IV Black Holes Workshop
Universidade de Aveiro

Regular black holes

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Aveiro, 19 December 2011

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1. Introduction

- Solutions of Einstein's equations and black holes

· From Einstein's equation ($G = 1\,, c = 1$)

$$G_{\mu\nu} = 8\pi\,\tau_{\mu\nu}$$

need to find solutions.
· Arbitrarily chosen spacetimes $g_{\mu\nu}$ usually give unphysical stress tensors, i.e., to matter which is of no interest. Finding solutions is a nontrivial task (Exact solutions book by Stephani et al, 2nd edition 2002).
· Facilitates finding solutions for two regions, an interior and an exterior, and then match through a smooth junction, a boundary surface (Israel NC 1966).
· Can also opt for a more drastic junction between both regions where a surface layer, i.e., a thin shell, is needed. Usually the solderings are through timelike surfaces, as in a surface of a star. Formalism applies also to spacelike surfaces. For a lightlike surface can extend (Barrabès and Hogan book 2003).

1. Introduction

· When $\tau_{\mu\nu} = 0$, vacuum solution. The Schwarzschild black hole is a vacuum solution. Spherically symmetric, has an event horizon at $r_{\rm h} = 2m$. Represents a wormhole, with two phases, the white hole and the black hole, harboring singularities and connecting two asymptotically flat universes (Misner, Thorne and Wheeler book 1973). In its amputated form, the solution represents a black hole shielding a singularity, with one asymptotically flat region, the black hole being formed from the collapse of a star or lump of matter. Generalizes to the Reissner-Nordström solution when there is q, to the Kerr solution when there is J, to the Kerr-Newman family when there is q and J (see also Griffiths and Podolsky book 2009).

· The outside of a black hole is visible. Potent telescopes and detectors watch with ease what is going on in jets and phenomena powered out by black holes. The outside of a black hole is well known classically (Stewart and Walker 1973). Quantically, black holes still pose problems for the outside. Related to the Hawking radiation and the Bekenstein-Hawking entropy. Although solution not at hand, the quantum outside problems are well posed and delineated.

1. Introduction

· The inside of a black hole is another story, it is not known at all. By definition the black hole interior is hidden, it encloses a mysterious unknown.
· The understanding of the inside of a black hole is one of the outstanding problems in gravitational theory.
· The Schwarzschild solution describes the black hole inside as an ever moving spacetime that ends on an all encompassing spacelike singularity.
· The Reissner-Nordström solution also has an ever moving inward spacetime that, instead, ends on a Cauchy horizon which can then be cruised into a region where a singularity can be seen but avoided. The Kerr and the Kerr-Newman solutions have analogous properties to the Reissner-Nordström solution.
· The event horizon for this class of black hole solutions harbors a singularity. What is a singularity? The singularity theorems (Penrose PRL 1965, Penrose 1978) do not tell. Impose some precise physical conditions. Then the theorems prove generically singularities are inevitable. But those precise physical conditions might not be upheld in the situations they are to be used, so the theorems are useless.

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· The existence of a singularity, by its very definition, means spacetime ceases to exist signaling a failure of the physical laws.

· So, if physical laws do exist at those extreme conditions, singularities should be substituted by some other object in a more encompassing theory. The extreme conditions, in one form or another, that may exist at a singularity, imply that one should resort to quantum gravity. Singularities are certainly objects to be resolved in the realm of quantum gravity (Wheeler 1964).

· There is no definite quantum gravity yet, so a line of work to understand the inside of a black hole and resolve its singularity is to study classical or semiclassical black holes, with regular, i.e., nonsingular, properties. These type of black holes can be motivated by quantum arguments. In this way, there has been a trend to find regular black hole solutions with special matter cores that would substitute the true singularities of the Schwarzschild, Reissner-Nordström, Kerr, and Kerr-Newman black holes.

1. Introduction

- Early considerations

· Sakharov (JETP 1966) and Gliner (JETP 1966) proposed that singularities, such as cosmological singularities, could be avoided by matter at superhigh densities with an inflationary equation of state, i.e., with a de Sitter core, with a matter equation of state $p = -\rho_{\rm matter}$, or, equivalently, $T_{\mu\nu}$ takes a lambda term or false vacuum form $T_{\mu\nu} = \Lambda g_{\mu\nu}$, Λ the cosmological constant. Zel'dovich (Sov. Phys. Usp. 1968) proposed that such a $T_{\mu\nu}$ arises naturally as a result of vacuum polarization processes in gravitational interactions.
· This indicates that an unlimited increase of spacetime curvature during a collapse process can lead to the halt of the collapse if quantum fluctuations dominate the process, putting an upper bound to the value of the curvature and obliging the formation of a central core.

1. Introduction

- The Bardeen regular black hole

Bardeen (Proc. 1968) realized concretely the idea of a central matter core, by proposing a solution of Einstein's equation in which there is a black hole with horizons but without a singularity, the first regular black hole. The matter field content was a kind of magnetic matter field, yielding a modification of the Reissner-Nordström metric. But near the center the solution tended to a de Sitter core solution. All the subsequent regular black hole solutions are based on Bardeen's proposal, although there has been a tremendous development on the implementation and on the analysis of the properties of regular black hole solutions.

1. Introduction

- Other regular black holes

· A useful way to classify the regular black hole solutions is 1. No junction: solution is continuous throughout spacetime. 2. Two simple regions: solution has boundary surfaces joining the two regions. 3. Two regions: more drastic, the solution has a surface layer, i.e., thin shell, joining the two regions.

1. Solutions with continuous fields

· Based on Gliner (1966, 1975) on how to avoid cosmological singularities, Dymnikova (GRG 1992) proposed a black hole model in which the core is de Sitter and gives way in a smooth manner into a Schwarzschild solution, with Cauchy and event horizons. Several subsequent works developing this idea followed (Dymnikova 1996, 2000, 2001, 2003, 2004, 2005, 2010, Gliner 1998).

· Next, Ayón-Beato and Garcia (PLB 2000, GRG 2005) invoked nonlinear fields and sources to generate from first principles the Bardeen model as a nonlinear magnetic monopole, (also attempted regular black holes from nonlinear electric fields (PRL 2000), criticized in Bronnikov (PRL 2000, PRD2001), Matyjasek (PRD2004) found the extremal limit)).

1. Introduction

· Bronnikov and collaborators (Fabris, Dehnen, Melnikov, Dymnikova, PRL 2006, GRG 2007, CQG 2007) produced several regular black holes in which the source are fields permeating the whole spacetime, the core is an expanding universe with de Sitter asymptotics and the exterior outer region tends to Schwarzschild. Matyjasek, Tryniecki, and Klimek (MPLA 2008) made a development along the same lines.

· Regular black holes in quadratic gravity have also been discussed by Berej, Matyjasek, Tryniecki, and Woronowicz, (GRG 206).

2.Solutions with boundary surfaces

Can construct regular black holes by filling the inner space with matter up to a certain surface and make a smooth junction, through a boundary surface, to the Schwarzschild solution as was done in (Mars CQG 1996, Magli RMP 1999, Elizalde and Hildebrandt PRD 2002, Conboy and Lake PRD 2005). The junction to Schwarzschild is made through a spacelike surface, rather than an usual timelike surface. This means the junction exists at a single instant of time. Regular black holes in which the boundary surface is lightlike or timelike have not been found in the literature.

1. Introduction

3. Solutions with boundary layers, i.e., thin shells

It is possible and of interest to make the transition from an inner de Sitter core to an outer Schwarzschild, Reissner-Nordström, or other spacetime, through surface layers, or thin shells. Regular black holes with thin shells of spacelike, lightlike, and timelike character have been found.

(a) Spacelike thin shells

· Following Zel'dovich's idea (1968), Markov (AP 1984) suggested an upper bound for the curvature, of the order of the Planck curvature. After it is achieved the matter turns into a de Sitter phase. The transition is made through a spacelike thin shell. It was developed by Lake and Zannias (PLA 1989), Frolov, Markov, and Mukhanov (PRD 1990), Balbinot and E. Poisson (PRD 1990), Balbinot (PRD 1990), Morgan (PRD 1991), Barrabes and V. P. Frolov (PRD "How many new worlds are inside a black hole?" 1996).

· See also Burinskii, Elizalde, Hildebrandt, and Magli (PRD 2002) for a general discussion including the Kerr-Newman metric.

1. Introduction

(b) Lightlike thin shells

· Even before Dymnikova (GRG 1992) developed her regular black hole with smooth features, Gonzalez-Diaz (LNC 1981) took interest in finding a regular black hole. He tried a solution by direct matching of de Sitter spacetime with the Schwarzschild solution on the horizon, a null surface.

· Shen and Zhu (PLA 1989) reanalyzed later this soldering of de Sitter spacetime with the Schwarzschild solution, while Shen and Tan in 1989 (PLA 1989) generalized the Gonzalez-Diaz idea to d dimensions. Daghigh, Kapusta, Hosotani (Arxiv 2000) argued that a Schwarzschild type matching can also be achieved within a more general parametrization of the static metric by two different functions due to the jump of the product $g_{tt}g_{rr}$.

· However, Gron and Soleng (LNC 1985, PLA 1989) showed that the direct matching onto Schwarzschild at the horizon is incorrect.

· Poisson and Israel (CQG 1988) reinforced this de Sitter spacetime cannot be soldered directly to an exterior Schwarzschild vacuum at the horizon, since the junction conditions would be violated. It is necessary to put a thin shell of noninflationary material at a junction outside the event horizon.

1. Introduction

· Gal'tsov and Lemos (CQG 2001) showed in a no-go theorem that the more general tentative matching proposed in Daghigh, Kapusta, Hosotani (Arxiv 2000) is also not possible (see also Bronnikov (PRD 2001) for more on this).
· Additional tries of the same type of matching, now extending to the Reissner-Nordström spacetime, were performed in by Shen and Zhu (GRG 1985, NC 1985). By including charge the matching problems occurring in a Schwarzschild matching may be avoided. Barrabès and Israel (PRD 1991) gave an example where there is the possibility of joining correctly at a null surface and gave interesting examples of a lightlike thin shell matching at the Cauchy horizon (see also (Barrabès and Hogan book 2003) for null matching).

(c) Timelike thin shells

For regular black holes with boundary layers or thin shells, timelike matching is not found in the literature. So it is of interest to study regular black hole solutions in such a case. Regular black holes either with a charged (usually magnetic) core or with a de Sitter core are known, but with electric charge and a de Sitter core together seem to have not been explored. To study such cases is a local motivation within the larger context.

1. Introduction

- General results on regular black holes

· Results related to the topology and causality of these solutions, were put forward by Borde in an important development (PRD 1994, PRD 1997)
· Also energy conditions and other properties have been studied by Mars, Martín-Prats, and Senovilla (CQG 1996) and Zaslavskii (PRD 2009, PRD 2010).
· The quasilocal energy of regular black holes has been analyzed by Balart (PLB 2010). Entropy and thermodynamics of regular black holes have been studied by Myung, Kim, and Park (PLB 2010, GRG 2008).

- Reviews on regular black holes

For a general review on regular black holes, including black holes with Gaussian sources, see (Ansoldi arXiv 2008), and for a motivation of these sources as well as a review on noncommutative black holes see Nicolini (IJMPA 2009).

1. Introduction

- Connections to other works

· An issue connected to regular black holes is quasiblack holes. Quasiblack holes are objects whose boundary is as near a horizon as one wants. For the outside they act as black holes, the inside properties are completely different (Lemos, Zaslavskii PRD 2000-2011). Based on a worked by Guilfoyle (GRG 1999) solutions of quasiblack holes with pressure, i.e., of relativistic charged spheres as frozen stars, have been found (Lemos, Zanchin PRD 2011). These solutions contain, unexpectedly, regular black holes. This is under study.

· There are interesting investigations on the dynamics of time-dependent bubbles, in which an outer observer describes the system as having a horizon and a black hole, and an observer in the inner region, made of false vacuum, sees a de Sitter universe (Blau, Guendelman, Guth PRD 1987, Berezin, Kuzmin, Tkachev PRD 1987, Alberghi, Lowe, Trodden JHEP 1999).

· Related to the inside of a black hole is mass inflation (Poisson and Israel, PRD 1990). The internal Cauchy horizon is unstable and a spacelike or null singularity emerges inside a charged Reissner-Nordström black hole.

· Black holes, and in particular charged black holes, singular or regular, as elementary charged particles is an issue in itself.

1. Introduction

- Our work

The main motivation is to have a clue of what the inside is. There are no solutions with timelike boundary. So we explored it in two papers. Then we resort to nonminimal theories and to stability:
· Lemos, Zanchin (PRD 2011) - Regular black holes - Electric charged in a thin shell.
· Lemos, Zanchin (TBP 2012) - Regular black holes - Electric charged Guilfoyle solutions (a plethora).
· Balakin, Lemos, Zayats (TBP 2012) - Regular black holes - Non-minimal Einstein–Yang-Mills theories.
· Flachi, Lemos (TBP 2012) - Regular black holes - quasinormal modes and stability.
I will briefly mention each work.

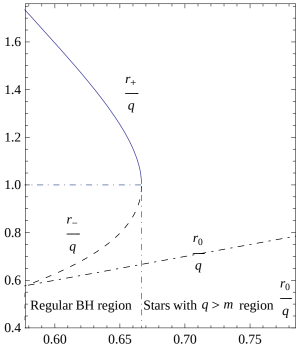
2. Regular black holes with electric charge in Einstein-Maxwell theory

Lemos, Zanchin (PRD 2011)

- The set up

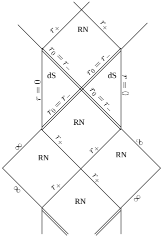
· The idea: a de Sitter core, an electric coat (thin-shell), a Reissner-Nordström spacetime outside. Implies: if there are horizons, the matter is inside the Cauchy horizon, boundary is timelike (like in a star).

· The configurations.



2. Regular black holes with a thin electric charge in Einstein-Maxwell theory

- Carter-Penrose diagrams



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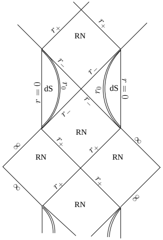

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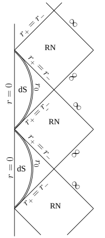

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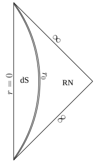


(iv)

2. Regular black holes with a thin electric charge in Einstein-Maxwell theory

- Several features

· (i) For a range of parameters, the solutions are thus regular electrically charged black hole solutions. They are built from false vacuum up to, but not at, r_0. The metric for $r < r_0$ is the de Sitter metric, where the isotropic pressure is constant ($p(r) = -\rho_{\rm m}(r) = 3/8\pi R^2$), and goes to zero at r_0. Furthermore, since the charge density $\rho_{\rm e}(r)$ is a Dirac delta function centered in $r = r_0$, the total charge q is distributed uniformly on the surface $r = r_0$. At r_0 there is thus a thin electrical layer of an energyless field, and exterior to it is pure Reissner-Nordström, with two horizons at r_- and r_+.

· (ii) The limit of zero charge of these solutions is a Minkowski spacetime, rather than a Schwarzschild spacetime.

· (iii) These regular charged black hole solutions have boundaries which are either timelike or, in one instance, lightlike.

· (iv) If the charge q is the elementary charge, i.e., the electron charge e, then the radius r_0 of the particle is of the order of the Planck radius and the mass m is of the order of the Planck mass. The solution could then be a model for a heavy elementary charged particle.

3. Regular black holes in Einstein-Maxwell with matter theory: Guilfoyle's solution

Lemos, Zanchin (TBP 2012)

- The solutions

The cold charged pressure fluid is bounded by a spherical surface of radius $r = r_0$, and in the electrovacuum region, for $r > r_0$, the metric and the electric potentials are given by extremal Reissner-Nordström solution.

For the inside: $$ds^2 = -B(r)\,dt^2 + A(r)\,dr^2 + r^2\,d\Omega\,.$$

Gauge field is: $$\mathscr{A}_\mu = -\phi(r)\,\delta^0_\mu\,, \qquad U_\mu = -\sqrt{B(r)}\,\delta^0_\mu\,.$$

Try,

$$B(r) = a\left[-\varepsilon\,\phi(r) + b\right]^2\,,$$

$$8\pi\,\rho_{\mathrm{m}}(r) + \frac{Q^2(r)}{r^4} = \frac{3}{R^2}\,,$$

$$A(r) = \left(1 - \frac{r^2}{R^2}\right)^{-1}\,,$$

where R is a constant to be determined by the junction conditions of the metric at the surface $r = r_0$.

3. Regular black holes in Einstein-Maxwell with matter theory: Guilfoyle's solution

Joining

$$\frac{1}{R^2} = \frac{1}{r_0^3}\left(2m - \frac{q^2}{r_0}\right) .$$

$$B(r) = \left[\frac{2-a}{a^{1+1/a}}\,F(r)\right]^{2a/(a-2)} ,$$

$$8\pi\rho_{\rm m}(r) = \frac{3}{R^2} - \frac{a}{(2-a)^2}\,\frac{k_0^2\,r^2}{F^2(r)} ,$$

$$Q(r) = \frac{\varepsilon\sqrt{a}}{2-a}\,\frac{k_0\,r^3}{F(r)},$$

$$8\pi p(r) = -\frac{1}{R^2} + \frac{a}{(2-a)^2}\,\frac{k_0^2\,r^2}{F^2(r)} + \frac{2k_0\,a}{2-a}\,\frac{\sqrt{1-r^2/R^2}}{F(r)} ,$$

3. Regular black holes in Einstein-Maxwell with matter theory: Guilfoyle's solution

where k_0 is an integration constant, and $F(r)$ and $Q(r)$ are defined respectively by

$$F(r) = k_0 R^2 \sqrt{1 - \frac{r^2}{R^2}} - k_1 \,,$$

$$Q(r) = 4\pi \int_0^r \rho_e(r) \frac{r^2 dr}{\sqrt{1 - \frac{r^2}{R^2}}} = \frac{r^2}{\sqrt{B(r)}} \sqrt{1 - \frac{r^2}{R^2}} \frac{d\phi(r)}{dr} \,,$$

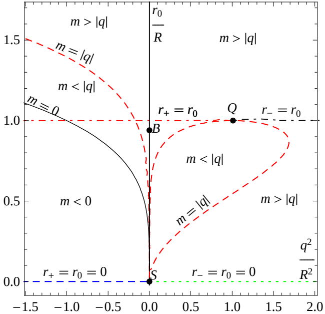
with k_1 being another integration constant. The integration constants k_0 and k_1 are determined by using the continuity of the metric potentials $A(r)$ and $B(r)$ and the first derivative of $B(r)$ with respect to r at the boundary $r = r_0$. The result is

$$k_0 = \frac{|q|\, a^{2/a}}{r_0^3} \left(\frac{m}{q} - \frac{q}{r_0} \right)^{1-2/a} ,$$

$$k_1 = \sqrt{1 - \frac{r_0^2}{R^2}} \left[k_0 R^2 - \frac{a^{1+1/a}}{2-a} \left(1 - \frac{r_0^2}{R^2} \right)^{-1/a} \right] .$$

3. Regular black holes in Einstein-Maxwell with matter theory: Guilfoyle's solution

- The plethora of solutions diplayed



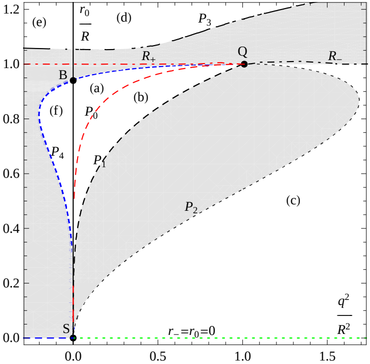
Region (i): Stars, dust stars, tension overcharged stars, regular black holes with timelike matching to exterior Reissner-Nordström

Region (ii): Regular black holes with spacelike matching to exterior Reissner-Nordström

Region (iii): Regular black holes with spacelike matching to exterior Reissner-Nordström

Region (iv): Gravitational instantons

3. Regular black holes in Einstein-Maxwell with matter theory: Guilfoyle's solution



Region (a): Charged stars. Region (b): Overcharged tension stars. Region (c): Regular black holes with timelike boundary. Region (d): Regular black holes with spacelike boundary. Line R_-: Extremal regular black holes. Curve P_0: Charged dust stars. Point Q: The quasiblack hole with pressure. Line SB: Neutral stars. Point B: The Buchdahl limit. Point S: Schwarzschild black holes. Line $r_0 = 0$: Special cases.

4. Regular black holes in non-minimal Einstein–Yang-Mills theories

Balakin, Lemos, Zayats (TBP 2012).

- The action and equations

The action is

$$S_{\text{NMEYM}} = \int d^4x\sqrt{-g}\,\left\{\frac{R+2\Lambda}{8\pi} + \frac{1}{2}F^{(a)}_{ik}F^{ik}_{(a)} + \frac{1}{2}\mathscr{R}^{ikmn}F^{(a)}_{ik}F_{(a)\,mn}\right\}.$$

The nonminimal susceptibility tensor $\mathscr{R}^{ikmn}$ is

$$\mathscr{R}^{ikmn} \equiv \frac{q_1}{2}R\,(g^{im}g^{kn} - g^{in}g^{km}) + \frac{q_2}{2}(R^{im}g^{kn} - R^{in}g^{km} + R^{kn}g^{im} - R^{km}g^{in}) + q_3R^{ikmn}\,.$$

We consider a Yang-Mills field taking values in the Lie algebra of the gauge group $SU(2)$

$$\mathbf{A}_m = -i\,\mathbf{t}_{(a)}A^{(a)}_m\,, \quad \mathbf{F}_{mn} = -i\,\mathbf{t}_{(a)}F^{(a)}_{mn}\,.$$

Here $\mathbf{t}_{(a)}$ are the Hermitian traceless generators of $SU(2)$ group.

4. Regular black holes in non-minimal Einstein–Yang-Mills theories

Static spherically symmetric space-time with the metric

$$ds^2 = \sigma^2 N dt^2 - \frac{dr^2}{N} - r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right) .$$

The gauge field has the special ansatz

$$\mathbf{A}_0 = \mathbf{A}_r = 0 , \quad \mathbf{A}_\theta = i\mathbf{t}_\varphi , \quad \mathbf{A}_\varphi = -i\nu \sin\theta \, \mathbf{t}_\theta .$$

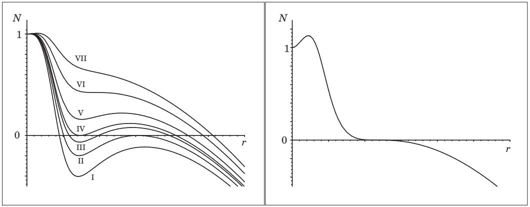
The parameter ν is a non-vanishing integer. The field strength tensor has only one non-vanishing component:

$$\mathbf{F}_{\theta\varphi} = i\nu \sin\theta \, \mathbf{t}_r .$$

Clearly, it is a magnetic type solution. The solution has arbitrary Λ, q_1, q_2 and q_3.

4. *Regular black holes in non-minimal Einstein–Yang-Mills theories*

- Solutions



Plots for the metric function $N(r)$ for nonminimal regular models with $\Lambda > 0$, $3q_1 + q_2 = q > 0$, $N(0) = 1$: All have the de Sitter asymptotic behavior, traps near the center and repulsive barriers. On panel (a) the curve II has a maximum coinciding with a double horizon, while the curve IV has a minimum coinciding with a double horizon. All plots here have a regular center and a sophisticated causal structure. There are nonminimal regular black holes with the Cauchy, event and cosmological horizons. In certain cases the plots describe regular nonminimal stars inside a cosmological horizon. One specific curve with a triple horizon of the cosmological type is displaced on the panel (b). On this plot four specific points coincide: the inflexion point and three zeros of the function $N(r)$.

5. Regular black holes, quasinormal modes and stability

Flachi and Lemos (TBP 2012).

- The action and solutions

$$S = \frac{1}{16\pi}\int d^4x\sqrt{g}\left(R - \mathscr{L}(F)\right) ,$$

where R is the scalar curvature and $\mathscr{L}(F)$ is a nonlinear function of the electromagnetic field strength with $F = F_{\mu\nu}F^{\mu\nu}/4$. In most examples $\mathscr{L}$ reproduces Maxwell's theory in the weak field limit.
Regular black hole solutions are presented in the form of spherically symmetric geometries,

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega_2^2 ,$$

where the lapse function $f \equiv f(r)$ depends on the specific form of $\mathscr{L}$ and on the parameters of the nonlinear electrodynamics. For example, the regular solution of Ayon-Beato and Garcia (PLB 1998) has f given by

$$f = 1 - \frac{2mr^2}{(r^2+q^2)^{3/2}} + \frac{q^2r^2}{(r^2+q^2)^2} .$$

5. Regular black holes, quasinormal modes and stability

This is obtained by introducing a nonlinear electrodynamics with Lagrangian density,

$$\mathscr{L} = \frac{X^2}{-2q^2}\frac{1-8X-3X^2}{(1-X)^4} - \frac{3m}{2q^3}\frac{X^{5/2}(3-2X)}{(1-X)^{7/2}},$$

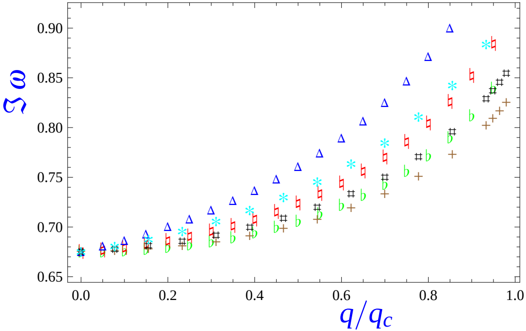
where $X = \sqrt{-2q^2F}$, and m and q are associated to mass and charge. All the solutions considered in this paper are summarized in the Table.

Lapse function	Ref.
$f = 1 - \frac{2mr^2}{r^3+2\alpha^2}$	(Hayward, PRL 2006)
$f = 1 - \frac{2mr^2}{(r^2+q^2)^{3/2}} + \frac{q^2r^2}{(r^2+q^2)^2}$	(Ayon-Beato-Garcia, PRL 1998)
$f = 1 - \frac{2mr^2}{(r^2+q^2)^{3/2}}$	(Ayon-Beato-Garcia, PLB 2000)
$f = 1 - \frac{4m}{\pi r}\left(\tan^{-1}\frac{r}{r_0} - \frac{rr_0}{r^2+r_0^2}\right)$	(Dymnikova CQG 2004)
$f = 1 - \frac{2m}{r}\left(1-\tanh\frac{r_0}{r}\right)$	(Bronnikov PRD 2001, Matyjasek et al GRG 2006)
$f = 1 + \frac{cr^2}{b^2} + \frac{\rho_0 r^2}{b^3}\left(\frac{b\sqrt{r^2-b^2}}{r^2} + \tan^{-1}\frac{\sqrt{r^2-b^2}}{b}\right)$	(Bronnikov, Fabris PRL 2006)

5. Regular black holes, quasinormal modes and stability

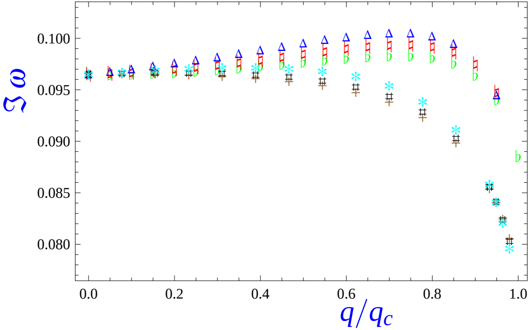
- The modes

Perturb the solution with a charged scalar field, $q_{\text{perturbation}}$.



The figure illustrates the behavior of the Real part of the QNF for $l = 3$ and $n = 0$ in the RN case and in the regular black hole case (Ayon-Beato). The symbols refer to: $q_{\text{perturbation}} = 0\,(+),\ 0.1\,(\sharp),\ 0.3\,(*)$ for the regular case, and $q_{\text{perturbation}} = 0\,(\flat),\ 0.1\,(\natural),\ 0.3\,(\Delta)$ for the RN case.

5. Regular black holes, quasinormal modes and stability



The figure illustrates the behavior of the Imaginary part of the QNF for $l = 3$ and $n = 0$ in the RN case and in the regular black hole case (Ayon-Beato).
The symbols refer to: $q_{\text{perturbation}} = 0\,(+),\ 0.1\,(\sharp),\ 0.3\,(*)$ for the regular case, and $q_{\text{perturbation}} = 0\,(\flat),\ 0.1\,(\natural),\ 0.3\,(\Delta)$ for the RN case.
The regular black hole is stable against perturbations.

6. Conclusions

· We have shown that we can make some progress in some understanding of the black hole interior.