A holographic model of the fractional quantum Hall effect

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Talk is divided into two parts

- PART I : Holographic quantum Hall fluid

- PART II : Holographic striped instability and zero sound

Outline

- Motivation
- Part I
  - Brief reminder of QHE
  - D3-D7' system as a FQH fluid
  - Fluctuation spectrum, e.g., magneto-roton
  - Phase structure
  - Conclusions and open questions
- Part II
  - Key observations about zero sound and stripes
  - D3-D7' system as a Fermi-like fluid
  - Fluctuation spectrum, e.g., zero-sound
  - Striped instability
  - Conclusions and outlook

Begin part I : Main references

- Oren Bergman, NJ, Gilad Lifschytz, and Matthew Lippert, "Quantum Hall Effect in a Holographic Model," JHEP **1010** (2010) 063 [arXiv:1003.4965 [hep-th]].

- NJ, GL, and ML, "Magneto-roton excitation in a holographic quantum Hall fluid," JHEP **1102** (2011) 104 [arXiv:1012.1230 [hep-th]].

General idea

- Use AdS/CFT.
- Learn about phenomena associated with low-energy physics in strongly coupled systems. (superconductivity, confinement, chiral symmetry breaking, FQHE,. . .)
- Maybe:
  - New computational tools
  - New effective models
- Maybe^2: New understanding

Point of view

- Start with a system we understand and explore phenomena which arise within it
- Extra criterion (maybe false): system with only fermion matter in the fundamental representation of some group
- More specific: a model with (2+1)-dimensional fermions interacting with (3+1)-dimensional gauge fields $\rightarrow$ D3-D7' systems

[S.J. Rey]
[J. Davis,P. Kraus,A. Shah]
[Myers,Wapler]
[J. Alanen,E. Keski-Vakkuri,P. Kraus,V. Suur-Uski]
[NJ et al.]

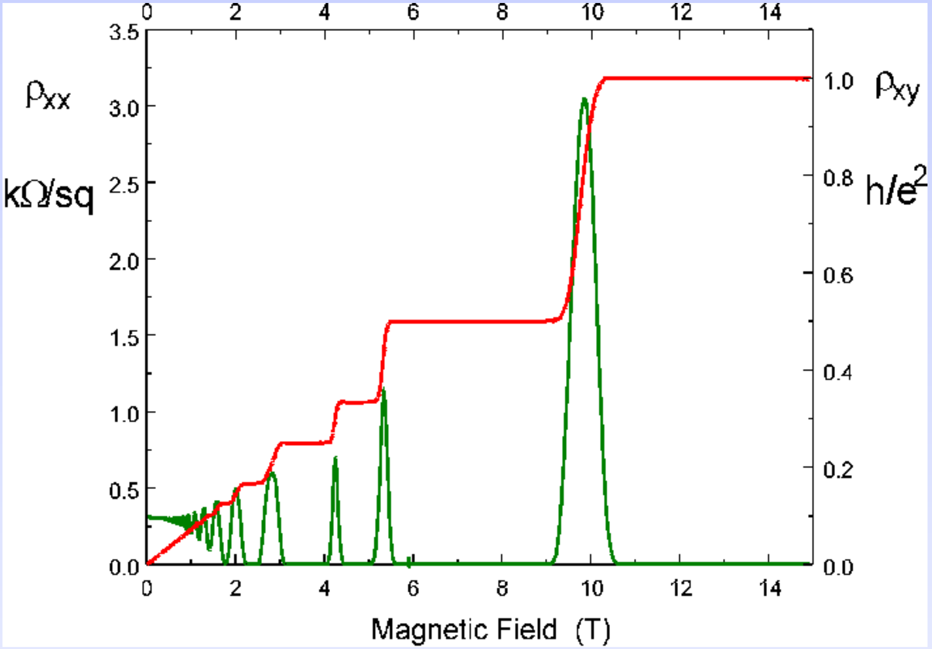
- The D3-D7' has a variety of interesting phenomena. First we will concentrate on QH fluid.

Quantum Hall Effect

- Charged particles in 2+1 dimensions with a perpendicular magnetic field. A series of plateaux

$$\sigma_{xx} = 0 \quad , \quad \sigma_{xy} = \nu \frac{e^2}{h}$$

$$\nu \equiv \frac{n_e}{2\pi B} = \begin{cases} \text{integers} \\ \text{certain fractions} \end{cases}$$



- Integer ν is understood from Landau levels plus impurities.
- Fractional ν is a strong coupling effect. Interesting phenomena include: fractional charge and anyonic statistics of charged excitations, independence of the material, incompressible fluid, edge states etc.
- Different descriptions: Laughling wave function, Chern-Simons theory, Jain's composite fermions

D3-D7' system

	t	x	y	z	r	ψ	S^2	S^2
D3	■	■	■	■				
D7	■	■	■		■		■	■

$$d\Omega_5^2 = d\psi^2 + \cos^2\psi d\Omega_2^{(1)2} + \sin^2\psi d\Omega_2^{(2)2}$$

- $\#ND = 6 \to$ only fundamental fermions.
- D7' embedding $z(r), \psi(r)$.
- Use DBI+CS.
- However, there is an instability.

The instability

$$\psi \sim \psi_\infty + A r^\Delta \qquad , \qquad \text{as } r \to \infty \,,$$

find Δ complex $\to \psi$ is tachyonic.

- One can turn on flux through the two S^2

$$2\pi\alpha' F = \frac{L^2}{2}\left(f_1 d\Omega_2^{(1)} + f_2 d\Omega_2^{(2)}\right) \,, \quad f_i = \frac{2\pi\alpha'}{L^2} n_i$$

  Study the equation of motion and find

$$(f_1^2 + 4\cos^4\psi_\infty)\sin^2\psi_\infty = (f_2^2 + 4\sin^4\psi_\infty)\cos^2\psi_\infty$$

  and

$$\Delta_\pm = -\frac{3}{2} \pm \frac{1}{2}\sqrt{9 + 16\frac{f_1^2 + 16\cos^6\psi_\infty - 12\cos^4\psi_\infty}{f_1^2 + 4\cos^6\psi_\infty}} \,.$$

- Turning on fluxes stabilizes the system and gives many more possibilities for ψ_∞.

More details

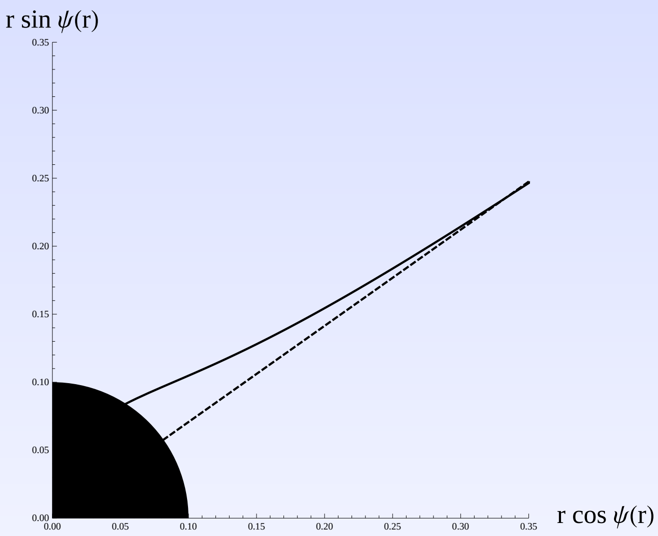
$$\psi \sim \psi_\infty + \underbrace{m}_{\text{mass } m_q} r^{\Delta_+} - \underbrace{c}_{\text{condensate } \langle \bar{q} q \rangle} r^{\Delta_-} , \qquad \text{as r} \to \infty$$

- Remark on dimensions.
- Solutions of the equations of motion for the D7' are either BH-type (enter the horizon at some r_T) or MN-type $\psi(r_0) = 0, \pi/2$ (one of the two S^2's shrinks at some r_0)
- For the latter one, the shrinking S^2 can not carry any flux, and we choose $f_1 = 0$. (This is why $S^2 \times S^2$ is better than S^4.) In this case $z(r)$ = constant.

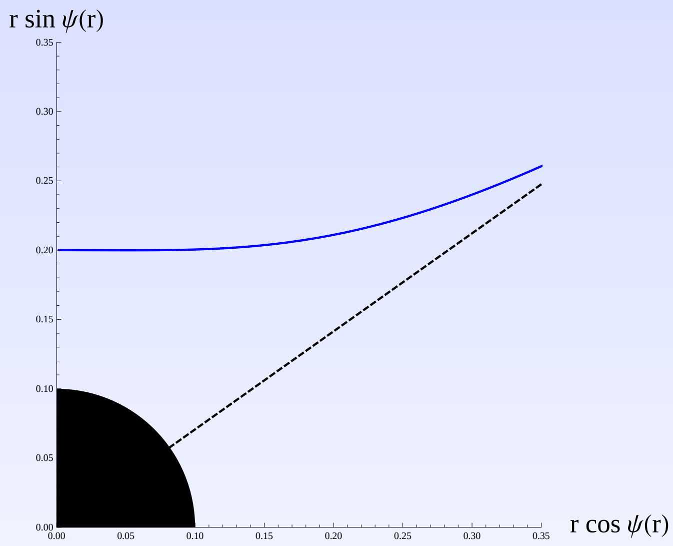
Embedding

$$S_{DBI} = -2\mathcal{N} \int dr r^2 \cos^2\psi \sqrt{(f_2^2 + 4\sin^4\psi)(1 + hr^2\psi'^2)}\,, \tag{1}$$

where $h(r) = 1 - \frac{r_T^4}{r^4}$.



D7 enters the horizon



D7 ends where S^2 shrinks

Add charge and magnetic field

- We can turn the $U(1)$ gauge fields on the D7' brane corresponding to charge density and magnetic field:

$$2\pi\alpha' F_{r0} = a_0'(r) \qquad , \qquad 2\pi\alpha' F_{xy} = b \, .$$

- This changes the DBI action but more importantly adds a CS term:

$$S_{CS} = 2\mathcal{N} \int drc(r)ba_0'(r) \, , \; c(r) = \psi(r) - \frac{1}{4}\sin(4\psi(r)) - \psi_\infty + \frac{1}{4}\sin(4\psi_\infty) \, .$$

- Usually in MN embedding it is not possible to add charge, since this is translated to having sources at the tip, which would pull the D7-brane down to the horizon.

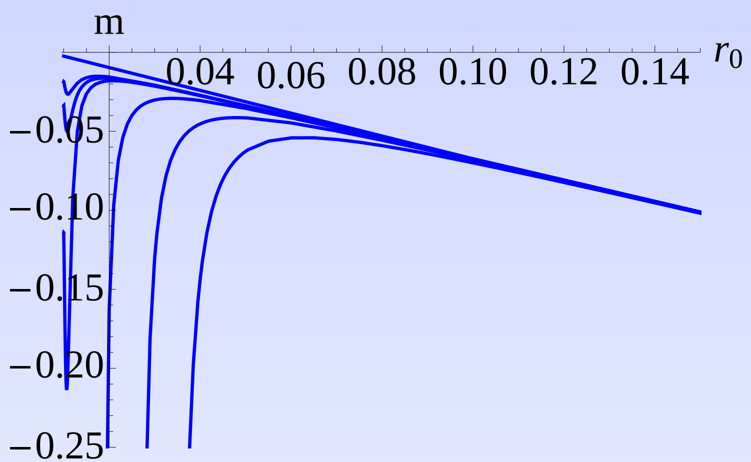
Add charge and magnetic field

- Due to CS term one can have non-zero charge without sources, but the amount of charge is fixed by the requirement of no source at the tip,

which means (d is the total charge, in some units)

$$d - 2bc(r_0) = 0\ .$$

In other words all of the charge is induced by the RR five-form.



- The filling fraction

$$\nu = \frac{2\pi D}{B} = \frac{2N_3 c(r_0)}{\pi} \qquad , \qquad 0.6972 \lesssim \frac{\nu}{N_3} \lesssim 0.8045\ .$$

- To change the filling fraction we need to change ψ_∞.
- Filling fraction is quantized because of f_2 and since it is the density of open strings divided by the density of D5-branes.

MN as QH fluid

- B and D are locked together.
- We can compute the longitudinal and transverse conductivities by turning on an electric field and requiring no sources at the tip:

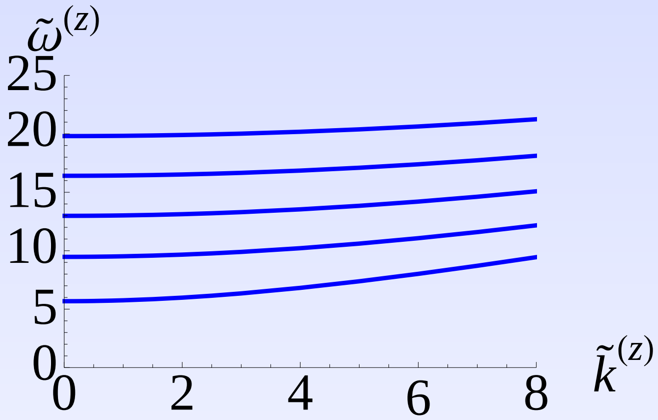
$$\sigma_{xx} = \frac{J_x}{E} = 0 \qquad , \qquad \sigma_{xy} = \frac{\nu}{2\pi} \; .$$

- So, charged excitations are gapped (which is also clear from the embedding). Expected mass from this picture is of order $(g_{YM}^2 N_3)^{1/4}\sqrt{B}$.
- Compute fluctuations of the D7'-brane, find that neutral excitations are also gapped $\rightarrow$ incompressible fluid. Mass computed of order $(g_{YM}^2 N_3)^{-1/4}\sqrt{B}$.

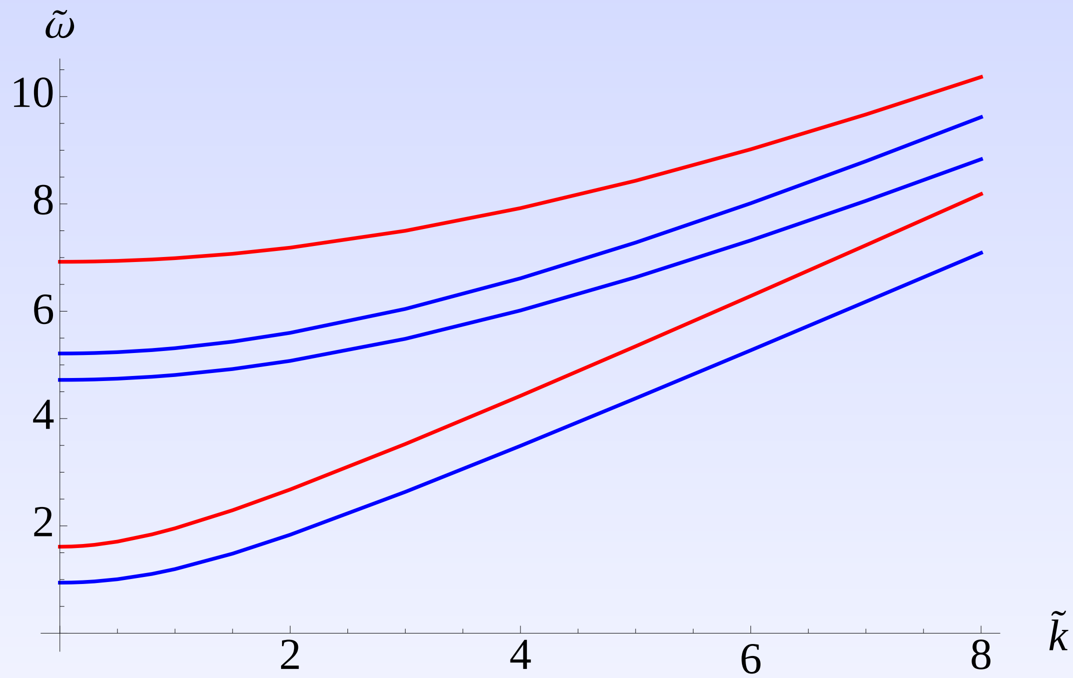
Small fluctuation analysis

- Spectra for small fluctuations, at zero temperature and $k_\mu = \sqrt{b}\tilde{k}_\mu/L$,

$$\tilde{\omega} = \sqrt{\tilde{\omega}_0^2 + c_s^2\tilde{k}^2}\ .$$



δz spectrum



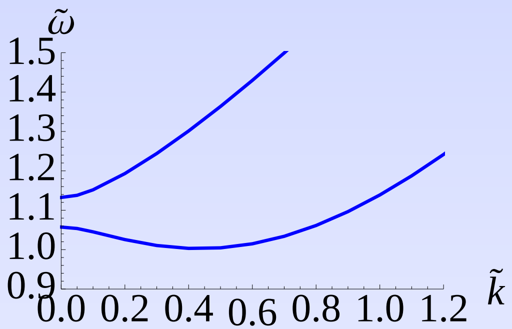
Other fluctuations couple at nonzero momentum.

- The system is stable and gapped.

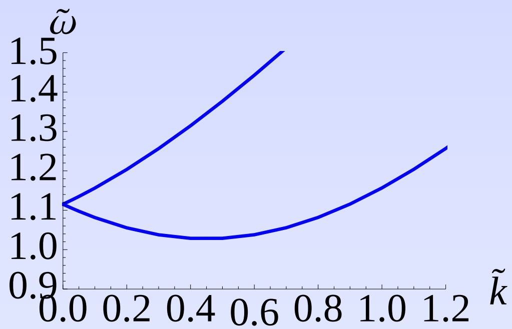
Magneto-roton excitation

- Depending on the density (magnetic field) one finds a magneto-roton:

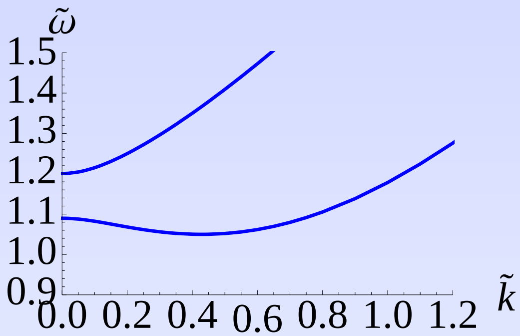
$$\tilde{\omega}^2 = \sqrt{\tilde{\omega}_*^2 + c_s^2(\tilde{k} - \tilde{k}_*)^2}\,.$$



Small intermediate b.

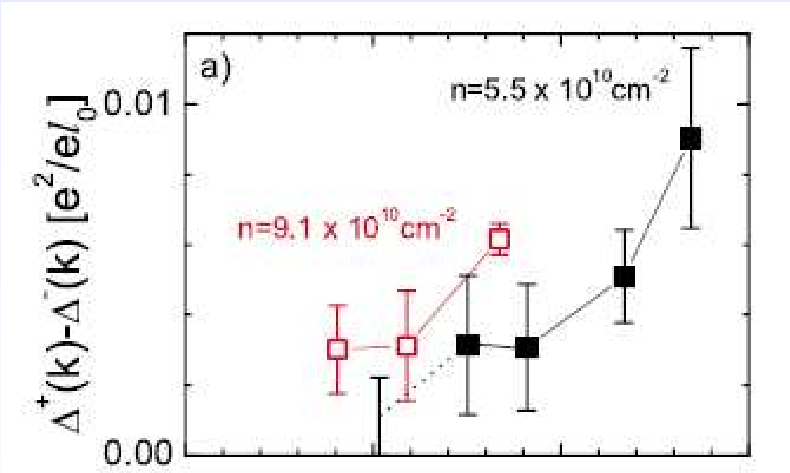


Intermediate b.



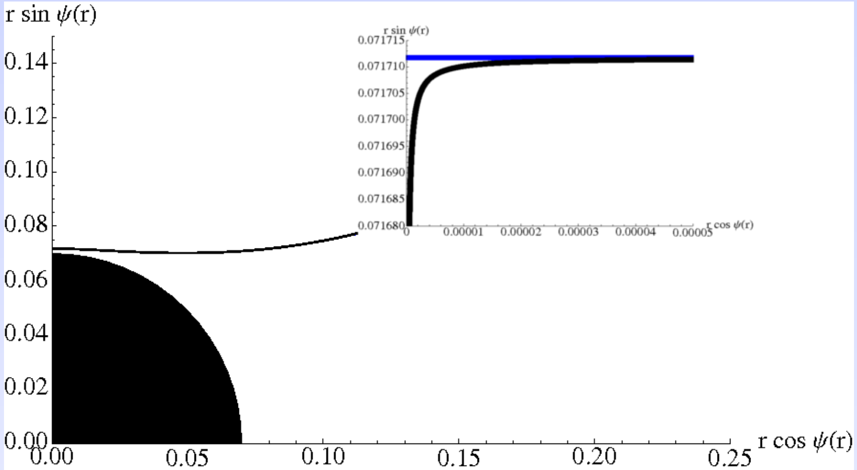
Large intermediate b.

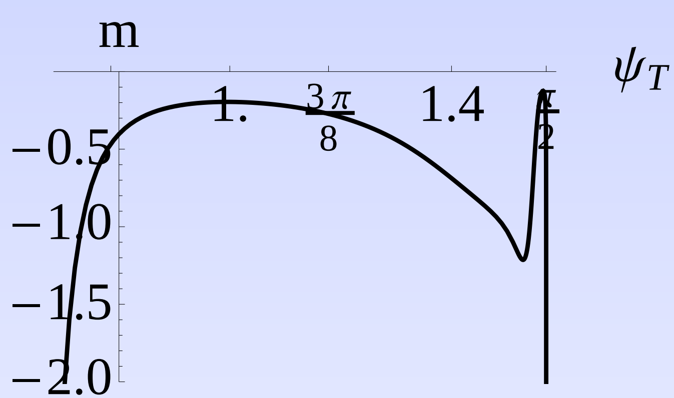
- Recently observed for $\nu = 1/3$.
  [Hirjibehedin,Dujovne,Pinczuk,Dennis,Pfeiffer,West '05]
- In our model we find it for all ν, but depends on the density (magnetic field).
- The roton minimum decreases with decreasing filling fraction.



BH is metallic

- When we change slightly the relationship between d and b, only BH embeddings exist, but (some of) those are “semi-smoothly” connected to the MN embeddings.





- The conductivities can be computed (note, QGP for $d = T = b = m = 0$)

$$\sigma_{xx} = \frac{N_3}{2\pi^2}\frac{r_T^2}{b^2 + r_T^4}\sqrt{\tilde{d}(r_T)^2 + (f_1^2 + 4\cos^4\psi(r_T))(f_2^2 + 4\sin^4\psi(r_T))(b^2 + r_T^4)}$$

$$\sigma_{xy} = \frac{N_3}{2\pi^2}\left(\frac{b}{b^2 + r_T^4}\tilde{d}(r_T) + 2c(r_T)\right) \quad , \quad \tilde{d}(r_T) = d - 2c(r_T)b\,.$$

- Anomalous QH for $\sigma_{xy}(b = 0) \neq 0$ and resistivity saturation $\rho_{xx}(T \to \infty) \to \mathrm{const.}$.

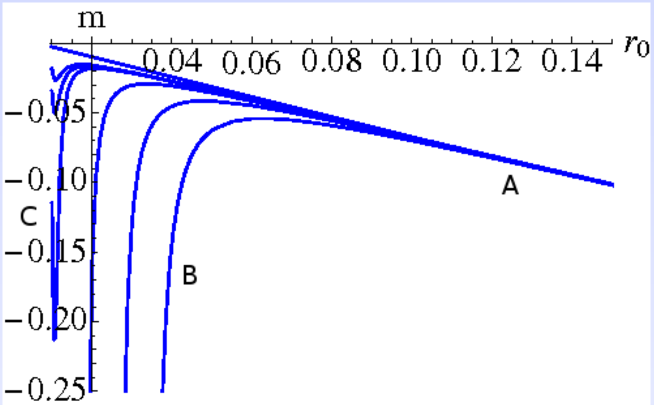
BH is magnetic

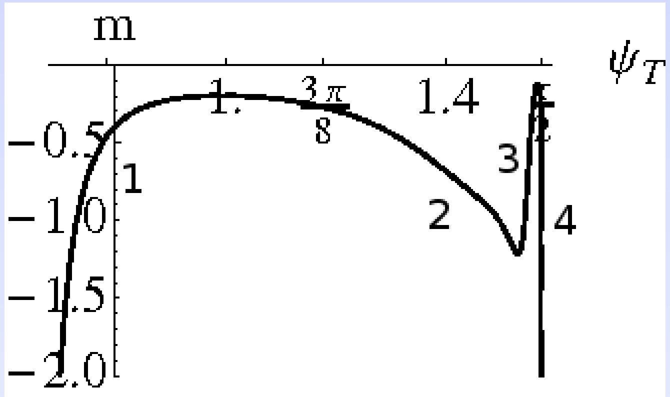
- For $m = 0$ and for small b, the system is paramagnetic for small T and diamagnetic for large T.
- For $m = 0$ and for large b, the system is diamagnetic for all T.
- Ferromagnet for $m \neq 0$:

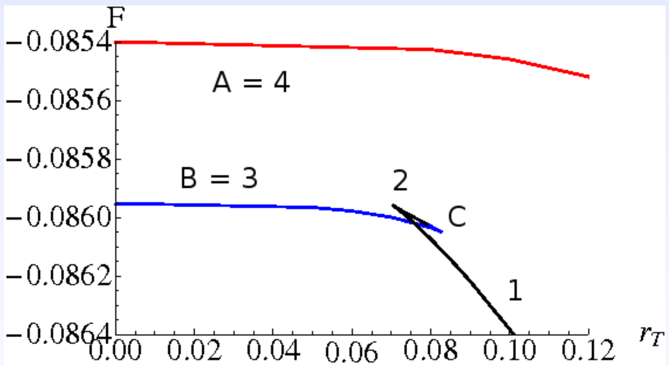
$$M(b=0) = 2\mathcal{N} \int dr c(\psi) a_0'(b=0) \ .$$

Different phases

- There are at least 2 MN and a few BH solutions. While as a function of a magnetic field MN deforms to a BH, as a function of temperature there is a first order phase transition.







- Small fluctuation analysis $\longrightarrow$ blue is stable but red is tachyonic. [NJ,Lifschytz,Lippert]

Part I : Conclusions and open questions

Holographic model of a QHF of strongly interacting charged fermions in 2+1 dimensions:

- States with $\sigma_{xx} = 0$ and quantized σ_{xy} (but not rational)
- Evolve into conducting states as the density varies relative to the magnetic field
- First order conductor/QHF phase transition at finite temperature
- Excitation spectra: stable and gapped. Also magneto-roton excitation.

Things to understand better:

- Fractional charge?
- Edge states?
- Plateaux - impurities?
- Transitions between plateaux?
- Universal properties? (Other Dp-Dq' systems are available via T-duality.)

Begin part II : Main references

- Oren Bergman, NJ, Gilad Lifschytz, and Matthew Lippert,
  "Quantum Hall Effect in a Holographic Model,"
  JHEP **1010** (2010) 063 [arXiv:1003.4965 [hep-th]].

- OB, NJ, GL, and ML,
  "Striped instability of a holographic Fermi-like liquid,"
  JHEP **1110** (2011) 034 [arXiv:1106.3883 [hep-th]].

- NJ, GL, and ML,
  to appear.

Key points

Zero sound

- In a system of interacting fermions at low temperature there exists a collective excitation known as the zero sound.

[Landau, 1957]

- Observed in liquid He-3 in the 60's.
- Temperature dependence, at low T, of the zero sound mode is described by Landau Fermi-liquid theory.
- D3-D7' zero sound at non-zero T behaves exactly as Landau Fermi-liquid zero sound.

Stripes

- D3-D7' in the BH phase is holographically dual to spatially modulated phase of holographic matter at large enough density.
- The spatial translation is spontaneously broken.
- The true ground state of the system resembles that of charge density wave and spin wave.

BH embedding as a Fermi-like fluid

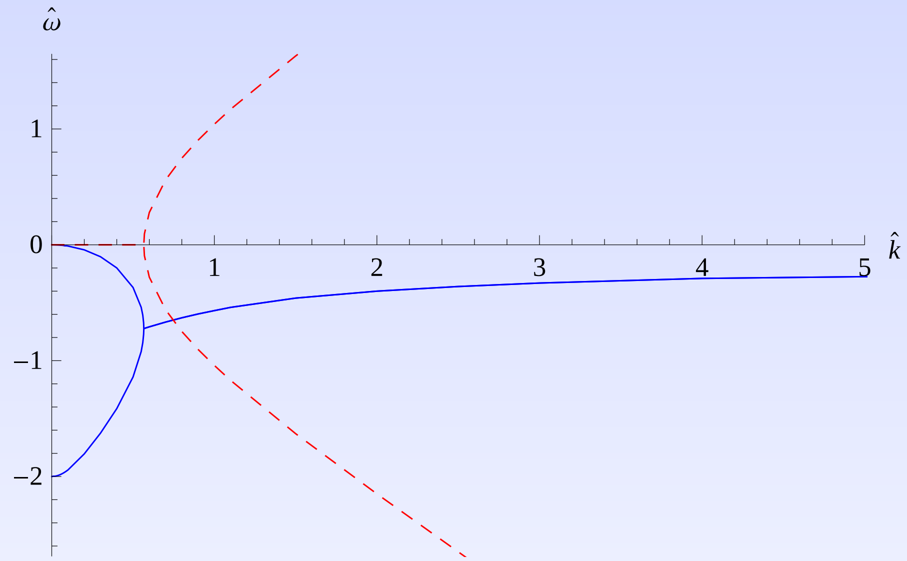
- Keep $f_1 \neq 0 \neq f_2$. We will thus only consider BH embeddings from now on.
- The BH embedding is gapless. This can be verified by the fluctuation analysis, *e.g.*, the diffusion mode.
- Recall the conductivities from before, at $b = 0$:

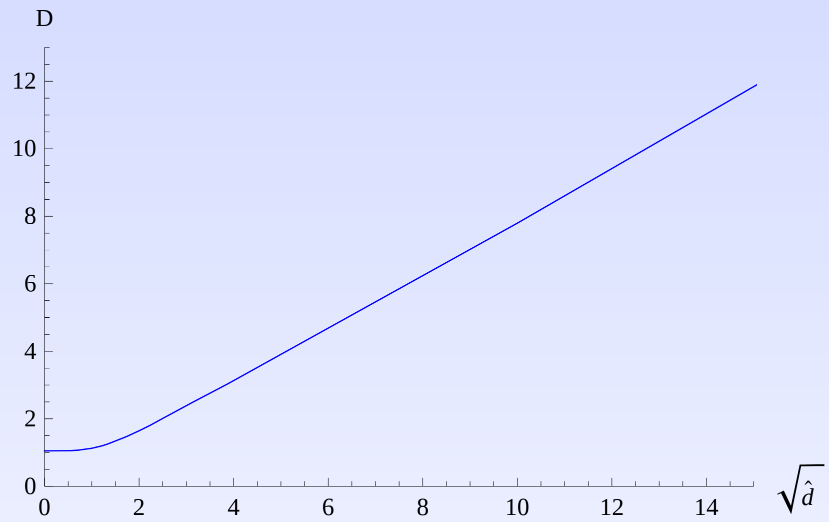
$$\sigma_{xx}(b=0) = \frac{N_3}{2\pi^2}\frac{1}{r_T^2}\sqrt{d^2 + r_T^4(f_1^2 + 4\cos^4\psi(r_T))(f_2^2 + 4\sin^4\psi(r_T))}$$
$$\sigma_{xy}(b=0) = \frac{N_3}{2\pi^2}\cdot 2c(r_T)\ .$$

- At low temperatures but non-zero densities we have $\sigma_{xx} \sim \frac{d}{T^2}$. This matches the expectation from the Fermi liquid theory.
- The heat capacity goes as $C_v \sim \frac{T^4}{d}$, unlike that for a regular Fermi liquid.

Zero sound at non-zero temperature

- The diffusion mode $\hat{\omega} \sim -iD\hat{k}^2 + \ldots$.
- Find $D \sim \sqrt{\hat{d}}$, where $\hat{d} = \frac{d}{r_T^2}$.



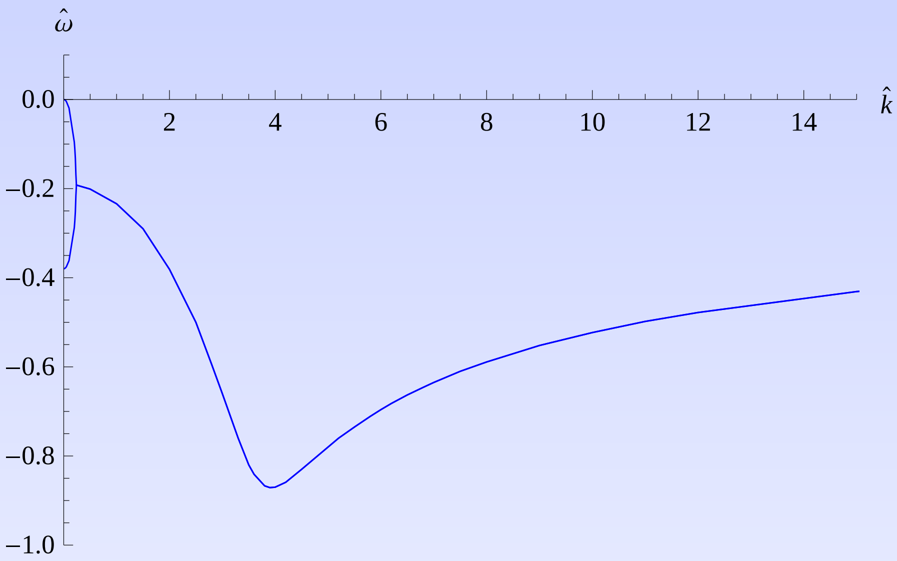


- Transition from the hydrodynamic regime to the collisionless regime. Noticed before in the susic D3-D7 system.

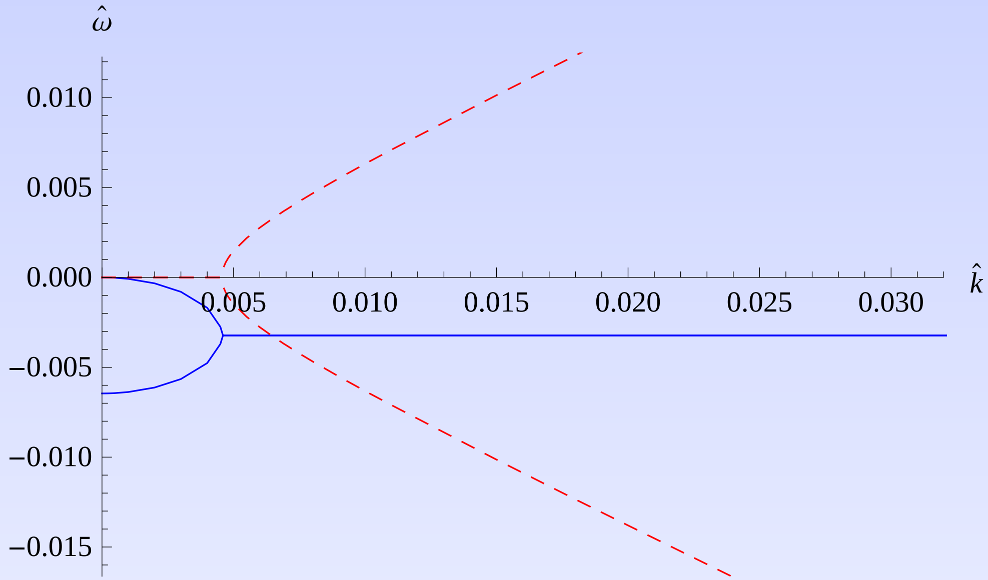
[Kaminski-Landsteiner-Mas-Shock-Tarrio]

Zero sound at non-zero temperature

- Interesting things happen when cooling down the system.



Intermediate temperature, $\hat{d} = 5$.



Very low temperature, $\hat{d} = 10^4$.

- One finds that the hydro mode has rendered into the zero sound:

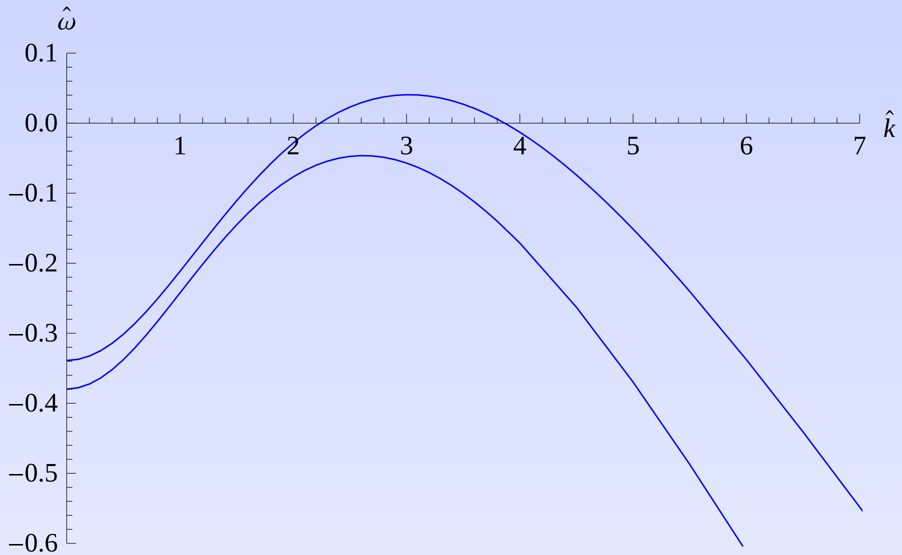
$$\hat{\omega} \sim v_s \hat{k} - ia\hat{k}^2 + \ldots\ ,$$

whose speed equals to that of normal sound, $v_s = \frac{1}{\sqrt{2}}$.

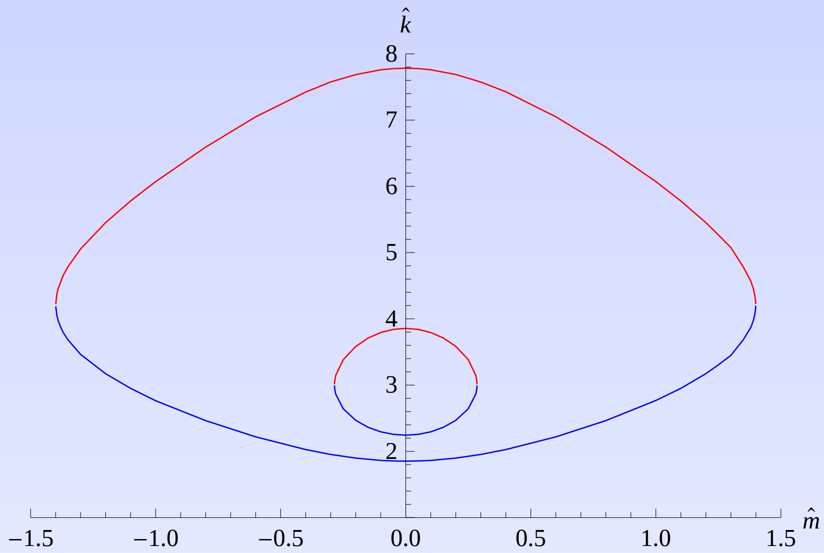
- By dialing up $b \neq 0$, the zero sound becomes massive above some critical value.

Striped instability

- The system is unstable towards decay to an inhomog. phase when $\hat{d} > \hat{d}_{critical}$.



The $\delta\psi, \delta a_y$ system at $\hat{m} = 0$, $\hat{d} = 5, 6$.



Domains of instability, $\hat{d} = 6, 10$.

- For $\hat{m} = 0$, the fermion bi-linear is spatially modulated $\longrightarrow$ spin-wave.
- As $|\hat{m}| > 0$ also the δa_t is involved $\longrightarrow$ charge-density wave.
- Notice, for $T = 0$, any non-zero density triggers the tachyon!
- Turning on b will eventually always tame the tachyon.

Generic phenomenon with CS terms

- Consider 4d Maxwell-axion Lagrangian:

$$\mathcal{L} = -\frac{1}{4}F_{IJ}F^{IJ} + \frac{1}{2}(\partial_I\Phi)^2 - \frac{1}{2}m^2\Phi^2 + \frac{\alpha}{2}\epsilon^{IJKL}\Phi F_{IJ}F_{KL}$$

  in the background electric field $F_{03} = E$.

- Plug in waves $\sim e^{-i\omega t + ikx}$ to the resulting equations of motion and find the dispersion relation:

$$\omega^2 = k^2 + \frac{1}{2}m^2 \pm \frac{1}{2}\sqrt{m^4 + 64\alpha^2 E^2 k^2}\ .$$

  Thus, there is a tachyon in the range $0 < k < \sqrt{16\alpha^2 E^2 - m^2}$.

- We believe that this is quite generic. Other examples include: D2-D8', Sakai-Sugimoto model, 11d SUGRA, etc.

[Donos-Gauntlett]
[Bayona-Peeters-Zamaklar]
[NJ-MJ-ML to appear]

Conclusions and outlook

- Showed that D3-D7', in the black hole phase, resembles Fermi-like fluid.
- Found that the system has a zero sound mode which renders into a hydrodynamic mode at non-zero temperature.
- There must be a Fermi surface. Should be able to find it (needs backreaction).
- Noticed, that the true ground state of the system is inhomogeneous, which resembles that of spin-wave and charge density wave. One should be able to construct it directly!