Inverse scattering construction of dipole black rings

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based on:

arXiv:1108.3527 with M. J. Rodriguez and A. Virmani (published in JHEP 11 (2011));
and ongoing work with M. J. Rodriguez and O. Varela.

Outline

1. Introduction
2. Review: rod structure & inverse scattering method
3. Inverse scattering construction of a dipole ring
4. Conclusion

Exact black hole solutions: a non-trivial pursuit

- Gravity in higher dimensions has attracted much attention in recent years.

  review by [Emparan & Reall, 2008]

- Gravity in higher dimensions is much richer:

  – multiple rotations;
  – non-spherical topologies;
  – non-uniqueness.

- The discovery of black rings in 5D [Emparan & Reall, 2001] brought a lot of excitement.

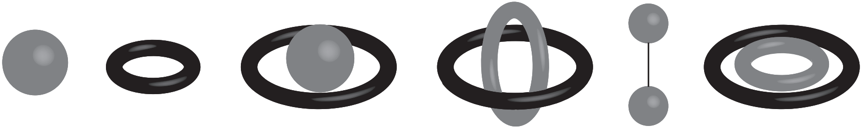
  horizon topology: $S^1 \times S^2$
  (a higher dimensional "bolo rei")

- Since then, many other exact BH solutions in 5D have been discovered.

Known solutions

- Focus on asymptotically flat solutions. (Also stationary and bi-axisymmetric.)

  Some known exact solutions in 5D vacuum gravity:



Myers-Perry black rings black saturn bicycling black ring double MP black di-ring

- All these solutions can be (and have been) generated using the Inverse Scattering Method.
  [Pomeransky, 2006], [Tomizawa et al., 2006], [Tomizawa & Nozawa, 2006], [Pomeransky & Senkov, 2006], [Elvang & Figueras, 2007], [Elvang & Rodriguez, 2008], [Herdeiro et al., 2008], [Evslin & Krishnan, 2009]

  The ISM is a powerful solution-generating technique to construct new solutions by adding rotation to simpler known solutions. (More later.)

- Including gauge fields: dipole rings, charged rings, supersymmetric rings. . .

  At present the most general regular, asymptotically flat solutions have 3 parameters.
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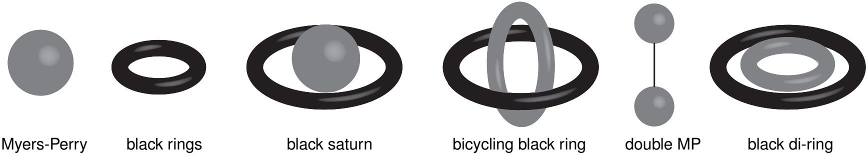

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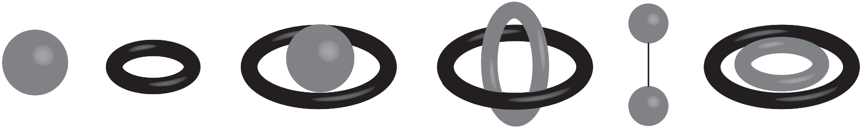

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Goal and motivation

- Motivation: a 5-parameter (mass, two angular momenta, electric charge and dipole charge) family of black rings has been conjectured to exist. [Elvang, Emparan & Figueras, 2005]

  — The original dipole black ring solution was constructed using educated guesswork. It is not understood how to systematically generate dipole charges. [Emparan, 2004]

  — There exists an algorithmic construction of a dipole black ring solution. However, it cannot generate multiple rotations. . . [Yazadjiev, 2006]

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Construct a dipole black ring in 5D Einstein-Maxwell-dilaton (using the ISM in 6D).

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Canonical form of the metric

- Consider stationary, axisymmetric solutions of Einstein eqs. in vacuum:

$$R_{\mu\nu} = 0\,.$$

- Assume $D-2$ commuting Killing vector fields, $\partial/\partial x^i$.

  Then metric can be written in canonical form: [Wald, 1984] [Emparan & Reall, 2002] [Harmark, 2004]

$$ds^2 = \sum_{i,j=0}^{D-3} G_{ij}(\rho,z)dx^i dx^j + e^{2\nu(\rho,z)}\left[d\rho^2 + dz^2\right]\,, \qquad \det G = -\rho^2\,,$$

- Metric only depends on coordinates (ρ, z) and has block diagonal form:

$$g_{\mu\nu} = \left(\begin{array}{c|cc} G_{ij} & \multicolumn{2}{c}{0} \\ \hline 0 & e^{2\nu} & 0 \\ & 0 & e^{2\nu} \end{array}\right)$$

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For G_{ij}:

$$\partial_\rho U + \partial_z V = 0\,,$$

where $\quad U \equiv \rho(\partial_\rho G)G^{-1}, \quad V \equiv \rho(\partial_z G)G^{-1}\,.$

For ν:

$$\partial_\rho \nu = -\frac{1}{2\rho} + \frac{1}{8\rho}\mathrm{Tr}(U^2 - V^2)\,,$$

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Static, axisymmetric solutions [Weyl, 1917] [Emparan & Reall, 2002]

- Obtaining static (diagonal) solutions is straightforward. Writing

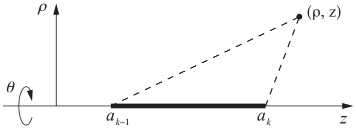
$$G = \text{diag}\{-e^{2U_0}, e^{2U_1}, e^{2U_2}, \dots\},$$

the problem reduces to finding $D-2$ solutions, $U_i(\rho, z)$, of the Laplace equation in an auxiliary (cylindrically symmetric) 3D flat space:

$$\nabla^2 U_i = 0\,, \qquad ds^2_{aux} = d\rho^2 + \rho^2 d\theta^2 + dz^2\,.$$

- Boundary conditions: zero-thickness rods act as sources for the Newtonian potentials U_i. E.g., for a finite rod:

$$U_i(\rho, z) = \frac{1}{2}\log(\mu_{k-1}/\mu_k)$$



- The potentials are entirely specified by the location of the rod endpoints, a_k. These appear in combinations known as solitons and anti-solitons:

$$\mu_k = \sqrt{\rho^2 + (z-a_k)^2} - (z-a_k)\,, \qquad \overline{\mu_k} = -\sqrt{\rho^2 + (z-a_k)^2} - (z-a_k)\,.$$

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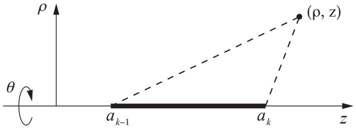

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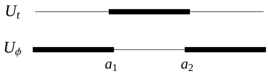
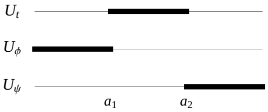
- The constraint $\det G = -\rho^2$ translates into $\sum_i U_i = \log \rho$.

  Meaning: sources must add up to give an infinite rod.

- Some examples:



4D Schwarzschild



5D Tangherlini

Conclusion

Vacuum solutions of the Einstein equations with $D-2$ orthogonal commuting KVFs are fully determined by rod-like sources, only subject to the above constraint. [Emparan & Reall, 2002]

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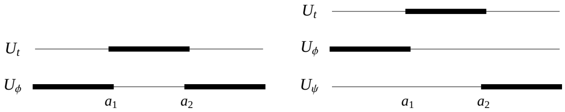

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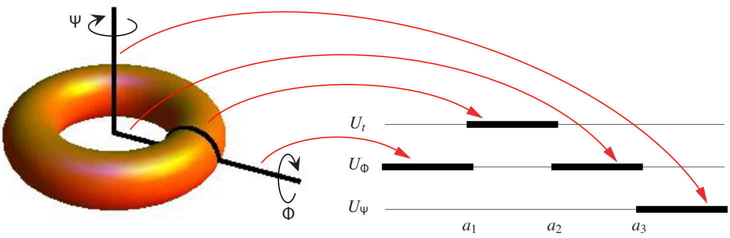
- Some thumb rules:

finite timelike rods $\longrightarrow$ event horizons
semi-infinite spacelike rods $\longrightarrow$ axes of rotation



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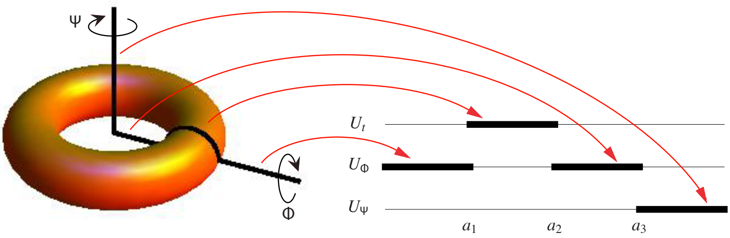

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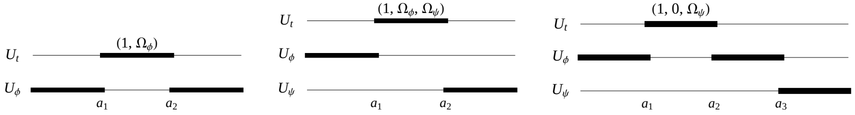

4D Kerr — 5D Myers-Perry — singly rotating black ring

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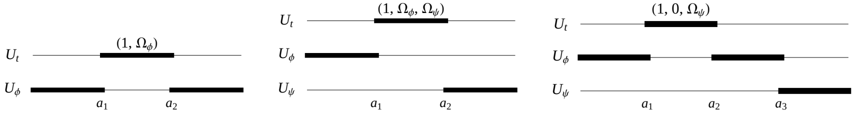

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The inverse scattering method [Belinski & Zakharov, 1979]

- The BZ approach consists in replacing the original (non-linear) equation for $G(\rho, z)$ by a system of linear equations (Lax pair) for a generating matrix $\Psi(\lambda, \rho, z)$, such that

$$\Psi(\lambda = 0, \rho, z) = G(\rho, z)\,.$$

New solutions are obtained by dressing the generating matrix Ψ_0 of a known seed G_0.

The BZ algorithm

If the seed is diagonal (static) and the 'dressing' procedure is restricted to the class of solitonic transformations, then *the whole scheme is purely algebraic*.

Input needed: the positions of the solitons a_k and the (constant) BZ vectors $m_0^{(k)}$.

Note: if the BZ vectors mix the time and spatial Killing directions, then this procedure yields a *rotating* version of the original static solution.

- Issue: Generically, after a solitonic transformation $\det G \neq -\rho^2$.

  Solution: [Pomeransky, 2006]

  The determinant of the new metric is independent of the BZ vectors.
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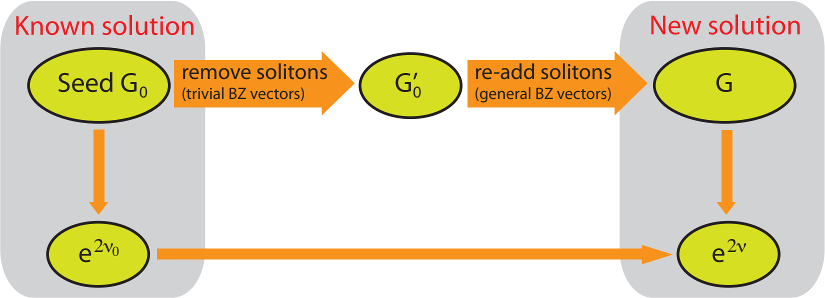

- Notes: – The seed solution need not be regular.
  – Might need to impose some constraints to generate a regular solution.

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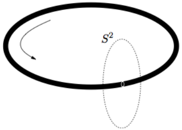
The set-up

- Consider 5D Einstein-Maxwell-dilaton theory, governed by the action

$$S = \frac{1}{16\pi G_N}\int d^5x\sqrt{-g}\left(R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4}e^{-a\phi}F_{\mu\nu}F^{\mu\nu}\right)\,, \qquad \text{with} \quad a = \frac{2\sqrt{2}}{\sqrt{3}}\,.$$

- The five-dimensional theory naturally supports magnetic one-branes and dipole black rings.

  Can define a local charge by $\mathcal{Q} = \frac{1}{4\pi}\int_{S^2} F\,.$



from [Emparan, 2004]

- This action can also be obtained from 6D vacuum gravity by performing a Kaluza-Klein reduction on S^1 using the ansatz

$$ds_6^2 = e^{\frac{\phi}{\sqrt{6}}}ds_5^2 + e^{-\frac{\sqrt{3}\phi}{\sqrt{2}}}(dw + A)^2.$$

  The sixth dimension is parametrized by w and $F = dA$.

- Strategy: Construct the dipole ring solution of this theory by applying the ISM in 6D and then reducing to 5D.

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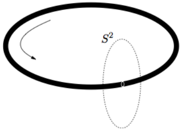

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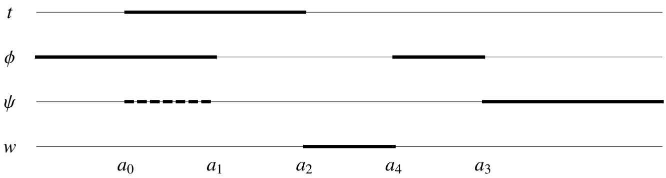
Seed metric [JVR, Rodriguez & Virmani, 2011]

The seed is taken to be the following static (diagonal) metric:



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- This solution is singular and not of direct physical interest, but satisfies $\det G_0 = -\rho^2$.
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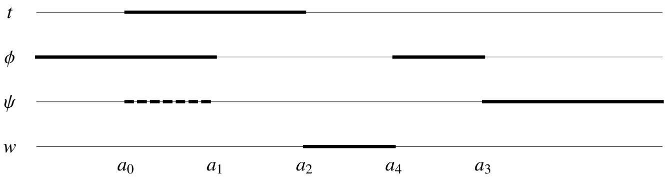

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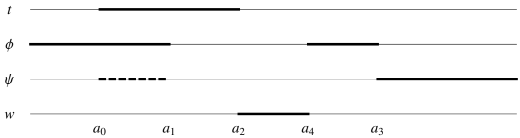
Soliton transformations [JVR, Rodriguez & Virmani, 2011]



We generate the 6D uplift of the dipole ring solution by a 2-soliton transformation:

1. Perform two 1-soliton transformations on the seed G_0 to obtain G_0':
   — remove an anti-soliton at $z = a_0$ with trivial BZ vector $(1, 0, 0, 0)$;
   — remove a soliton at $z = a_4$ with trivial BZ vector $(0, 0, 0, 1)$.

2. Perform now a 2-soliton transformation on G_0' to obtain G:
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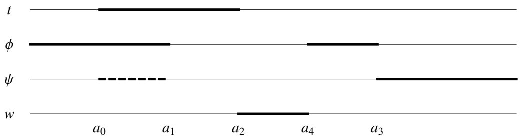
3. Construct $e^{2\nu}$. The result $(G, e^{2\nu})$ is the 6D solution we want.

Appropriately tuning c_1 and c_2 and KK reducing along the w direction we obtain the smooth 5D dipole black ring solution of the theory under consideration.

Soliton transformations [JVR, Rodriguez & Virmani, 2011]



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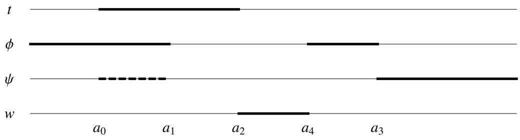

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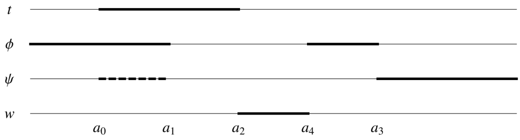

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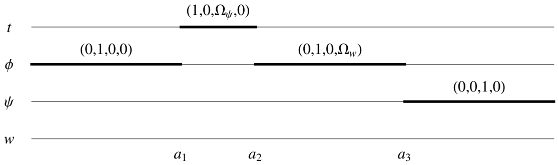
We arrive at a metric described by the following rod diagram:



$$\Omega_\psi = \sqrt{\frac{(a_1 - a_0)}{2(a_2 - a_0)(a_3 - a_0)}}\,,$$

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- The parameter c_1 must be fixed to avoid a divergence as $z \to a_0$ along the rod $(-\infty, a_1]$. Equally, c_2 must be fixed so that the rod along w merges with the finite rod along ϕ.
- The general solution has a conical deficit, but the balanced solution is regular.
- Parameter counting:

$$\underbrace{\#a_i}_{5} + \underbrace{\#c_j}_{2} - \underbrace{(\text{translational invariance in z})}_{1} - \underbrace{(\text{regularity conditions})}_{2} - \underbrace{(\text{balance condition})}_{1} = 3$$

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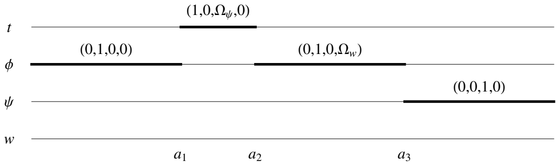

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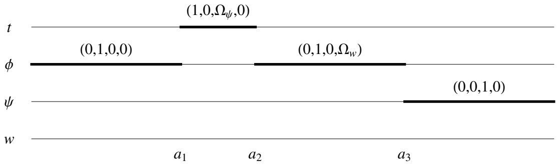

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Dipole black ring solution [JVR, Rodriguez & Virmani, 2011]

To confirm we have indeed reproduced the dipole ring solution we have to:

1. Convert from Weyl canonical coordinates (ρ, z) to ring coordinates (x, y);
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We obtain precise agreement with the 5D line element ds_5^2, the vector potential A and the dilaton ϕ of [Emparan, 2004].

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Outline

1. Introduction
2. Review: rod structure & inverse scattering method
3. Inverse scattering construction of a dipole ring
4. Conclusion

Conclusion and Outlook

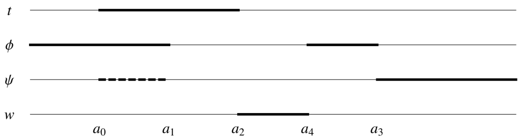
- Summary: We have re-derived the dipole ring solution of 5D Einstein-Maxwell-dilaton theory (with a specific coupling constant).

- Take-home message: The ISM can be used to generate dipole charge.

- Possible extensions: generating more general black rings in the above mentioned theory.

  E.g., adding an independent rotation or multi-horizon solutions. (*Work in progress.*)

  — Start with same seed but perform a 4-soliton transformation to generate dipole charge, rotation on the S^1 and rotation on the S^2.



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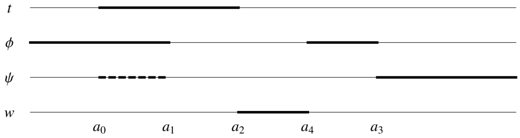

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