Holographic DC conductivities from the open string metric

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Outline

What is Gauge/gravity duality?

- A holographic duality between a weakly-coupled theory of gravity in certain spacetime and a strongly-coupled field theory living on the boundary of that spacetime.
- A powerful new tool for investigating dynamics of strongly-coupled field theories in the dual gravity side.
- A new window towards understanding real-world physics: QCD, CMT, etc.
- Two complementary approaches: bottom-up and top-down.

Two complementary approaches:

Bottom-up

- Toy-models coming from simple gravity theory;
- Basic ingredients: $g_{\mu\nu}, A_\mu, \psi$ and/or dilaton ϕ;
- Advantage(s): simplicity and universality;
- Disadvantage(s): the dual field theory is unclear.

Top-down

- Configurations originated from string/M theory;
- Exact solutions of SUGRA or Dp/Dq-branes;
- Advantage(s): good understanding on the field theory;
- Disadvantage(s): complexity.

Three main approaches

Retarded Green's function method(Son, Starinets '02)

- General, resulting in many transport coefficients;
- The bulk retarded Green's function encodes a retarded correlator of its dual (field theory) operator;
- Kubo's formula $\Rightarrow$ transport coefficients.

The membrane paradigm (Iqbal, Liu '08)

- Hydrodynamic behavior of boundary field theory $\Leftrightarrow$ those at the stretched horizon of the black hole;
- Transport coefficients $\Leftrightarrow$ quantities at the horizon;
- This elegantly explains universalities of transport coefficients.

Three main approaches Cont'd

The real action method (Karch, O'Bannon '07)

- DC conductivity only, not applicable to other transport coefficients;
- Probe D-brane systems only;
- Non-linear current (electric field dependent conductivity).

These properties stem from the DBI action

$$S_{\mathrm{DBI}} = -T_p \int d^{p+1}\xi \sqrt{P[G] + \mathcal{F}}$$

by requiring that the on-shell action should be real.

Using open string metric

- The open string membrane paradigm with external electromagnetic fields, by K.Y.Kim, J.P.Shock and J.Tarrio, arXiv: 1103.4581[hep-th]
- a membrane paradigm method based on open string metric;
- DC conductivity of a D3/D7 system;
- We will see more generalizations in the current work.

Why open strings?

- When background Kalb-Ramond fields or world-volume gauge fields on a probe D-brane are turned on, the fluctuations of open strings on the probe D-brane do not feel simply the background geometry that they are probing;
- The open string metric (OSM) describes precisely the effective geometry felt by open strings in the presence of external fields.
- We may understand the dynamics of these fluctuating fields in terms of the OSM. In some sense, the background gauge fields are *geometrized*.

The definitions

- DBI+WZ

$$\mathcal{L} = \sqrt{-\mathrm{det} P[G] + \mathcal{F}} + P[C] \wedge \mathcal{F},$$

P[ ]-pull-back, $\mathcal{F} = \tilde{F} + \tilde{f}$, f-fluctuations. Quantities with tildes-those multiplied by $2\pi\alpha'$.

- Define the OSM as follows

$$\begin{aligned} \gamma_{mn} &\equiv P[G] + \tilde{F}, \\ \gamma^{mn} &= (\gamma_{mn})^{-1} = s^{mn} + \theta^{mn}, \end{aligned} \quad (1)$$

The definitions Cont'd

- s^{mn}-the symmetric part, θ^{mn}-the anti-symmetric part.
- The OSM s_{mn} is defined as

$$s_{mn} = g_{mn} - (\tilde{F} g^{-1} \tilde{F})_{mn}, \tag{2}$$

- Notice that $s_{mn} s^{np} = \delta_m^p$.

Black hole embedding

Steps to compute non-linear DC conductivity:

- compute the linear conductivity using OSM and membrane paradigm;
- compute the singular shell position r_s from $\xi(r_s) = 0$ with finite $\tilde{E}$;
- apply the same formula obtained in step 1 at $r = r_s$.

Minkowski embedding

- We cannot apply the real-action method since there is no singular shell on the world volume;
- From the OSM point of view, the geometry is regular everywhere and there seems to be no reason to introduce the current;
- We still require regularity on the gauge field configuration;
- This was proposed in arXiv: 1003.4965[hep-th] (Bergman, Jokela, Lifshytz and Lippert).

General assumptions

- Consider probe Dq-branes sharing t, x, y field theory space. The induced metric and gauge field

$$\begin{aligned} ds^2 &= g_{tt}dt^2 + \sum_{i=1}^{2} g_{ii}dx_i^2 + g_{rr}dr^2 + ds_{(l)}^2, \\ 2\pi\alpha' A &= \tilde{A}_t dt + \tilde{B}xdy + 2\pi\alpha' a, \end{aligned} \tag{3}$$

$ds_{(l)}^2$-the metric of the internal space, $l = q - 3$.

- There may be nontrivial background RR fields and fluxes through the internal space in concrete examples.

the DBI term

- Assume the matrix $\gamma = g + \tilde{F}$ is a direct sum of the submatrix in the bulk spacetime $m = t, 1, 2, r$ and the internal space $\alpha = 4, \cdots, q+1$. $\det\gamma = \det\gamma_{ab}\det\gamma_{\alpha\beta}$, where

$$\det\gamma_{\alpha\beta} \sim \Theta(r) \times \text{a function of } \xi^{\alpha}.$$

- The DBI action becomes

$$\begin{aligned} S_{\rm DBI} &= -N_f T_{Dq} V_{(I)} \int dtd\vec{x}dre^{-\phi}\sqrt{\Theta}\sqrt{-\det\gamma_{mn}}, \\ &\equiv \mathcal{N} \int dtd\vec{x}dr\mathcal{L}_{\rm DBI} \end{aligned} \tag{4}$$

the DBI term Cont'd

- The normalization constant

$$\mathcal{N} = N_f T_{Dq} V_{(I)}, \quad \mathcal{N}' \equiv (2\pi\alpha')^2 \mathcal{N}, \tag{5}$$

$\mathcal{N}'$ is defined for later convenience.

- The leading order Lagrangian

$$\mathcal{L}^{(0)}_{\mathrm{DBI}} = -e^{-\phi}\sqrt{\Theta\kappa}\sqrt{-g_{tt}g_{rr} - \tilde{A}'^2_t}, \tag{6}$$

$$\kappa \equiv \det\gamma_{ij} = \tilde{B}^2 + g_{xx}g_{yy}, \quad i,j = 1,2. \tag{7}$$

the DBI term Cont'd

- The conserved quantity

$$\hat{J}_t \equiv \frac{\partial \mathcal{L}}{\partial \tilde{A}_t'} = \frac{e^{-\phi}\tilde{A}_t'\Theta\kappa}{\sqrt{-(g_{tt}g_{rr} + \tilde{A}_t'^2)\Theta\kappa}}, \tag{8}$$

$$\tilde{A}_t' = \sqrt{-\frac{\hat{J}_t^2 g_{tt}g_{rr}}{\hat{J}_t^2 + e^{-2\phi}\Theta\kappa}}, \tag{9}$$

- The sub-leading action

$$S_{\rm DBI}^{(2)} = -\mathcal{N}' \int dtd\vec{x}dr \left[\frac{\sqrt{-s}}{4g_4^2}s^{mp}s^{nq}f_{mn}f_{pq} + \frac{1}{8}\epsilon^{mnpq}f_{mn}f_{pq}Q\right], \tag{10}$$

the DBI term Cont'd

- The effective coupling

$$g_4^2 = \frac{\sqrt{-s}}{e^{-\phi}\sqrt{-\det\gamma_{mn}}\sqrt{\Theta}}.$$

- The non-vanishing components of θ

$$\theta^{tr} = -\frac{e^{\phi}\hat{J}_t}{\sqrt{-\det\gamma_{mn}}\sqrt{\Theta}}, \quad \theta^{xy} = -\frac{\tilde{B}}{\kappa}, \tag{11}$$

- The OSM (recall (2))

$$s_{mn}dx^m dx^n = g_{tt}\mathcal{G}^2 dt^2 + g_{rr}\mathcal{G}^2 dr^2 + \frac{\kappa}{g_{yy}}dx^2 + \frac{\kappa}{g_{xx}}dy^2, \tag{12}$$

the DBI term Cont'd

$$\mathcal{G}^2 = \frac{e^{-2\phi}\Theta\kappa}{\hat{J}_t^2 + e^{-2\phi}\Theta\kappa},$$

$$Q = -\frac{1}{8}e^{-\phi}\sqrt{-\det\gamma_{mn}}\sqrt{\Theta}\epsilon_{mnpq}\theta^{mn}\theta^{pq} = -\frac{\tilde{B}\hat{J}_t}{\kappa},$$

with $\epsilon_{txyr} = 1$.

- The effects of density $\hat{J}_t$ and magnetic field $\tilde{B}$ are geometrized through $\mathcal{G}$ and κ.

The conductivity

The conductivity can be obtained following Iqbal, Liu ’08

- For a black hole embedding

$$\sigma^{ij} = \mathcal{N}' \left[\frac{1}{g_4^2} \sqrt{\frac{s}{s_{tt} s_{rr}}} s^{ij} - Q\epsilon^{ij} - C_{q-3}\epsilon^{ij} \right] , \qquad (13)$$

- This is a conductivity which is electric field independent (a linear conductivity).

The conductivity Cont'd

- For the nonlinear conductivity, we first determine the position of the singular shell r_s,

$$\det\gamma_{\mu\nu}(r_s) = [\tilde{B}^2 g_{tt} + \tilde{E_x}^2 g_{yy} + \tilde{E_y}^2 g_{xx} + g_{tt} g_{xx} g_{yy}]_{r\to r_s} = 0, \tag{14}$$

  then we evaluate (13) at $r = r_s$.
- For the Minkowski embedding, the regularity of the gauge fields at r_0 requires $f_{rt}(r_0) = \tilde{A}'_t(r_0) = 0$, therefore

$$\sigma^{ij} = -\mathcal{N}' C_{q-3}(r_0)\epsilon^{ij} = \mathcal{N}' \frac{\hat{J}_t}{\tilde{B}}\epsilon^{ij}, \tag{15}$$

General assumptions

- The logic of 3+1 dimensions is the same as that of 2+1 dimensions.
- The induced metric and gauge fields

$$\begin{aligned} ds^2 &= g_{tt}dt^2 + \sum_{i=1}^{3} g_{ii}dx_i^2 + g_{rr}dr^2 + ds_{(I)}^2, \\ \tilde{A} &= \tilde{A}_t(r)dt + \tilde{B}_y z dx + \tilde{B}_z x dy + \tilde{B}_x y dz + \tilde{a}, \end{aligned} \quad (16)$$

where the field theory directions are t, x, y, z and $I = q - 4$.

- We keep all the components of the magnetic field for generality.

The conductivity

The conductivity

$$\sigma^{ii} = \frac{\mathcal{N}'}{g_5^2}\sqrt{\frac{s}{s_{tt}s_{rr}}}\frac{1}{s_{ii}}\Big|_{r\to r_s},$$
$$\sigma^{ij} = -\mathcal{N}' Q_k \epsilon^{kij}\Big|_{r\to r_s}, \qquad (17)$$

where

$$g_5^2 = \frac{\sqrt{-s}}{e^{-\phi}\sqrt{-\det\gamma_{mn}}\sqrt{\Theta}}, \quad \epsilon_{txyzr} = 1$$
$$Q_l = -\frac{1}{8}e^{-\phi}\sqrt{-\det\gamma_{mn}}\sqrt{\Theta}\epsilon_{mnpql}\theta^{mn}\theta^{pq} = \frac{\tilde{B}_l g_{ll}\hat{J}_t}{\kappa}. \qquad (18)$$

D3-D7' model

- The model was proposed in arXiv:1003.4965[hep-th] (Bergman, Jokela, Lifshytz and Lippert).
- The configuration

	0	1	2	3	4	5	6	7	8	9
D3	•	•	•	•						
D7	•	•	•		•	•	•	•	•	

- The configuration is non-supersymmetric and unstable.

D3-D7' model Cont'd

- To ensure the stability, we assume that D7-brane wraps $S^2 \times S^2$ inside S^5 and we introduce the following magnetic fluxes through S^2's

$$\tilde{F} = \frac{1}{2}(f_1 d\Omega_2^{(1)} + f_2 d\Omega_2^{(2)}), \quad f_i = 2\pi\alpha' n_i, \tag{19}$$

  $d\Omega_2^{(i)} = \sin\theta_i \wedge d\phi_i, n_i$ are integers.
- The gauge field

$$\tilde{A} = \tilde{A}_t dt + \tilde{B} x dy,$$

- Assuming that the scalars $z(= x_3)$ and $\psi(= x_9)$ are functions of r only,

D3-D7' model Cont'd

the induced metric and the RR 4-form

$$\begin{aligned} ds^2_{D7} &= r^2(-f(r)dt^2 + dx^2 + dy^2) + (\frac{1}{r^2 f(r)} + r^2 z'^2(r) \\ &\quad + \psi'^2(r))dr^2 + \cos^2\psi (d\Omega_2^{(1)})^2 + \sin^2\psi (d\Omega_2^{(2)})^2, \\ C_4 &= r^4 dt \wedge dx \wedge dy \wedge dr + \frac{1}{2}c(r) d\Omega_2^{(1)} \wedge d\Omega_2^{(2)}, \end{aligned} \tag{20}$$

where $f(r) = 1 - r_H^4/r^4$ and

$$c(r) = \frac{1}{8\pi^2}\int_{S^2\times S^2} C_4 = \psi(r) - \frac{1}{4}\sin 4\psi(r) - \psi(\infty) + \frac{1}{4}\sin 4\psi(\infty). \tag{21}$$

D3-D7' model Cont'd

For black hole embedding, the conductivity reads

$$\sigma^{xx} = \frac{\mathcal{N}' r_s^2}{\tilde{B}^2 + r_s^4} \times \sqrt{\bar{J}_t^{\,2} + \left(\cos^4\psi + \frac{1}{4} f_1^2\right)\left(\sin^4\psi + \frac{1}{4} f_2^2\right)(\tilde{B}^2 + r_s^4)},$$

$$\sigma^{xy} = -\mathcal{N}' \left(\frac{\tilde{B}\bar{J}_t(r_s)}{\tilde{B}^2 + r_s^4} + \frac{c(r_s)}{2} \right), \tag{22}$$

where $\bar{J}_t = \hat{J}_t + c(\psi)/2 \times \tilde{B}$.
For Minkowski embedding,

D3-D7' model Cont'd

$$\sigma^{xx} = 0, \quad \sigma^{xy} = -\frac{1}{2}\mathcal{N}' c(r_0) = \mathcal{N}' \frac{\hat{J}_t}{\tilde{B}}, \tag{23}$$

- The results obtained by OSM method agree to arXiv: 1003.4965, where the real-action method was used.
- Black hole embedding-metal phase, Minkowski embedding-fractional QHE phase.

Motivations

Properties of “strange” metals (obtained from experiments):

- the resistivity $\rho \sim T$,
- the AC conductivity $\sigma(\omega) \sim \omega^{-0.65}$,
- the Hall angle $\frac{\sigma^{xy}}{\sigma^{xx}} \sim \frac{1}{T^2}$

Point 1 can be realized in many holographic models while point 3 cannot be realized without introducing spatial anisotopic symmetry.

1 and 3 can be realized simultaneously by considering probe D7-branes in light-cone Schwarzschild-AdS(Kim, Kiritsis and Panagopoulos’10).

AdS space in light-cone frame

Such a metric can be obtained by the transformation $x^+ = b(t+x), x^- = 1/(2b)(t-x)$.

$$
\begin{aligned}
ds^2 &= g_{++}dx^{+2} + 2g_{+-}dx^+dx^- + g_{--}dx^{-2} \\
&\quad + g_{yy}dy^2 + g_{zz}dz^2 + g_{rr}dr^2 \\
&\quad + R^2\cos^2\theta d\Omega_3^2 + R^2\sin^2\theta d\phi^2, \qquad (24)
\end{aligned}
$$

$$
\begin{aligned}
g_{++} &= \frac{(1-f(r))r^2}{4b^2R^2},\ g_{+-} = -\frac{1+f(r)r^2}{2R^2},\ g_{--} = \frac{(1-f(r))b^2r^2}{R^2}, \\
g_{yy} &= g_{zz} = \frac{r^2}{R^2},\ g_{rr} = \frac{R^2}{r^2f(r)},\ f(r) = 1-\frac{r_H^4}{r^4}, \qquad (25)
\end{aligned}
$$

where R is AdS_5 radius and b is the parameter related to the rapidity.

The conductivity from OSM

Here the OSM is off-diagonal

$$ds^2 = s_{tt} dt^2 + s_{rr} dr^2 + s_{xx} dx^2 + 2s_{tx} dtdx + s_{yy} dy^2 + s_{zz} dz^2,$$

The conductivity (Kim, Shock and Tarrio, '11)

$$\begin{aligned} \sigma^{ii} &= \frac{\mathcal{N}'}{g_5^2} \frac{\sqrt{-s}}{\sqrt{s_{rr}}\sqrt{-s_{tt}s_{xx} + s_{tx}^2}} \frac{\sqrt{s_{xx}}}{s_{ii}}\Big|_{r\to r_s}, \\ \sigma^{ij} &= -\mathcal{N}' Q_k \epsilon^{kij}\Big|_{r\to r_s}, \end{aligned} \qquad (26)$$

which reproduces the results in Kim, Kiritsis and Panagopoulos'10 .

Summary and discussion

- We studied the holographic DC conductivities of various systems using the OSM method.
- We proposed a new method to compute the DC conductivity based on OSM. We showed that all results obtained by the OSM method agreed to the results obtained by the real-action method.
- OSM can be used to study other transport coefficients and effective temperature induced by the effective world volume horizon, contrary to the real-action method.

Thank you for your attention!