

# Black hole dynamics in non-asymptotically flat spacetimes

(Work in progress, Phys.Rev.D81:084052, Phys.Rev.D82:104037)

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Portugal

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- 1 Motivation
- 2 Black holes in a box
- 3 Black holes in de Sitter
- 4 Black holes on a 5D cylinder
- 5 Final remarks

# Outline

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# Why numerical relativity

## Study of systems with strong and dynamical gravitational fields

- Gravitational radiation
  - Astrophysics, gravitational wave astronomy
- Mathematical and theoretical Physics:
  - Cosmic censorship
  - Instabilities (Black hole interior, Myers-Perry)
- High-energy particle systems:
  - AdS/CFT correspondence;
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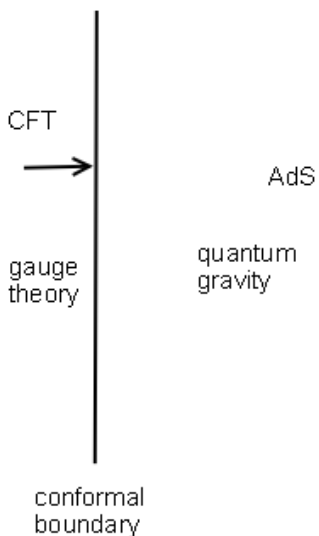
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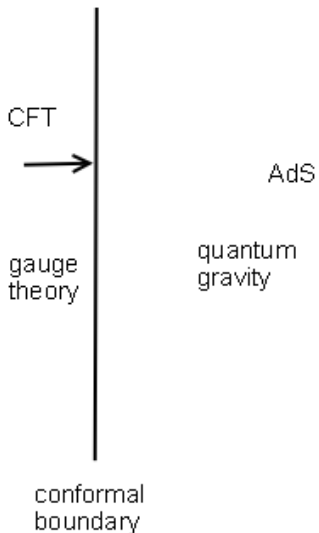


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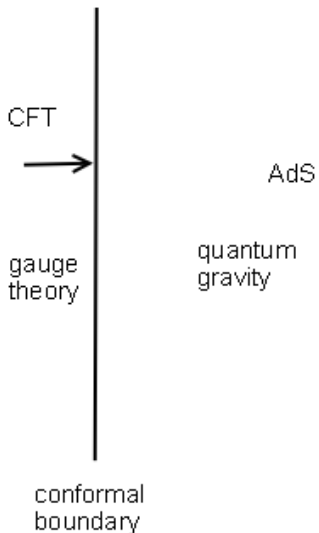


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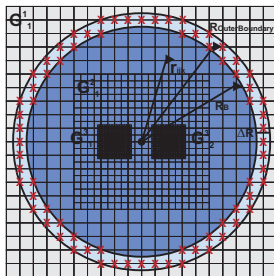
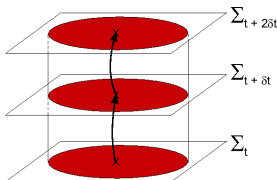
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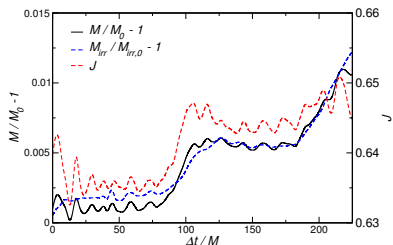
# Black holes in a box

Witek et al, Phys.Rev.D82:104037

- puncture initial data (equal-mass, non-spinning BHs)
- BSSN evolution scheme
- impose reflecting boundary conditions
- inspiraling BHB  $\Rightarrow$  spinning BH  
head-on collision  $\Rightarrow$  non-spinning BH



# Results

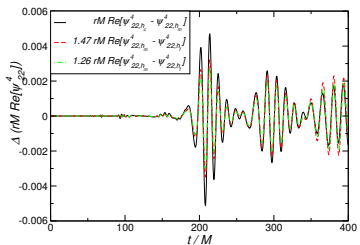


Time evolution of the BH mass

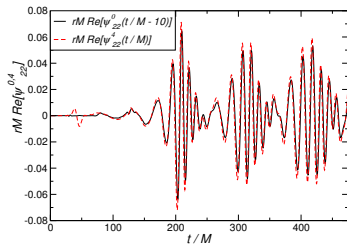
## Inspiral

- increasing mass and area of AH
- absorption of  $\approx 15\%$  of radiated energy per cycle

# Results



Convergence analysis



Real part of the  $l = m = 2$  mode of  $rM\Psi_0$  and  $rM\Psi_4$

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- Accelerated bodies emit gravitational radiation
- Detected **indirectly** by measurements of the Hulse-Taylor binary system (1993 Nobel Prize)
- Interact weakly with matter  $\Rightarrow$  carry unique information about astronomical phenomena
  - $\Rightarrow$  New window to the universe
- Observations suggest we live in an approximately de Sitter Universe;
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# Formalism

## Einstein equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 0$$

## Evolution equations

$$(\partial_t - \mathcal{L}_\beta)\gamma_{ij} = -2\alpha K_{ij}$$

$$(\partial_t - \mathcal{L}_\beta)K_{ij} = -D_i\partial_j\alpha + \alpha\left({}^{(3)}R_{ij} - 2K_i^k K_{jk} + K_{ij}K - \Lambda\gamma_{ij}\right)$$



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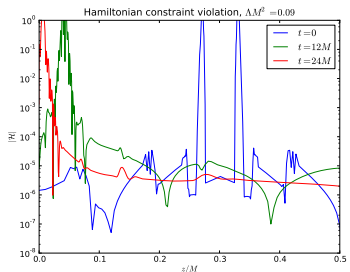
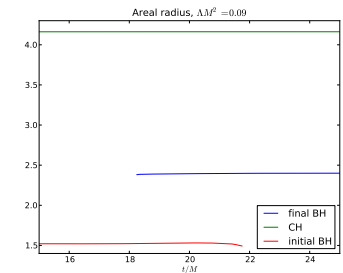
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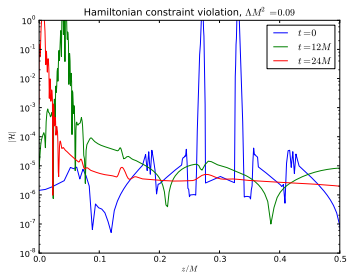
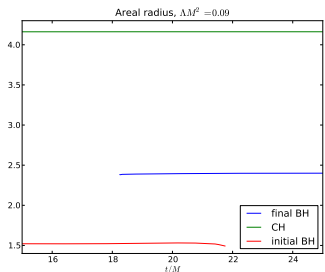
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- Evolution is stable and the constraints are preserved;
- Successfully monitor the apparent horizons;

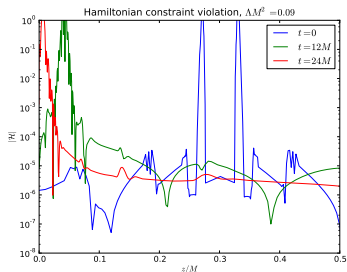
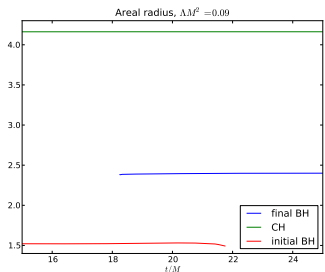
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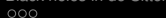


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- arise in gauge/gravity duality and braneworld scenarios;
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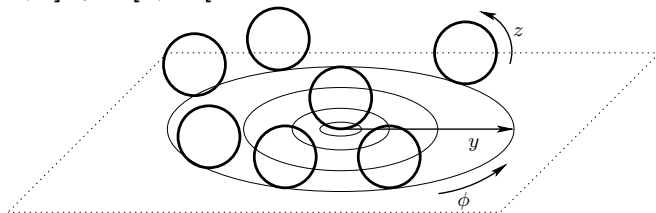
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# $D = 5$ black holes on a cylinder

In the absence of black holes, we have  $\mathbb{M}^{1,3} \times \mathcal{S}^1$ :

$$ds^2 = \underbrace{-dt^2 + dx^2 + dy^2 + y^2 d\phi^2}_{\mathbb{M}^{1,3}} + \underbrace{dz^2}_{\mathcal{S}^1}$$

$z \in [-L, L]$ ,  $\phi \in [0, 2\pi[$





# Formalism

Most general metric element

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + \lambda d\Omega_{D-4}^2$$

$\mu = 0, 1, 2, 3.$

$D$ -dimensional vacuum Einstein equations imply

$$R_{\mu\nu} = \frac{D-4}{2\lambda} \left( \nabla_\mu \partial_\nu \lambda - \frac{1}{2\lambda} \partial_\mu \lambda \partial_\nu \lambda \right)$$

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The resulting system is

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→ effective 3 + 1 system with source terms

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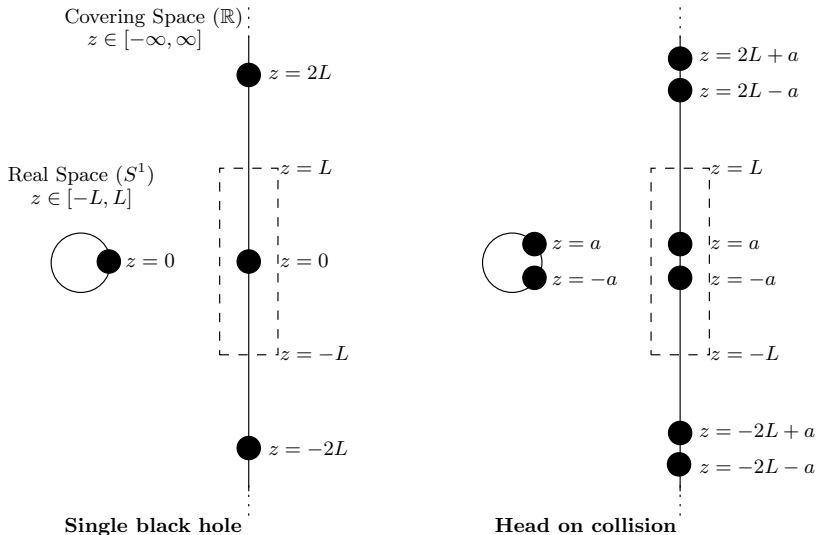
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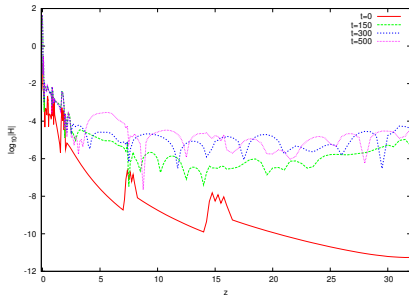
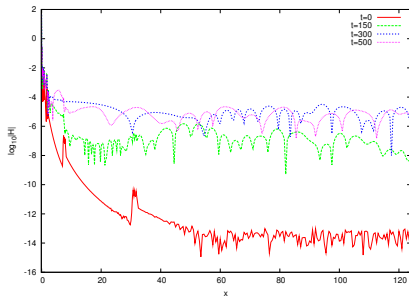
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# Initial data

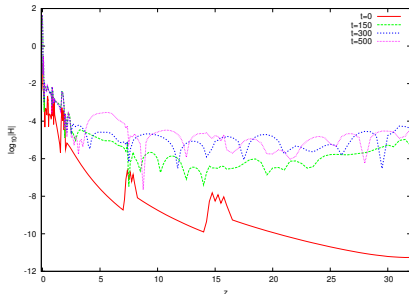
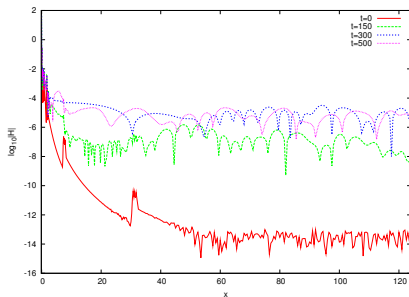


# $L = 32$ single black hole evolution – constraint violation



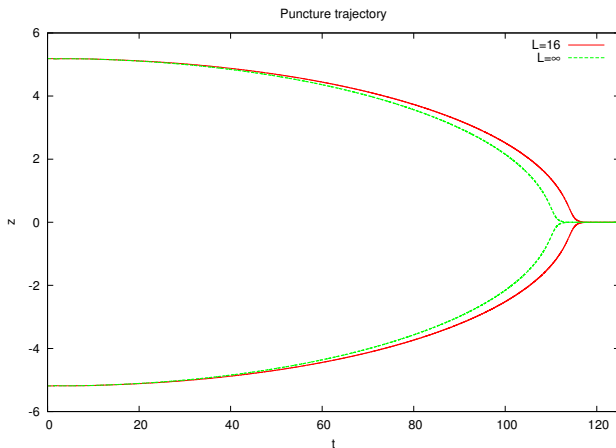
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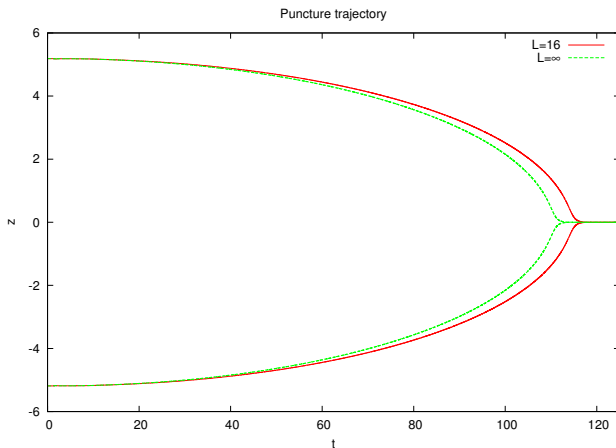
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# $L = 16$ head-on collision – trajectory



→ longer collision time for the cylindrical case

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# Conclusions

- BH in a box
  - Studied stability of rotating BH and gravitational radiation in boxed spacetime;
  - Results consistent with expectations for wavepacket of radiation travelling back and forth
- BH in de Sitter
  - Evolved head-on collision of BHs in asymptotically de Sitter spacetime and monitored apparent horizons;
  - ToDo: Study formation of common apparent horizon as function of initial separation;
- 5D cylinder
  - Successfully evolved head-on collision of BHs in a 5D cylindrical spacetime using dimensional reduction procedure;
  - ToDo: deformation of the apparent horizon; radiated energy; smaller compactification radius;

# The group



<http://blackholes.ist.utl.pt/>



the end