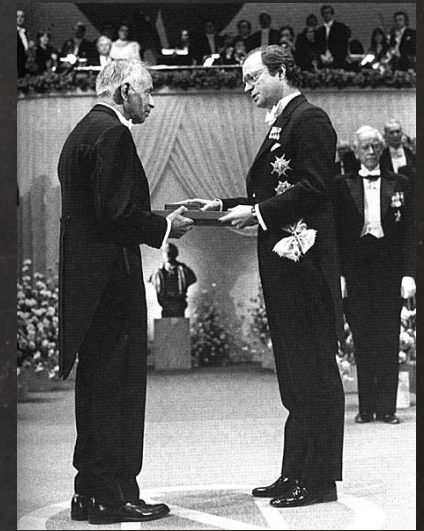
$$E = E_F + E_G$$

$$\approx \underbrace{\frac{\hbar c N^{1/3}}{R}}_{\text{Fermi energy}} - \underbrace{\frac{GNm_b^2}{R} + \frac{3\kappa N m_b^2}{16\pi R^3}}_{\text{Gravitational energy per fermion}}$$

Fermi energy

Gravitational energy per fermion

$$\nabla^2 \Phi = 4\pi G \rho + \frac{\kappa}{4} \nabla^2 \rho$$

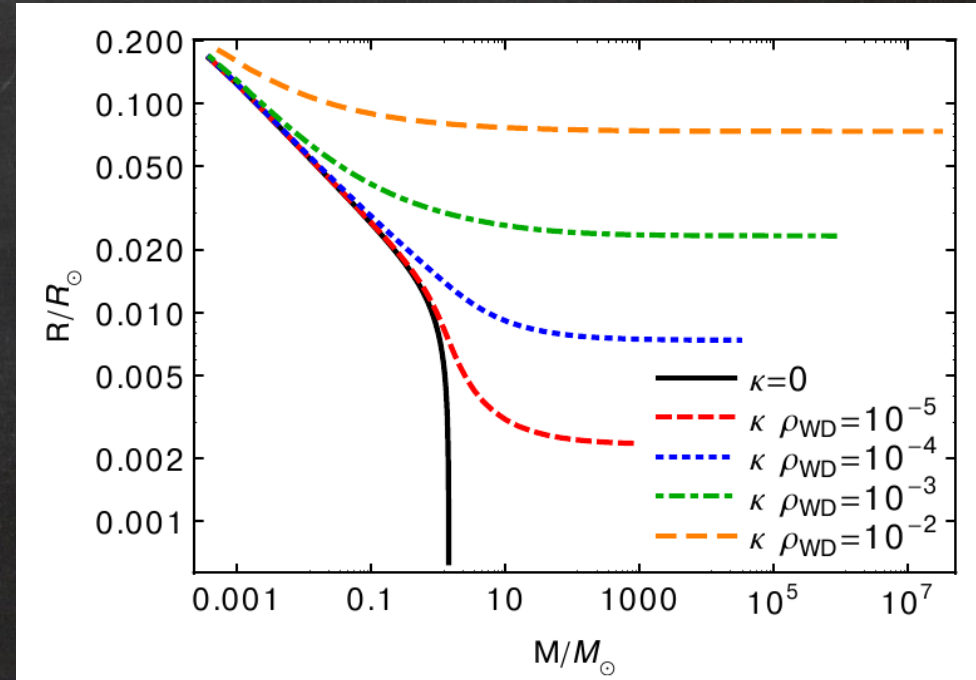


- If $\kappa=0$ (Chandra's result)

$$N_{\max} \approx \left(\frac{c\hbar}{Gm_b^2} \right)^{3/2} \Rightarrow M_{\max} \approx 1.4M_{\odot}$$

- If $\kappa>0$ (Eddington-inspired gravity)

$$R_{\min} \approx \frac{3}{4} \sqrt{\frac{\kappa}{\pi G}} \sqrt{1 - \frac{N_{\max}^{2/3}}{N^{2/3}}}$$

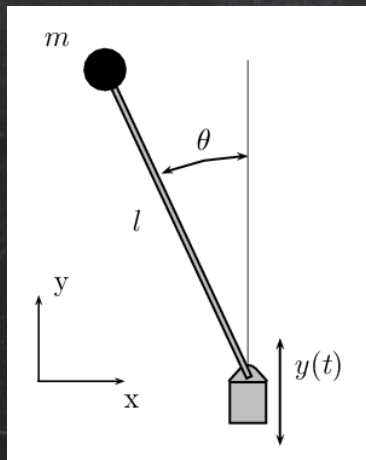


Open issues

- Collapse when $M > 1.4 M_{\text{sun}}$?

BHs are **vacuum solutions**, but can be formed in dynamical scenarios?

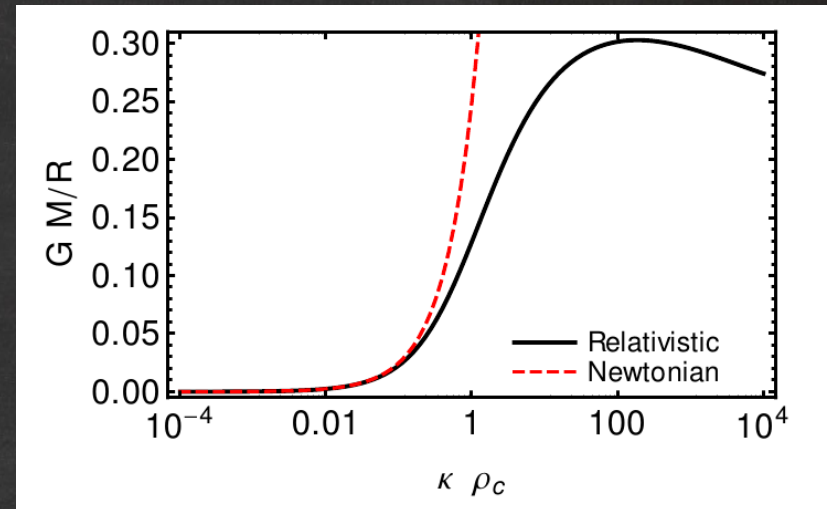
~ inverted pendulum:



- Are **BHs stable**?

Pressureless stars:

Maximum mass



To answer these questions, we need a **fully relativistic theory** which reduces to

$$\nabla^2 \Phi = 4\pi G \rho + \frac{\kappa}{4} \nabla^2 \rho$$

in the non-relativistic limit

Born-Infeld-Eddington (BEI) gravity

[Banados, Ferreira 2010]

$$S_{BEI}[g, \Gamma, \Psi] = \frac{2}{\kappa} \int d^4x \left[\sqrt{|g_{ab} + \kappa R_{ab}(\Gamma)|} - \lambda \sqrt{g} \right] + S_m[g, \Psi_m]$$

$$\Lambda = \frac{\lambda - 1}{\kappa}$$

Field equations:

$$q_{\mu\nu} = g_{\mu\nu} + \kappa R_{\mu\nu}^{(q)}$$

$$\sqrt{-q} q^{\mu\nu} = \lambda \sqrt{-g} g^{\mu\nu} - \kappa \sqrt{-g} T^{\mu\nu}$$

- q is the **affine metric**
- Equivalent to GR in **vacuum**

- Deviations only occur when coupled to matter

- Small κ limit $\rightarrow R_{\mu\nu}^{(q)} \approx \Lambda g_{\mu\nu} + T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu} + \underbrace{\kappa \left[S_{\mu\nu} - \frac{1}{4} S g_{\mu\nu} \right]}_{\text{Quadratic in the matter fields}} + \dots$

Derivatives corrections

$$S_{\mu\nu} = T_{\mu}^{\alpha} T_{\alpha\nu} - \frac{1}{2} T T_{\mu\nu}$$

- Non-relativistic limit $\rightarrow \nabla^2 \Phi = 4\pi G \rho + \frac{\kappa}{4} \nabla^2 \rho$

- Nonetheless, matter is minimally coupled to gravity $\rightarrow \nabla_{\mu}^{(g)} T^{\mu\nu} = 0$

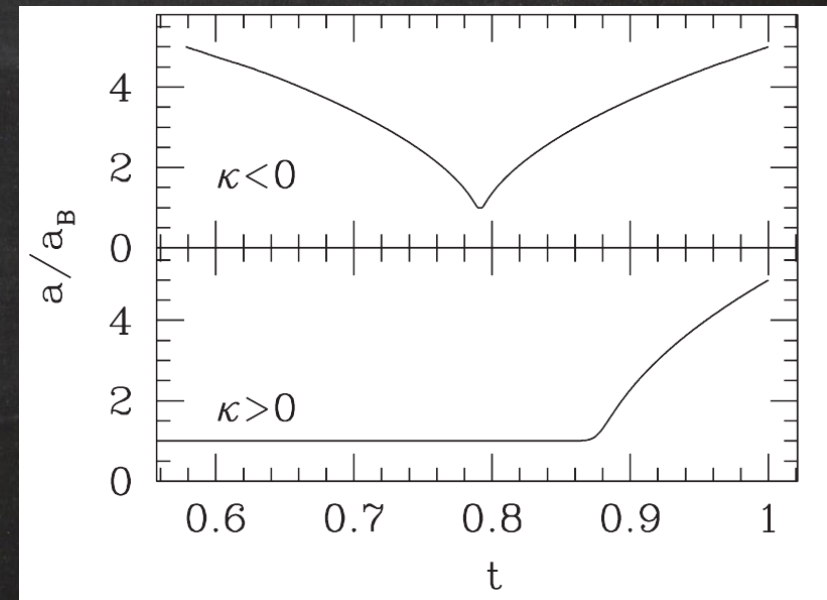
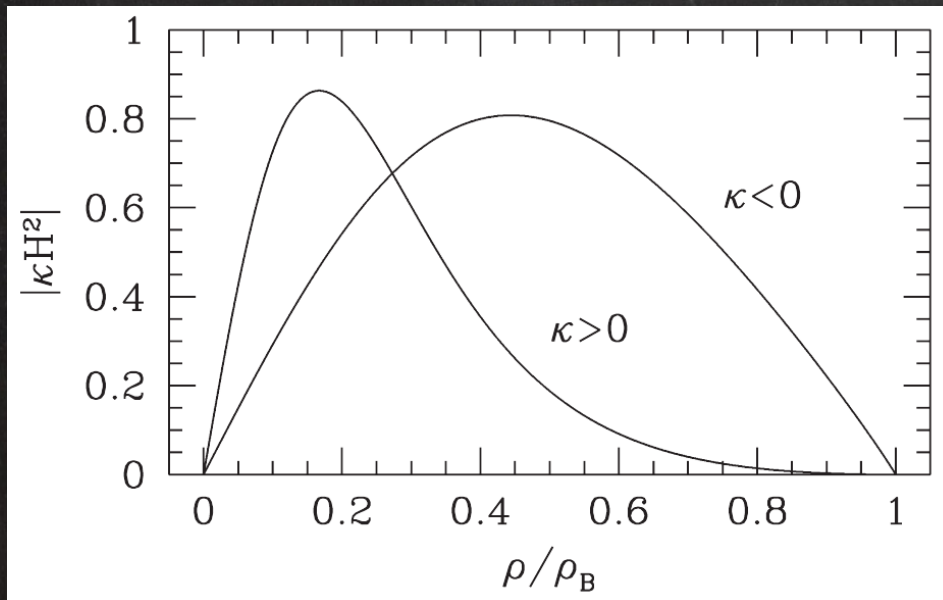
Cosmology in BLE gravity

[Banados, Ferreira PRL 105, 2010]

- Same energy conservation
- Different Friedmann equation
- At early times:

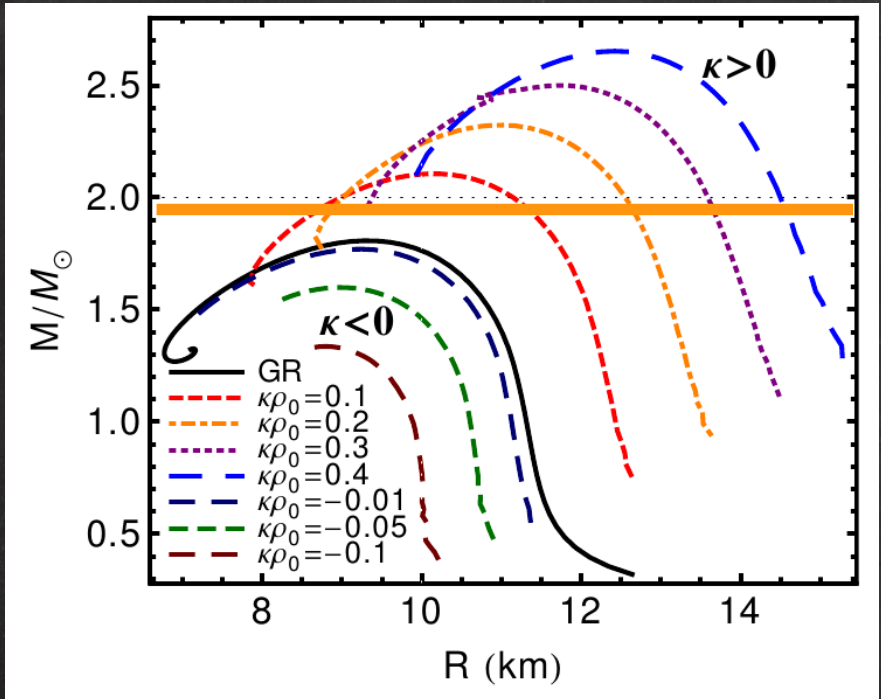
$$3H^2(\rho) = \frac{1}{\kappa} \left[\kappa\rho - 1 + \frac{1}{3\sqrt{3}} \sqrt{(1 + \kappa\rho)(3 - \kappa\rho)^3} \right]$$

No Big Bang!

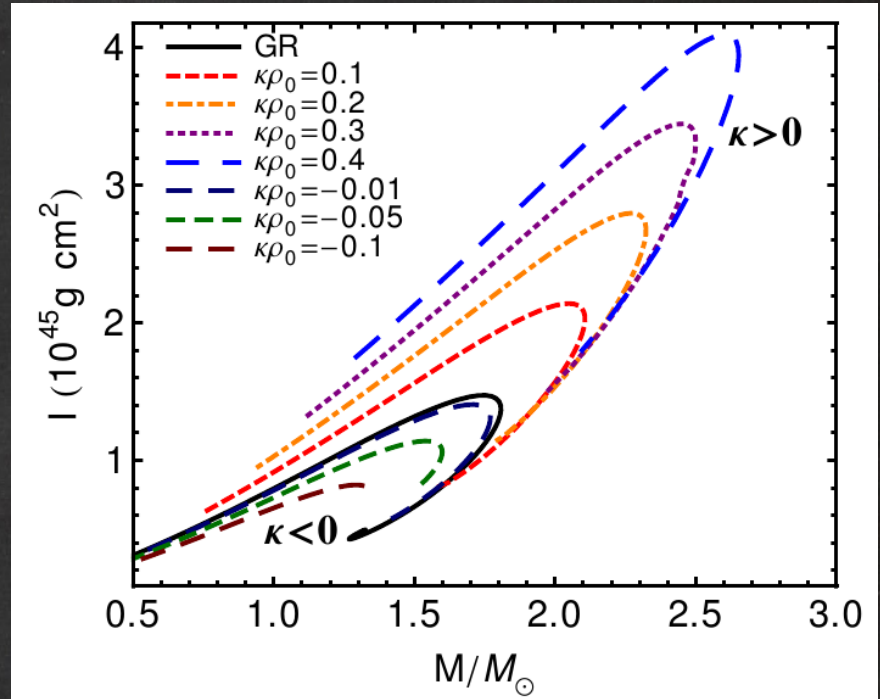


Standard neutron stars in BLE gravity:

Mass-radius relation



Moment of inertia



[Pani, Cardoso, Delsate, PRL 107, 2011]

Positive κ contribute to **enhance** the relativistic effects

- **Degeneracy** between different equations of state!
- Can explain recent observations **without assuming exotic EOS**

- **No compact objects** when: $\left\{ \begin{array}{ll} P_c \kappa < 1 & \kappa > 0 \\ \rho_c |\kappa| < 1 & \kappa < 0 \end{array} \right.$

Conclusion

- Did we test the matter-gravity sector of GR enough??
- Currently hidden sectors of GR will be tested in the near-future
- Singularities in GR can be avoided modifying the coupling to matter
- Rich and viable phenomenology even in the non-relativistic limit
- Born-Infled-Eddington gravity has a very appealing features
 - Non-singular cosmology
 - Stable dark matter stars
 - Modified non-relativistic limit
 - Non-singular Newtonian collapse
 - Higher maximum mass in neutron stars
 - Constraints from solar physics
- Important to understand the relativistic collapse
- Non-linear, strong-field effects are “smoking guns” for next experiments

Boas férias e até o próximo ano!

BHs V??



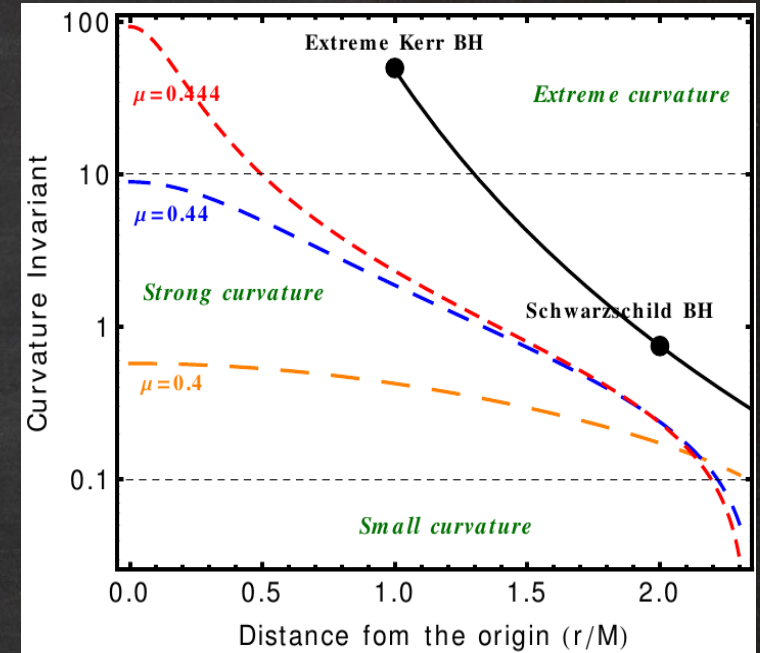
Backup slides

*"Nothing is More Necessary
than the Unnecessary"*



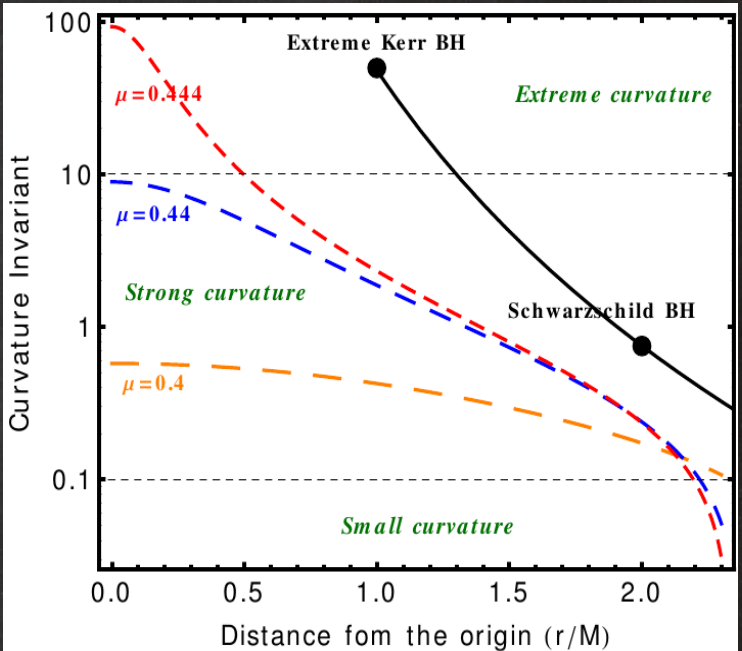
Compact stars as strong-field probes

- **Intimately related:** collapse, Chandra, etc..
- Even **stronger curvatures than BHs**
- New physics even at **non-relativistic level**
- Neutron stars (NSs) are **common objects**
- **More accessible** than black holes (BHs)

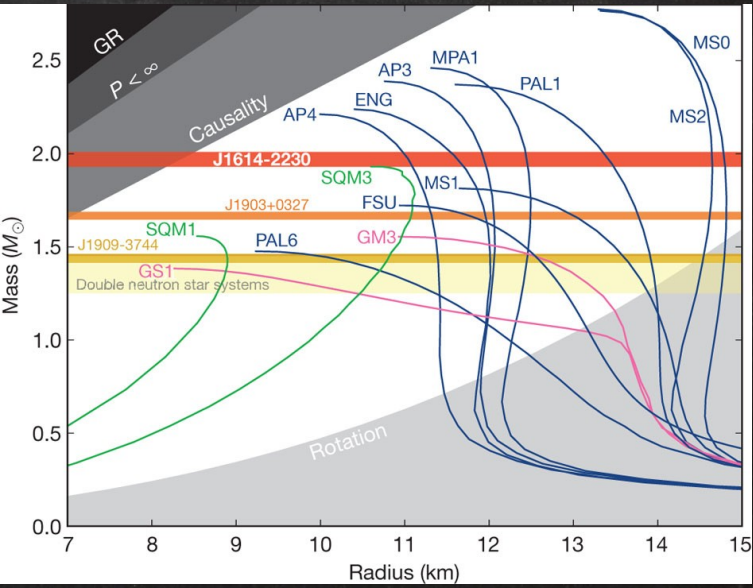


Compact stars as strong-field probes

- **Intimately related:** collapse, Chandra, etc..
- Even **stronger curvatures than BHs**
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- Neutron stars (NSs) are **common objects**
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- **However:**
 - BHs are simple objects, NSs are not!
 - **Equation of state** of a NS?
- **Future experiments (NICER)**
- **Theoretical insights may be EOS independent**



Demorest et al. Nature 2010

Part II

Coupling to scalars

Based on:

P. Pani, E. Berti, V. Cardoso, J. Read

Phys. Rev. D **84**, 104035 (2011)

P. Pani, E. Berti, V. Cardoso, J. Read, M. Salgado

Phys. Rev. D **83**, 081501 (2011)

Quadratic corrections to GR

$$S = \underbrace{\kappa \int d^4x \sqrt{-g} R}_{\text{General Relativity}} - \underbrace{\frac{1}{2} \int d^4x \sqrt{-g} [g^{ab} \nabla_a \phi \nabla_b \phi + V(\phi)]}_{\text{free scalar field}} + \underbrace{\alpha_{\text{CS}} \int d^4x \sqrt{-g} \phi^* R R}_{\text{Chern-Simons term}}$$

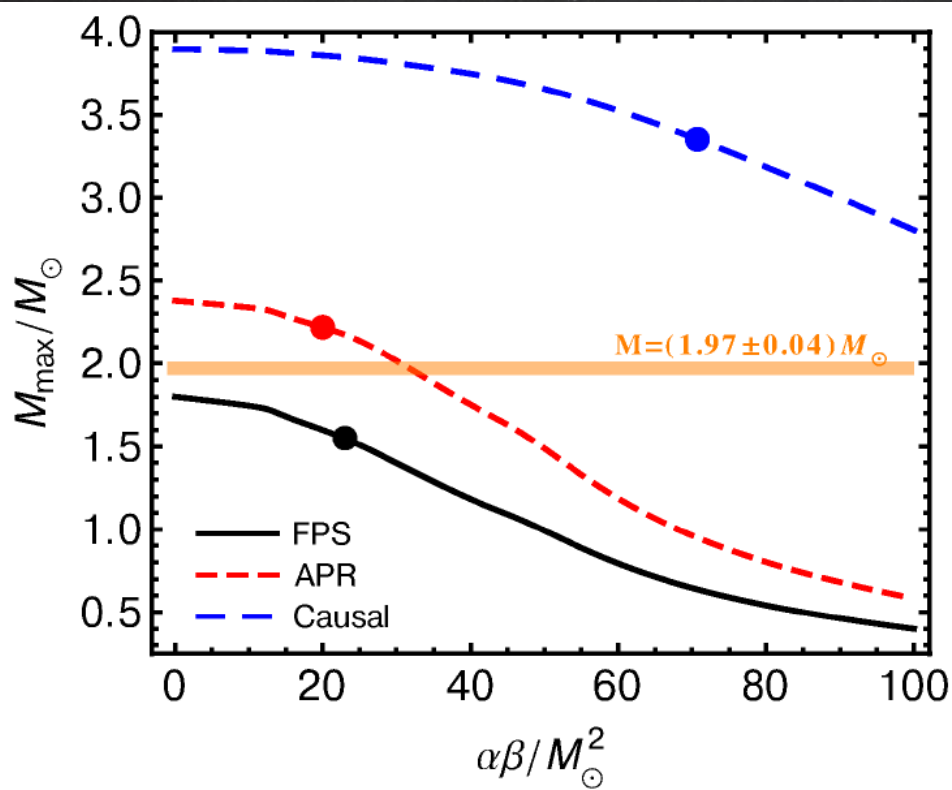
$$+ \underbrace{\alpha \int d^4x \sqrt{-g} e^{\beta \phi} (R^2 - 4R_{ab} R^{ab} + R_{abcd} R^{abcd})}_{\text{Gauss-Bonnet term}}$$

- **Well motivated** (from HEP, string theory, etc..)
- **Scalar field encoding modifications beyond GR**
- **Chern-Simons gravity** [Gualtieri's next talk]
 - Modified rotating solutions [Alexander & Yunes, Sopuerta & Yunes, ...]
[Yunes & Pretorius (2009)]
[Ali-Haimoud & Chen (2011)]
- **Gauss-Bonnet gravity**
 - Modified static models [Pani, Berti, Cardoso, Read (2011)] [Pani, Macedo, Crispino, Cardoso, (2011)]
- **Stability?** [Pani, Cardoso (2009)] [Motohashi, Suyama (2011)]

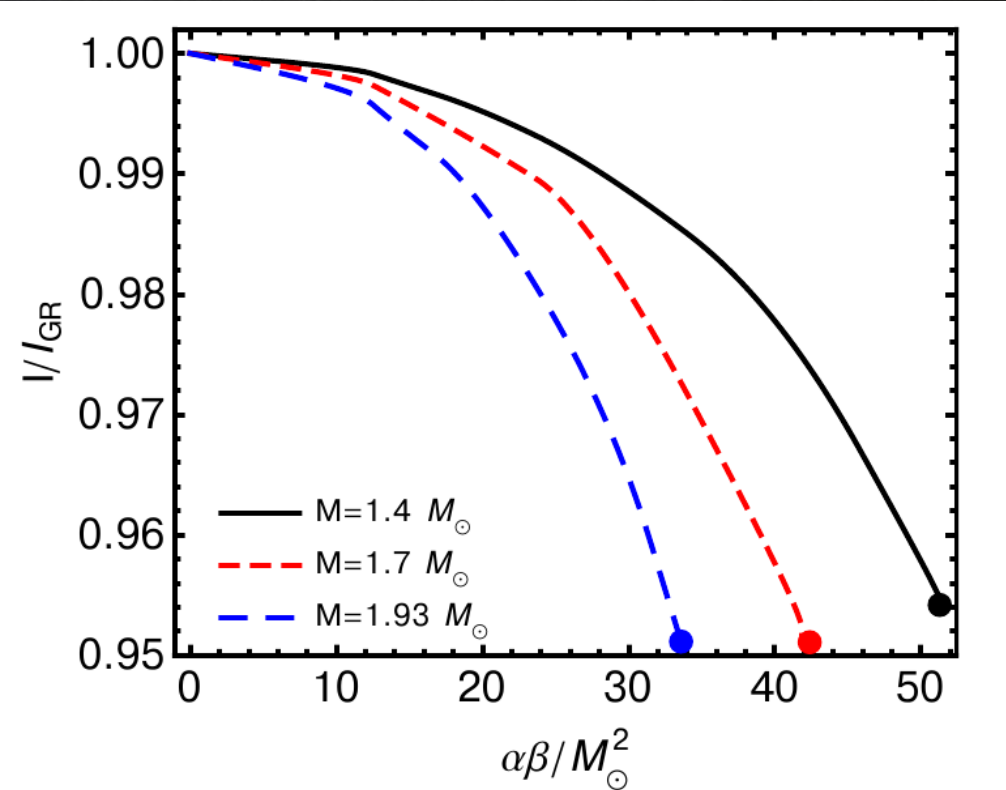
Neutron stars in Gauss-Bonnet gravity

[Pani, Berti, Cardoso, Read (2011)]

Maximum mass



Moment of inertia



Need to wait new observations to **disentangle** the effects of different **EOS** and put constraints

Scalar-tensor theories

[Fuji & Maeda book]

[Damour & Esposito-Farese, 90s]

“Physical” Jordan frame:

$$S_{(J)} = \frac{1}{16\pi} \int d^4x \sqrt{-g} [F(\phi)R - Z(\phi)g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - 2U(\phi)] + S_{\text{mat}}(\Psi_m; g_{\mu\nu})$$

Conformal transformation → Einstein frame:

$$S_{(E)} = \int d^4x \sqrt{-g^{(E)}} \left(\frac{R^{(E)}}{16\pi} - \frac{1}{2} g_{\mu\nu}^{(E)} \partial^\mu \Phi \partial^\nu \Phi - \frac{V(\Phi)}{16\pi} \right) + S_{\text{mat}}(\Psi_m; F(\Phi)g_{\mu\nu})$$

Spontaneous scalarization

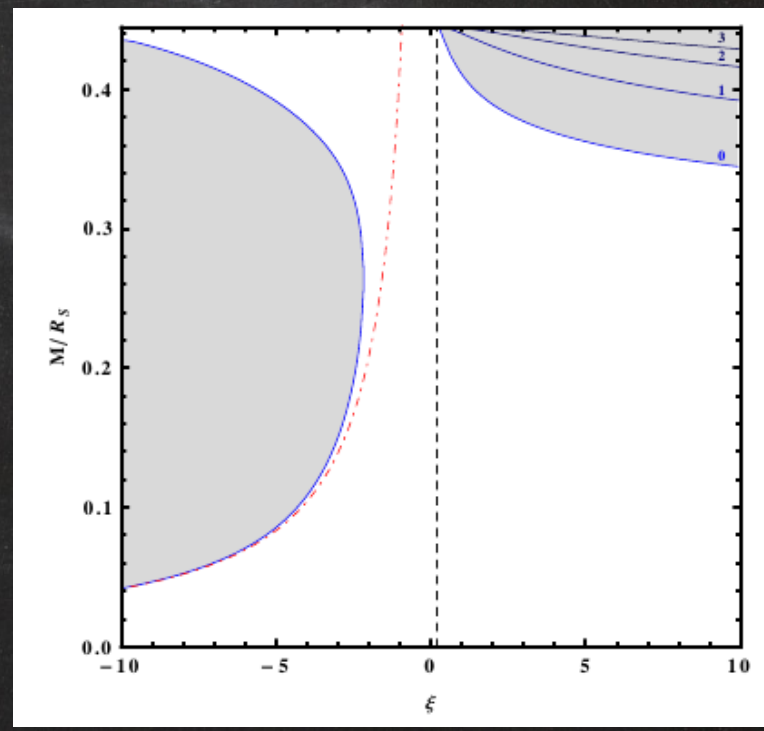
[Damour & Esposito-Farese (1992)]

[Harada 1997, Novak 1998]

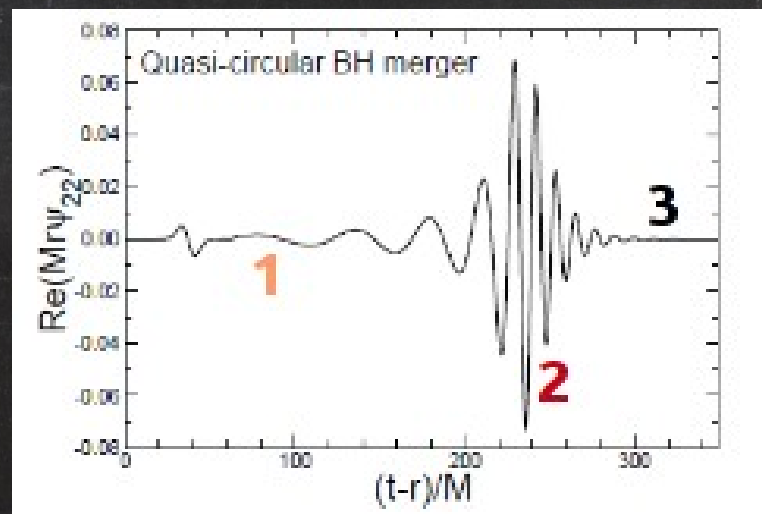
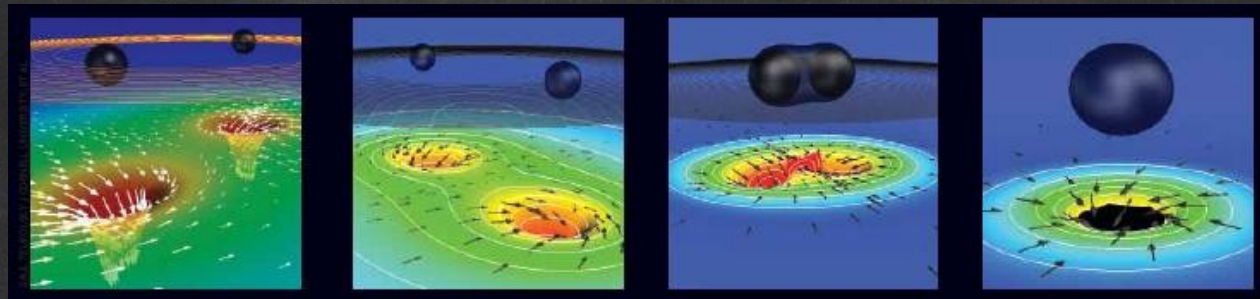
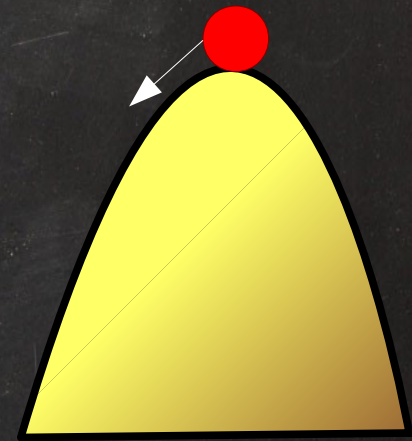
[Lima, Matsas, Vanzella, (2010)]

$$F(\phi) = 1 - \xi \phi^2 \quad Z(\phi) = 1 \quad U(\phi) = 0$$

- Phenomenologically viable
- Scalar instability
- New “scalarized” star



[Pani, Berti, Cardoso, Read, Salgado, (2011)]



Back to 1935:

- Meeting of the Royal Astronom. Society
- Chandrasekhar → theory of NS collapse
- Eddington → strong opposition



*"[...] there should be a law of Nature to prevent a star
from behaving in this absurd way!"*

- Chandrasekhar had to move to U.S.A.
- Devastating impact for the development of astrophysics
- He was eventually right → standard theory of NSs

The Observatory, Vol. 58, p. 33-41 (1935)

THE OBSERVATORY,

A MONTHLY REVIEW OF ASTRONOMY.

VOL. LVIII.

FEBRUARY, 1935.

No. 729.

MEETING OF THE ROYAL ASTRONOMICAL
SOCIETY.

Friday, 1935 January 11.

Professor F. J. M. STRATTON, M.A., D.S.O., *President*,
in the Chair.

Secretaries : W. M. SMART, M.A., D.Sc.
W. M. H. GREAVES, M.A.

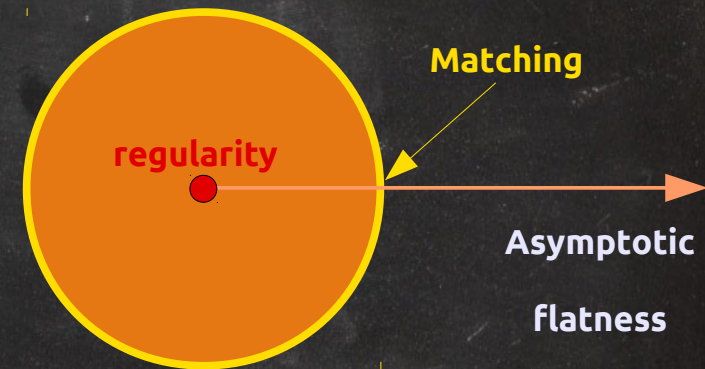


Relativistic stellar models

- It's not “just” modified gravity → many subtleties
 - **Well-posedness** of the field equations (cf. Palatini $f(R)$ theories)
 - **Matching conditions** at the stellar surface
- Relativistic stellar collapse?
- Let us start with **static configurations**:

$$ds_q^2 = q_{ab} dx^a dx^b = -p(r) dt^2 + h(r) dr^2 + r^2 d\Omega^2$$

$$ds_g^2 = g_{ab} dx^a dx^b = -F(r) dt^2 + B(r) dr^2 + A(r) r^2 d\Omega^2$$



- **Slowly-rotating models**

$$q_{t\varphi} = -\eta(r) r^2 \sin^2 \theta$$

$$g_{t\varphi} = -\zeta(r) r^2 \sin^2 \theta$$

→ Field eqs can be solved perturbatively [Hartle '67]

Stars in modified Newtonian gravity

- Pressureless stars are the **end-point** of non-relativistic, $P=0$ collapse

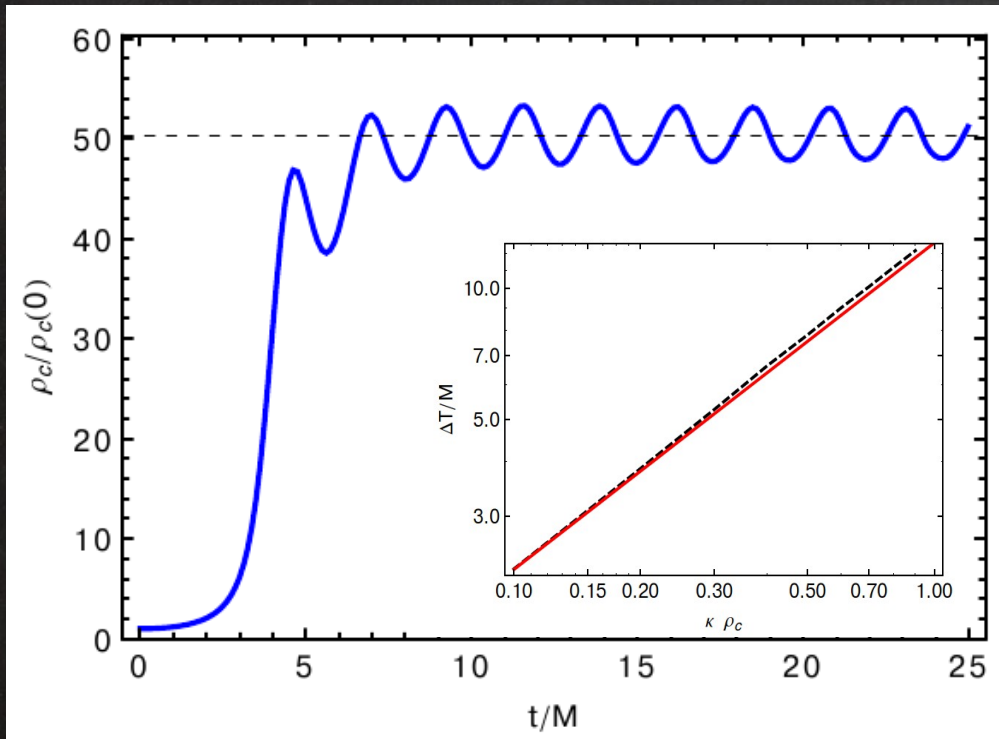
$$\ddot{\xi} - \frac{1}{\rho} \left[\frac{\gamma P}{r^2} (r^2 \xi)' \right]' + \frac{4}{\rho r} \xi P' = - \frac{\kappa}{4} \left[\frac{2}{r} \xi \rho' - \xi' \rho' - \left[\frac{\rho}{r^2} (r^2 \xi)' \right]' \right]$$

$\xi \sim e^{i\omega t} \rightarrow$ Lagrangian displacement

$\gamma \rightarrow$ Adiabatic index of perturbations

Oscillation period:

$$\frac{\Delta T}{M} \approx \frac{\pi^{5/4}}{4.4} \left(\frac{\kappa}{M^2} \right)^{3/4}$$



- Dissipation** would lead the system to a **stationary configuration**