Constraining Grand Unification Scenarios using the First and Second Generation Sfermion Masses

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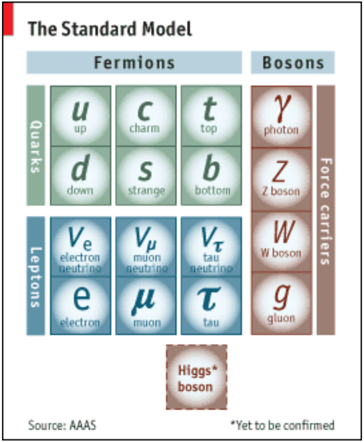
Hypothesis of Grand Unification

All forces and all matter become **one** at high energies regardless of how different they behave at low energy (apart from gravity)

The Standard Model of Particle Physics

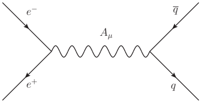


QED

- Quantum Electrodynamics (QED) is a Relativistic Quantum Field Theory describing the electromagnetic interaction
  - Phenomena involving electrically charged particles interacting by photon exchange
- Extremely well tested theory with a remarkable agreement with experiments (Lamb shift, hyperfine splitting, cross sections...)

QED Lagrangian and covariant derivative

$$\begin{aligned} \mathscr{L}_{QED} &= \overline{\psi}\left(i\gamma^{\mu}D_{\mu}-m\right)\psi-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ &= \overline{\psi}\left(i\gamma^{\mu}\partial_{\mu}-m\right)\psi-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\underbrace{q_e\overline{\psi}\gamma^{\mu}\psi A_{\mu}}_{\text{interaction term}} \quad , \\ D_{\mu} &= \partial_{\mu}+iq_eA_{\mu} \quad \text{and} \quad F_{\mu\nu}=\partial_{\mu}A_{\nu}-\partial_{\nu}A_{\mu} \end{aligned}$$



- Local gauge invariance: redefinition of the fields at every point without changing the physics
  - $\mathscr{L}_{QED} = \mathscr{L}'_{QED}$
    - $\psi \to \psi' = \exp(iQ\alpha(x))\psi$
    - $A_{\mu} \to A'_{\mu} = A_{\mu} - \partial_{\mu}\alpha(x)$
- Electromagnetic gauge (phase rotation $\alpha(x)$) forms the abelian group $U(1)_Q$
- **QED is a $\mathbf{U(1)}$ gauge theory**

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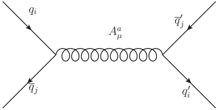
QCD

- Quantum Chromodinamics (QCD) is a Relativistic Quantum Field Theory describing the strong interaction between quarks and gluons
- Three kinds of charge (as opposed to one in QED) designated as ”colour charge”
  - Analogy with the three kinds of colours perceived by humans

QCD Lagrangian and covariant derivative

$$\mathscr{L}_{QCD} = \overline{\Psi}_i \left(i\gamma^\mu \left(D_\mu\right)_{ij} - m\delta_{ij} \right) \psi_j - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}$$

$$\left(D_\mu\right)_{ij} = \partial_\mu + i g_3 \left(T^a\right)_{ij} A^a_\mu$$



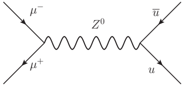
- Local gauge invariance $\mathscr{L}_{QCD} = \mathscr{L}'_{QCD}$: $\mathbf{U} = \exp(i g_3 \alpha_a(x) T^a)$
  - $\mathbf{U}$ are 3×3 complex unitary matrices, $\longrightarrow$ form a $SU(3)$ group
    - $\left[T^a, T^b\right] = i f^{abc} T^c \longrightarrow$ Non-abelian or non-comutative algebra
    - $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_3 f^{abc} A^b_\mu A^c_\nu$
    - Allows interactions between gauge fields (gluons) as opposed to QED eg: $-g_3 f^{abc} \partial_\mu A^a_\nu A^{\mu b} A^{\nu c}$
- **QCD is a non-abelian $\mathbf{SU(3)}$ gauge theory**

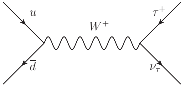
Weak Interactions and Electroweak Unification

- Weak force responsible for radioactive decay and triggers stellar nucleosynthesis (via β^+ decay)
- Phenomena involving the exchange of **massive** W and Z bosons
- Electromagnetic and weak forces very different at ”everyday” low energy
- **Above ~ 100 GeV ($\mathbf{T_{Universe}} > \mathbf{10^{15}}$ K) they merge/unify into a single electroweak force**

Electroweak Lagrangian

$$
\begin{aligned}
\mathscr{L}_{EW} &= \mathscr{L}_{scalar} + \frac{1}{4}\left(f_{\mu\nu}f^{\mu\nu} + F^k_{\mu\nu}F^{k\mu\nu}\right) + \mathscr{L}_{matter} \\
F^k_{\mu\nu} &= \partial_\mu W^k_\nu - \partial_\nu W^k_\mu + g\varepsilon^{ijk}W^i_\mu W^j_\nu \\
f_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu
\end{aligned}
$$





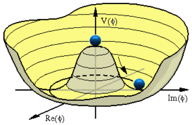
- Imposing local gauge invariance leads to a $\mathbf{SU(2)_L \otimes U(1)_Y}$ **gauge theory**
- Left-handed fields are weakly interacting
- Experiment tells us that weak bosons are massive! Is the theory actually gauge invariant?

Electroweak Symmetry Breaking and the Higgs Mechanism

- $SU(2)_L \otimes U(1)_Y$ prediction of massless gauge bosons is not seen experimentally
- Mass terms of the form $M_A^2 A_\mu A^\mu$ forbidden by gauge invariance
- Explicit mass terms for fermions of the form $\mathscr{L}_{mass} = -m\left(\overline{\psi}_L \psi_R + \overline{\psi}_R \psi_L\right)$ also violate gauge invariance
  - ψ_L and ψ_R transform differently under $SU(2)_L$
- **Problem of mass generation solved by the presence of a scalar filed ϕ, the Higgs field**
  - Expected to have a non-zero value in the *vacuum* state (min energy configuration of the Universe)
  - Higgs particle present in the vacuum in contrast with all other fundamental particles

$$\mathscr{L}_{scalar} = \left(D^\mu \phi\right)^\dagger \left(D_\mu \phi\right) - V(\phi), \quad \text{with} \quad \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad SU(2)_L \text{ doublet}$$

$$\langle \phi \rangle_0 = \begin{pmatrix} 0 \\ v \end{pmatrix} \quad \text{(vacuum)}$$



- Radial perturbations around the vacuum, $\phi = \begin{pmatrix} 0 \\ h+v \end{pmatrix}$, generate mass of all particles
- Electroweak symmetry is broken surviving a remnant one, QED
  - $SU(2)_L \otimes U(1)_Y \to U(1)_Q$

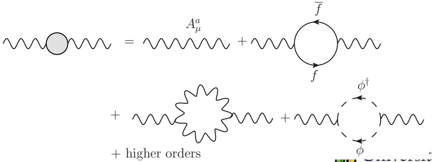
Evolution of Couplings with Scale

- From $\mathscr{L}_{SM}$ one obtains coupled non-linear equations
  - cannot be solved analytical
- SM observables calculated using perturbation theory
  - Series expansion in the couplings, $g_a, y_f, \cdots$
  - Pictorially done with **Feynman diagrams**
    - 'Loop' diagrams $\longrightarrow$ unobserved internal interaction with radiation and re-absorption of a particle
- Loop diagrams represent integrals that depend on the energy scale Q
- Different choices of $Q \longrightarrow$ **different values of the couplings**
- Variation described by the **Renormalization** Group Equations (RGE)

Evolution of a coupling

$$Q\frac{dg}{dQ} = \beta(g)$$

$\beta(g) \longrightarrow$ **β-function** may depend on other couplings, $\beta(g_a, y_f, \cdots)$



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The couplings and masses of G_{SM} "run" with Q

Motivations for Physics Beyond the Standard Model

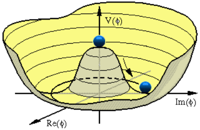
- For many years the SM proved to be the most accurate description of Particle Physics, however theoretical and experimental disagreements:
  - Neutrino oscillations require mass $\longrightarrow$ **not predicted** by the SM
  - Flavour symmetry not explained
  - Incompatible with the theory of General Relativity
  - No dark matter candidates
  - **Hierarchy problem**

The Hierarchy Problem

- The Higgs field expected to have a non-zero value, v, in the vacuum

$$V(\phi) = -\mu^2\phi^\dagger\phi + \lambda\left(\phi^\dagger\phi\right)^2 \quad \text{minimization} \rightarrow |\phi|^2 = \frac{\mu^2}{2|\lambda|} \equiv \frac{v^2}{2}$$

$$\langle\phi\rangle_0 = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix} \quad (\text{vacuum})$$



- Scale of SM masses set by v
- Radial perturbations around the vacuum, $h(x)$: $\phi = \begin{pmatrix}0\\ \frac{h+v}{\sqrt{2}}\end{pmatrix}$

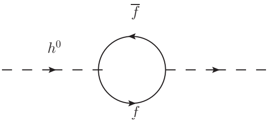
$$V(\phi) \longrightarrow \frac{1}{2}\left(2\mu^2\right)h^*h$$

- Higgs boson mass $m_{h^0} = 2\mu^2$
- **Just tree level so far...**

- The SM is a renormalizable theory
  - One can extend virtual momenta in loop integrals all the way to infinity
- New physics must be revealed at least at the Plank scale: $Q_p \sim 10^{19}$ GeV

One-loop corrections to the Higgs mass:

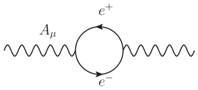
$$\Delta^1 m_{h^0} = \frac{\lambda_f^2}{8\pi^2}\left(-\Lambda^2 + 3m_f^2 \log\frac{\Lambda^2}{m_f^2} - 2m_f^2\right)$$

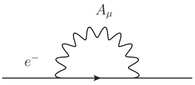


Λ is a cut-off scale (new physics expected)

- Correction to the Higgs mass will be quadratically divergent: $m^2_{h^0,phy} = m^2_{h^0} + \underbrace{\Delta^1 m_{h^0}}_{\sim -\Lambda^2}$
- $m_{h^0,phy}$ at the order of Q_{EW}
  - If new physics only at $Q_p \longrightarrow$ **Remarkable cancellation needed** (not natural)
- **Less severe if new physics at the low scale (500 GeV - few TeV)**
- **How to eliminate quadratic divergences?**

The Example of QED





Vacuum Polarization

- Correction to the photon mass
- Gauge invariance forbids photon mass
- Divergent but only **logarithmically**

Electron Self Energy

- Correction to the electron mass
- Chiral symmetry for fermions as their mass goes to zero
- Divergent but only **logarithmically**

- **Gauge and Chiral symmetries remove dangerous divergences**
- **SUSY associates...**
  - to each fermion a scalar (sfermion) $\longrightarrow$ **Chiral** supermultiplet
  - to each gauge boson a fermion (gaugino) $\longrightarrow$ **Gauge**/vector supermultiplet

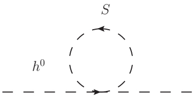
Quadratic Divergence Cancellation

- Consider a 1-loop correction to the Higgs propagator due to a scalar S:

  Extra correction term to the Higgs mass:

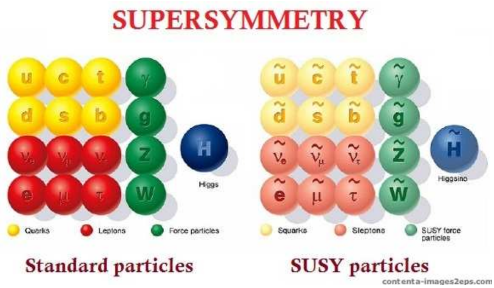
$$\Delta^2 m_{h^0} = \frac{\lambda_S}{16\pi^2}\left(\Lambda^2 - m_S^2 \log\frac{\Lambda^2 + m_S^2}{m_S^2}\right)$$

$$\Delta m_{h^0} = \Delta^1 m_{h^0} + \Delta^2 m_{h^0} = \frac{1}{8\pi^2}\left[\left(\frac{\lambda_S}{2} - \lambda_f^2\right)\Lambda^2 + \cdots\right]$$



- **Supersymmetry** (SUSY) requires $n_b = n_f$ in each supermultiplet
- $\frac{\lambda_S}{2} = \lambda_f^2$
- Dangerous quadratic divergences cancelled $\longrightarrow$ hierarchy stabilized
- Only Logarithmic dependence $\longrightarrow$ **SUSY solves the SM hierarchy problem**

Supersymmetry



- SUSY is a **global** space time symmetry
  - Contains the Poincaré algebra P^{μ}, $M^{\mu\nu}$
- If realized as a **local** symmetry $\longrightarrow$ P^{μ} vary from point to point
- Local SUSY is a **theory of gravity** $\longrightarrow$ SUPERGRAVITY

Motivations for GUTs: *The Idea of Grand Unification*

- The Standard Model of Strong and Electroweak interactions is described by the gauge group $G_{SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$
- The main idea is to embed G_{SM} into a larger simple group
  - $SU(N),\ SO(2N),\ SO(2N),\ SO(2N+1),\ Sp_{2N},\ G_2,\ F_4,\ E_6,\ E_7,\ E_8$
- We will consider standard $SU(5)$, $SO(10)$ and E_6 candidates

The RG Evolution of the Gauge Couplings in the SM: G_{SM} Charges

Matter fields spin $\frac{1}{2}$ (3 copies)

$Q_L = (\mathbf{3},\mathbf{2})_{\frac{1}{6}}$
$u_R^\dagger = (\overline{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}}$
$d_R^\dagger = (\overline{\mathbf{3}},\mathbf{1})_{\frac{1}{3}}$
$L = (\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$
$e_R^\dagger = (\mathbf{1},\mathbf{1})_1$

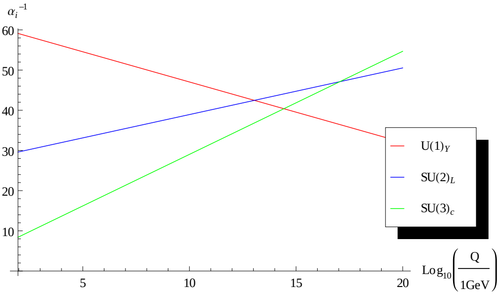
Higgs field spin 0 (1 copy)

$H_u = (\mathbf{1},\mathbf{2})_{\frac{1}{2}}$

Gauge fields spin 1

$g = (\mathbf{8},\mathbf{1})_0$
$W^{1,2,3} = (\mathbf{1},\mathbf{3})_0$
$B = (\mathbf{1},\mathbf{1})_0$

- Use these fields to study the RG evolution of the electroweak and strong gauge couplings
- At one-loop order: $\frac{d}{dt}\left(\alpha_i^{-1}\right) = -\frac{b_i}{2\pi}$ with $(b_1,b_2,b_3) = (44/10,-19/6,-7)$
- $b_N = \frac{11}{3}N - \frac{1}{3}n_f - \frac{1}{6}n_s$ for a generic $SU(N)$
- $b_1 = -\frac{2}{3}\sum_f X_f^2 - \frac{1}{3}\sum_S X_S^2$ for a generic $U(1)_X$
- $\alpha_i = \frac{g_i^2}{4\pi}$ (linear running)
- $t = \log\frac{Q}{Q_0}$



- Precise EW measurements dictate that gauge couplings do not meet within the SM
- Need something else to overcome this problem...
- This is an other motivation to go beyond the SM
- What if we include SUSY?

The MSSM RG Evolution: G_{SM} Charges

- The minimal extension of the particle content of the SM includes:

Squarks and Sleptons spin 0 (3 copies)

$\widetilde{Q}_L = (\mathbf{3},\mathbf{2})_{\frac{1}{6}}$

$\widetilde{u}_R^* = (\overline{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}}$

$\widetilde{d}_R^* = (\overline{\mathbf{3}},\mathbf{1})_{\frac{1}{3}}$

$\widetilde{L} = (\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$

$\widetilde{e}_R^* = (\mathbf{1},\mathbf{1})_1$

An extra Higgs doublet spin 0 (1 copy)

$H_d = (\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$

Higgsinos fields spin $\frac{1}{2}$ (1 copy)

$\widetilde{H}_u = (\mathbf{1},\mathbf{2})_{\frac{1}{2}}$

$\widetilde{H}_d = (\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$

Gauginos fields spin $\frac{1}{2}$

$\widetilde{g} = (\mathbf{8},\mathbf{1})_0$

$\widetilde{W}^{1,2,3} = (\mathbf{1},\mathbf{3})_0$

$\widetilde{B} = (\mathbf{1},\mathbf{1})_0$

- Use this **extended** particle content to study the RG flow of the electroweak and strong gauge couplings

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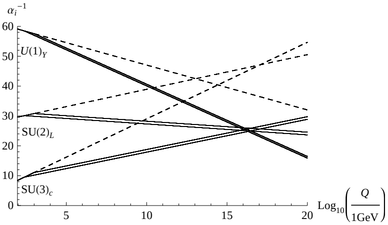
Running of the gauge couplings in the MSSM



$$\alpha_i^{-1}(t) = \alpha_i^{-1}(t_G) + \frac{b_i}{2\pi}(t_G - t) \qquad b_i = \begin{cases} (44/10, -19/6, -7) & \text{SM} \\ (33/5, 1, -3) & \text{MSSM} \end{cases}$$

- The gauge couplings tend to unify at a scale $Q_{GUT} \sim 1.2 \times 10^{16}\text{GeV}$
- SUSY mass thresholds in the interval $Q_{SUSY} \sim 250\text{GeV}$ and 1TeV
- Good reason towards Supersymmetric Grand Unified Theories

Some desirable properties for SUSY GUTs

- Flavor symmetry $\rightarrow$ Fermion mass hierarchy
- Natural explanation for neutrino masses (See-Saw mechanism)
- Charge quantization
- Proton stability
- Dark matter candidates (LSP)
- **SUSY GUTs: *natural extension of the SM***

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$SU(5)$ Grand Unification —$SU(5)$ Group Theory

$SU(5)$ is the simplest unification picture embedding G_{SM}

$SU(5) \supset SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$

- The $SU(5)$ operators U are 5×5 complex matrices such that $U^\dagger U = 1$ and $det(U) = 1$
- They may be represented by $U = exp(iT_a \omega_a)$ with T_a the generators
  - Gauge transformations on the fields
    - $\psi_i \rightarrow \psi_i' = \mathbf{U}\psi_i$
    - $\mathbf{A}_\mu \rightarrow \mathbf{A}_\mu' = \mathbf{U}\mathbf{A}_\mu\mathbf{U}^{-1} - \frac{i}{g_5}\partial_\mu \mathbf{U}\mathbf{U}^{-1}$
- $Tr(T_a) = 0$, $T_a^\dagger = T_a$, $a = 1, \ldots 24$
- The generators obey the commutation relation $[T_a, T_b] = i f_{abc} T_c$
- Choose the usual normalization $Tr(T_a T_b) = \frac{1}{2}\delta_{ab}$

The 24 $SU(5)$ generators

$$SU(3)_C: \quad T_{a_3} = \begin{pmatrix} \frac{1}{2}\lambda_{a_3} & 0 \\ 0 & 0 \end{pmatrix}, \; a_3 = 1, \ldots, 8$$

$$SU(2)_L: \quad T_{a_2} = \begin{pmatrix} 0 & 0 \\ 0 & \frac{1}{2}\sigma_{a_2-20} \end{pmatrix}, \; a_2 = 21, 22, 23$$

$$U(1)_Y: \quad T_{24} = \sqrt{\frac{3}{5}} \begin{pmatrix} -\frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

And 12 off-diagonal generators T_{a_4} with $a_4 = 9, \ldots, 20$

- 12 super-heavy gauge bosons $\longrightarrow$ mediate proton decay
- Highly suppressed by the GUT scale

- The **unified** $SU(5)$ covariant derivative may be written as $D^5_\mu = \partial_\mu + i g_U T_a \mathbf{G}^a_\mu$
- $g_U T_a G^a_\mu \supset g_s T_{a_3} \mathbf{G}^{a_3}_\mu + g T_{a_2} \mathbf{W}^{a_2}_\mu + g' \sqrt{\frac{5}{3}} T_{24} \mathbf{B}_\mu$

$SU(5)$ embedding of G_{SM}: *The* $\mathbf{10}$, $\overline{\mathbf{5}}$, $\mathbf{5}'$ *and* $\overline{\mathbf{5}}'$ *reps*

- The matter content of G_{SM} is unified in a $\overline{\mathbf{5}} \oplus \mathbf{10}$
- The two Higgs $SU(2)$ doublets are unified in a $\mathbf{5}'$ and a $\overline{\mathbf{5}}'$
  - Doublet-triplet splitting problem assumed to be solved by some mechanism (e.g. orbifold compactification) [Kawamura, 0012125]

The $\overline{\mathbf{5}}$ superpartners

$$\overline{\mathbf{5}} \to (\mathbf{1},\mathbf{2})_{-\frac{1}{2}} \oplus \left(\overline{\mathbf{3}},\mathbf{1}\right)_{\frac{1}{3}} = \widetilde{L} \oplus \widetilde{d}_R^*$$

The $\mathbf{10}$ superpartners

$$\mathbf{10} \to (\mathbf{1},\mathbf{1})_1 \oplus \left(\overline{\mathbf{3}},\mathbf{1}\right)_{-\frac{2}{3}} \oplus (\mathbf{3},\mathbf{2})_{\frac{1}{6}} = \widetilde{e}_R^* \oplus \widetilde{u}_R^* \oplus \widetilde{Q}_L$$

The $\mathbf{5}'$ Higgs

$$\mathbf{5}' \to (\mathbf{1},\mathbf{2})_{\frac{1}{2}} \oplus \left(\mathbf{3},\mathbf{1}\right)_{-\frac{1}{3}} = H_u \oplus (T_u)$$

The $\overline{\mathbf{5}}'$ Higgs

$$\overline{\mathbf{5}}' \to (\mathbf{1},\mathbf{2})_{-\frac{1}{2}} \oplus \left(\overline{\mathbf{3}},\mathbf{1}\right)_{\frac{1}{3}} = H_d \oplus (T_d)$$

$SO(10)$ embedding of G_{SM}: *The* $\mathbf{16}$ *and* $\mathbf{10}$ *reps*

Maximal subalgebra of $SO(10)$

$$SO(10) \to SU(5) \otimes U(1)_x$$

$\mathbf{16}$ and $\mathbf{10}$ branching rules

$$\mathbf{10} \to \mathbf{5}_2 \oplus \overline{\mathbf{5}}_{-2}$$
$$\mathbf{16} \to \mathbf{10}_{-1} \oplus \overline{\mathbf{5}}_3 \oplus \mathbf{1}_{-5}$$

From the branching rules of $SU(5)$ down to G_{SM} we see that:

- $\mathbf{10}$ contains the $SU(5)$ Higgs doublets and the colored Higgs triplets
- $\mathbf{16}$ contains the full $SU(5)$ superpartners and an extra singlet $\mathbf{1}_5$
- Extra abelian gauge group $U(1)_x$

Right handed sneutrino

$$\mathbf{1}_5 \to (\mathbf{1},\mathbf{1})_{(0,\ 5)} = \tilde{N}_R$$

- **A $SO(10)$ GUT naturally contains a right-handed neutrino/sneutrino**

E_6 embedding of G_{SM}: *The E_6SSM 27 representation*

We consider as E_6 SUSY GUTs the exceptional supersymmetric model E_6SSM
[King, Moretti and Nevzorov, 0510419, 0701064] [Athron, King, Miller, Moretti and Nevzorov, 0904.2169]

Maximal subalgebra of E_6

$$E_6 \to SO(10) \otimes U(1)_\psi$$

Branching rule for 27

$$\mathbf{27} \to \mathbf{1}_4 \oplus \mathbf{10}_{-2} \oplus \mathbf{16}_1$$

- E_6SSM predicts additional matter

Ordinary squarks and sleptons

$$\mathbf{10}_1 \to (\mathbf{3},\mathbf{2})_{(\frac{1}{6},\ 1)} \oplus (\overline{\mathbf{3}},\mathbf{1})_{(-\frac{2}{3},\ 1)} \oplus (\mathbf{1},\mathbf{1})_{(1,\ 1)} = Q_L \oplus \tilde{u}_R^* \oplus \tilde{e}_R^*$$

$$\overline{\mathbf{5}}_2 \to (\mathbf{1},\mathbf{2})_{(-\frac{1}{2},\ 2)} \oplus (\overline{\mathbf{3}},\mathbf{1})_{(\frac{1}{3},\ 2)} = L \oplus \tilde{d}_R^*$$

$$\mathbf{1}_0 \to (\mathbf{1},\mathbf{1})_{(0,\ 0)} = \tilde{N}_R$$

Higgs and exotics

$$\overline{\mathbf{5}}_{-3} \to (\mathbf{1},\mathbf{2})_{(-\frac{1}{2},\ -3)} \oplus (\overline{\mathbf{3}},\mathbf{1})_{(\frac{1}{3},\ -3)} = H_1 \oplus \overline{D}$$

$$\mathbf{5}_{-2} \to (\mathbf{1},\mathbf{2})_{(\frac{1}{2},\ -2)} \oplus (\mathbf{3},\mathbf{1})_{(-\frac{1}{3},\ -2)} = H_2 \oplus D$$

$$\mathbf{1}_5 \to (\mathbf{1},\mathbf{1})_{(0,\ 5)} = S$$

Soft Supersymmetry Breaking

- If SUSY exists it has to be an exact symmetry spontaneously broken (SSB) in a **Hidden sector** [Martin, 9709356]
- Many breaking scenarios proposed
- Parametrize the unknown realistic scenario of SSB
  - Introduce terms that explicitly break supersymmetry
  - Couplings should be of positive mass dimensions $\longrightarrow$ **renormalizable theory**, and given at the **low scale**
  - **SOFT TERMS**

Generic soft SUSY Lagrangian

$$\mathscr{L}_{soft} = -\left(\frac{1}{2}M_a\lambda^a\lambda^a + \frac{1}{6}a^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}b^{ij}\phi_i\phi_j + t^i\phi_i\right) + h.c. - \left(m^2\right)^i{}_j\,\phi^{j*}\phi_i$$

First and Second Generation Masses: *1-Loop RGEs*

[Ananthanarayan and Pandita, 0412125]

Squark and Slepton Soft Masses RGE

$$16\pi^2 \frac{dm^2_{\tilde{Q}_L}}{dt} = -\frac{32}{3} g_3^2 M_3^2 - 6 g_2^2 M_2^2 - \frac{2}{15} g_1^2 M_1^2 + \frac{1}{5} g_1^2 S$$

$$16\pi^2 \frac{dm^2_{\tilde{u}_R}}{dt} = -\frac{32}{3} g_3^2 M_3^2 - \frac{32}{15} g_1^2 M_1^2 - \frac{4}{5} g_1^2 S$$

$$16\pi^2 \frac{dm^2_{\tilde{d}_R}}{dt} = -\frac{32}{3} g_3^2 M_3^2 - \frac{8}{15} g_1^2 M_1^2 + \frac{2}{5} g_1^2 S$$

$$16\pi^2 \frac{dm^2_{\tilde{L}_L}}{dt} = -6 g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2 - \frac{3}{5} g_1^2 S$$

$$16\pi^2 \frac{dm^2_{\tilde{e}_R}}{dt} = -\frac{24}{5} g_1^2 M_1^2 + \frac{6}{5} g_1^2 S$$

- No Yukawa and trilinear couplings contributions → **possible to solve analytically**
- $t \equiv \log(Q/Q_0)$, $M_{1,2,3}$ running gaugino masses and $g_{1,2,3}$ are de usual G_{SM} gauge couplings
- S is a D-term contribution
  - $S \equiv Tr(Ym^2) = m^2_{H_u} - m^2_{H_d} + \sum_{generations} \left(m^2_{\tilde{Q}_L} - 2m^2_{\tilde{u}_R} + m^2_{\tilde{d}_R} - m^2_{\tilde{L}_L} + m^2_{\tilde{e}_R} \right)$
  - $\frac{dS}{dt} = \frac{66}{5} \frac{\alpha_1}{4\pi} S \Rightarrow S(t) = S(t_G) \frac{\alpha_1(t)}{\alpha_1(t_G)}$

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Solution of the RGEs

Squark and Slepton Running Masses

$$m^2_{\tilde{u}_L}(t) = m^2_{\tilde{Q}_L}(t_G) + C_3 + C_2 + \frac{1}{36}C_1 + \Delta_{u_L} - \frac{1}{5}K$$
$$m^2_{\tilde{d}_L}(t) = m^2_{\tilde{Q}_L}(t_G) + C_3 + C_2 + \frac{1}{36}C_1 + \Delta_{d_L} - \frac{1}{5}K$$
$$m^2_{\tilde{u}_R}(t) = m^2_{\tilde{u}_R}(t_G) + C_3 + \frac{4}{9}C_1 + \Delta_{u_R} + \frac{4}{5}K$$
$$m^2_{\tilde{d}_R}(t) = m^2_{\tilde{d}_R}(t_G) + C_3 + \frac{1}{9}C_1 + \Delta_{d_R} - \frac{2}{5}K$$
$$m^2_{\tilde{e}_L}(t) = m^2_{\tilde{L}_L}(t_G) + C_2 + \frac{1}{4}C_1 + \Delta_{e_L} + \frac{3}{5}K$$
$$m^2_{\tilde{\nu}_L}(t) = m^2_{\tilde{L}_L}(t_G) + C_2 + \frac{1}{4}C_1 + \Delta_{\nu_L} + \frac{3}{5}K$$
$$m^2_{\tilde{e}_R}(t) = m^2_{\tilde{e}_R}(t_G) + C_1 + \Delta_{e_R} - \frac{6}{5}K$$

- $C_i(t) = M_i^2(t_G)\left[A_i\frac{\alpha_i^2(t_G)-\alpha_i^2(t)}{\alpha_i^2(t_G)}\right] = M_i^2(t_G)\overline{c}_i(t),\ i = 1,2,3$ [Ananthanarayana and Pandita, 0706.2560]
- $K(t) = \frac{1}{2b_1}S(t_G)\left(1 - \frac{\alpha_1(t)}{\alpha_1(t_G)}\right)$
- $\Delta_\phi = M_Z^2(T_{3\phi} - Q_\phi \sin^2\theta_W)\cos 2\beta$
    - $SU(2)_L \otimes U(1)_Y \to U(1)_Q$ D-term

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Universal Boundary Conditions

- Common scalar mass $m^2_{\tilde{Q}_L}(t_G) = m^2_{\tilde{u}_R}(t_G) = m^2_{\tilde{d}_R}(t_G) = m^2_{\tilde{L}_L}(t_G) = m^2_{\tilde{e}_R}(t_G) = m_0^2$
- $m^2_{H_u} = m^2_{H_d}$
- Common gaugino mass $M_1^2(t_G) = M_2^2(t_G) = M_3^2(t_G) = M^2_{1/2}$
- Since $S(t_G) = 0$, then $S(t)$ is identically 0 at all scales, hence $K = 0$
- We are left with three unknowns: m_0, $M_{1/2}$ and $\cos 2\beta$
  - **Can be determined by measuring three sfermion masses, eg. $\tilde{u}_L$, $\tilde{d}_L$ and $\tilde{e}_R$**

$$\begin{pmatrix} M^2_{\tilde{u}_L} \\ M^2_{\tilde{d}_L} \\ M^2_{\tilde{e}_R} \end{pmatrix} = \begin{pmatrix} 1 & c_{\tilde{u}_L} & \delta_{\tilde{u}_L} \\ 1 & c_{\tilde{d}_L} & \delta_{\tilde{d}_L} \\ 1 & c_{\tilde{e}_R} & \delta_{\tilde{e}_R} \end{pmatrix} \begin{pmatrix} m_0^2 \\ M^2_{1/2} \\ \cos 2\beta \end{pmatrix}$$

- $\Delta_\phi \equiv \delta_\phi \cos 2\beta$
- $c_{\tilde{u}_L} \equiv \overline{c}_3(M_{\tilde{u}_L}) + \overline{c}_2(M_{\tilde{u}_L}) + \frac{1}{36}\overline{c}_1(M_{\tilde{u}_L})$
- $c_{\tilde{d}_L} \equiv \overline{c}_3(M_{\tilde{d}_L}) + \overline{c}_2(M_{\tilde{d}_L}) + \frac{1}{36}\overline{c}_1(M_{\tilde{d}_L})$
- $c_{\tilde{u}_L} \equiv \overline{c}_1(M_{\tilde{e}_R})$

Once m_0, $M_{1/2}$ and $\cos 2\beta$ determined through $M_{\tilde{u}_L}$, $M_{\tilde{d}_L}$ and $M_{\tilde{e}_R}$, it is possible to obtain all the other low scale masses

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$SU(5)$ Boundary Conditions

Common $m_{\mathbf{10}}$ for matter in a $\mathbf{10}$

$$m^2_{\tilde{Q}_L}(t_G) = m^2_{\tilde{u}_R}(t_G) = m^2_{\tilde{e}_R}(t_G) = m^2_{\mathbf{10}}$$

Common gaugino mass $M_{1/2}$

$$M_1^2(t_G) = M_2^2(t_G) = M_3^2(t_G) = M^2_{1/2}$$

Common $m_{\bar{\mathbf{5}}}$ for matter in a $\bar{\mathbf{5}}$

$$m^2_{\tilde{L}_L}(t_G) = m^2_{\tilde{d}_R}(t_G) = m^2_{\bar{\mathbf{5}}}$$

Higgs soft masses unrelated

$m^2_{H_u}(t_G) = m^2_{\mathbf{5}'}$ and $m^2_{H_d}(t_G) = m^2_{\bar{\mathbf{5}}'}$

- $S(t_G) = m^2_{\mathbf{5}'} - m^2_{\bar{\mathbf{5}}'} \Rightarrow K \neq 0$
- Five unkowns: $m_{\bar{\mathbf{5}}}$, $m_{\mathbf{10}}$, $M_{1/2}$, $\cos 2\beta$ and K
- **Can be determined by measuring five sfermion masses, eg. $\tilde{u}_L$, $\tilde{d}_L$, $\tilde{e}_R$, $\tilde{u}_R$ and $\tilde{d}_R$**

$$\begin{pmatrix} M^2_{\tilde{u}_L} \\ M^2_{\tilde{d}_L} \\ M^2_{\tilde{e}_R} \\ M^2_{\tilde{u}_R} \\ M^2_{\tilde{d}_R} \end{pmatrix} = \begin{pmatrix} 0 & 1 & c_{\tilde{u}_L} & \delta_{\tilde{u}_L} & -\frac{1}{5} \\ 0 & 1 & c_{\tilde{d}_L} & \delta_{\tilde{d}_L} & -\frac{1}{5} \\ 0 & 1 & c_{\tilde{e}_R} & \delta_{\tilde{e}_R} & -\frac{6}{5} \\ 0 & 1 & c_{\tilde{u}_R} & \delta_{\tilde{u}_R} & \frac{4}{5} \\ 1 & 0 & c_{\tilde{d}_R} & \delta_{\tilde{d}_R} & -\frac{2}{5} \end{pmatrix} \begin{pmatrix} m^2_{\bar{\mathbf{5}}} \\ m^2_{\mathbf{10}} \\ M^2_{1/2} \\ \cos 2\beta \\ K \end{pmatrix}$$

- $c_{\tilde{u}_R} \equiv \bar{c}_3(M_{\tilde{u}_R}) + \frac{4}{9}\bar{c}_1(M_{\tilde{u}_R})$
- $c_{\tilde{d}_R} \equiv \bar{c}_3(M_{\tilde{d}_R}) + \frac{1}{9}\bar{c}_1(M_{\tilde{d}_R})$

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$SO(10)$ Boundary Conditions

- Breaking $SO(10) \to SU(5) \otimes U(1)_x \to G_{SM}$ the rank is reduced from 5 to 4
  - D-term contributions from the additional $U(1)_x$ of the form $\Delta m_a^2 = -\sum_k Q_{ka} g_k^2 D_k$

    [Kolda and Martin, 9503445]

- Consider that the Higgs are embedded in a **10** of $SO(10)$

Common sfermion mass $m_{\mathbf{16}}$

$$m^2_{\tilde{Q}_L}(t_G) = m^2_{\tilde{u}_R}(t_G) = m^2_{\tilde{e}_R}(t_G) = m^2_{\mathbf{16}} + g_{10}^2 D$$
$$m^2_{\tilde{L}_L}(t_G) = m^2_{\tilde{d}_R}(t_G) = m^2_{\mathbf{16}} - 3g_{10}^2 D$$
$$m^2_{\tilde{N}_e}(t_G) = m^2_{\mathbf{16}} + 5g_{10}^2 D$$

Common Higgs mass $m_{\mathbf{10}}$

$$m^2_{\tilde{H}_u}(t_G) = m^2_{\mathbf{10}} - 2g_{10}^2 D$$
$$m^2_{\tilde{H}_d}(t_G) = m^2_{\mathbf{10}} + 2g_{10}^2 D$$

- $S(t_G) = -4g_{10}^2 D$
- Five unknowns: $m_{\mathbf{16}}$, $g_{10}^2 D$, $M_{1/2}$, $\cos 2\beta$ and K
- **Can be determined by measuring five sfermion masses, eg. $\tilde{u}_L$, $\tilde{d}_L$, $\tilde{e}_R$, $\tilde{u}_R$ and $\tilde{d}_R$**

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$$\begin{pmatrix} M^2_{\tilde{u}_L} \\ M^2_{\tilde{d}_L} \\ M^2_{\tilde{e}_R} \\ M^2_{\tilde{u}_R} \\ M^2_{\tilde{d}_R} \end{pmatrix} = \begin{pmatrix} 1 & 1 & c_{\tilde{u}_L} & \delta_{\tilde{u}_L} & -\frac{1}{5} \\ 1 & 1 & c_{\tilde{d}_L} & \delta_{\tilde{d}_L} & -\frac{1}{5} \\ 1 & 1 & c_{\tilde{e}_R} & \delta_{\tilde{e}_R} & -\frac{6}{5} \\ 1 & 1 & c_{\tilde{u}_R} & \delta_{\tilde{u}_R} & \frac{4}{5} \\ 1 & -3 & c_{\tilde{d}_R} & \delta_{\tilde{d}_R} & -\frac{2}{5} \end{pmatrix} \begin{pmatrix} m^2_{\mathbf{16}} \\ g^2_{10}D \\ M^2_{1/2} \\ \cos 2\beta \\ K \end{pmatrix}$$

- $K(t) = \frac{-4g^2_{10}D}{2b_1}\left(1 - \frac{\alpha_1(t)}{\alpha_1(t_G)}\right)$
- Masses are further constrained throught this relation

More explicitly and given that $X_5 = c_{\tilde{d}_L} - c_{\tilde{e}_R} + c_{\tilde{u}_L} - c_{\tilde{u}_R}$

$$\begin{aligned} K &= \frac{1}{6X_5(\sin^2\theta_W - 1)}\Big[3c_{\tilde{u}_R}(M^2_{\tilde{d}_L} - 2M^2_{\tilde{e}_R} + M^2_{\tilde{u}_L}) + 3(c_{\tilde{d}_L} + c_{\tilde{u}_L})\,(M^2_{\tilde{e}_R} - M^2_{\tilde{u}_R}) \\ &\quad -3c_{\tilde{e}_R}(M^2_{\tilde{d}_L} + M^2_{\tilde{u}_L} - 2M^2_{\tilde{u}_R}) + 2\Big(c_{\tilde{u}_R}(M^2_{\tilde{d}_L} + 3M^2_{\tilde{e}_R} - 4M^2_{\tilde{u}_L}) - c_{\tilde{d}_L}(4M^2_{\tilde{e}_R} - 5M^2_{\tilde{u}_L} + M^2_{\tilde{u}_R}) \\ &\quad +c_{\tilde{u}_L}(-5M^2_{\tilde{d}_L} + M^2_{\tilde{e}_R} + 4M^2_{\tilde{u}_R}) + c_{\tilde{e}_R}(4M^2_{\tilde{d}_L} - M^2_{\tilde{u}_L} - 3M^2_{\tilde{u}_R})\Big)\sin^2\theta_W\Big] \\ g^2_{10}D &= \frac{1}{20X_5}\Big[-c_{\tilde{u}_R}(2M^2_{\tilde{d}_L} - 5M^2_{\tilde{d}_R} + M^2_{\tilde{e}_R} + 2M^2_{\tilde{u}_L}) - c_{\tilde{e}_R}(-3M^2_{\tilde{d}_L} + 5M^2_{\tilde{d}_R} - 3M^2_{\tilde{u}_L} + M^2_{\tilde{u}_R}) \\ &\quad +(c_{\tilde{d}_L} + c_{\tilde{u}_L})(5M^2_{\tilde{d}_R} - 3M^2_{\tilde{e}_R} - 2M^2_{\tilde{u}_R}) + 5c_{\tilde{d}_R}(M^2_{\tilde{d}_L} - M^2_{\tilde{e}_R} + M^2_{\tilde{u}_L} - M^2_{\tilde{u}_R})\Big] \end{aligned}$$

- This was obtained for a particular choice of the Higgs in a **10**-plet
- If Higgs in a **120**, **126** or combinations? Different constraints?

E_6SSM First and Second Generation Sfermion Masses

- Extended $G_{SM} \otimes U(1)_N$ at the low scale
- RGEs with an extra S' D-term contribution, additional fields contributing to the loops and a D-term from $U(1)_N$ breaking

Solution of the E_6SSM 1-Loop RGEs

$$m^2_{\tilde{u}_L}(t) = m^2_{\tilde{Q}_L}(t_G) + C_3^{E_6} + C_2^{E_6} + \frac{1}{36}C_1^{E_6} + \frac{1}{4}C_1' + \Delta_{u_L} - \frac{1}{5}K - \frac{1}{20}K' - g_1'^2 D$$
$$m^2_{\tilde{d}_L}(t) = m^2_{\tilde{Q}_L}(t_G) + C_3^{E_6} + C_2^{E_6} + \frac{1}{36}C_1^{E_6} + \frac{1}{4}C_1' + \Delta_{d_L} - \frac{1}{5}K - \frac{1}{20}K' - g_1'^2 D$$
$$m^2_{\tilde{u}_R}(t) = m^2_{\tilde{u}_R}(t_G) + C_3^{E_6} + \frac{4}{9}C_1^{E_6} + \frac{1}{4}C_1' + \Delta_{u_R} + \frac{4}{5}K - \frac{1}{20}K' - g_1'^2 D$$
$$m^2_{\tilde{d}_R}(t) = m^2_{\tilde{d}_R}(t_G) + C_3^{E_6} + \frac{1}{9}C_1^{E_6} + C_1' + \Delta_{d_R} - \frac{2}{5}K - \frac{1}{10}K' - 2g_1'^2 D$$
$$m^2_{\tilde{e}_L}(t) = m^2_{\tilde{L}_L}(t_G) + C_2^{E_6} + \frac{1}{4}C_1^{E_6}C_1' + \Delta_{e_L} + \frac{3}{5}K - \frac{1}{10}K' - 2g_1'^2 D$$
$$m^2_{\tilde{\nu}_L}(t) = m^2_{\tilde{L}_L}(t_G) + C_2^{E_6} + \frac{1}{4}C_1^{E_6}C_1' + \Delta_{\nu_L} + \frac{3}{5}K - \frac{1}{10}K' - 2g_1'^2 D$$
$$m^2_{\tilde{e}_R}(t) = m^2_{\tilde{e}_R}(t_G) + C_1^{E_6} + C_1' + \Delta_{e_R} - \frac{6}{5}K - \frac{1}{20}K' - g_1'^2 D$$

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- $C_i^{E_6}(t) = M_i^2(t_G)\left[A_i^{E_6}\frac{\alpha_i^2(t_G)-\alpha_i^2(t)}{\alpha_i^2(t_G)}\right] = M_i^2(t_G)\overline{c}_i^{E_6}(t)$
- $D_N = \frac{1}{20}K' + g_1'^2 D$

- Common scalar mass $m^2_{\tilde{Q}_L}(t_G) = m^2_{\tilde{u}_R}(t_G) = m^2_{\tilde{d}_R}(t_G) = m^2_{\tilde{L}_L}(t_G) = m^2_{\tilde{e}_R}(t_G) = m^2_{27}$
- Five unknowns: $m_{\mathbf{27}}$, D_N, $M_{1/2}$, $\cos 2\beta$ and K
- **Can be determined by measuring five sfermion masses, eg.** $\tilde{u}_L$, $\tilde{d}_L$, $\tilde{e}_R$, $\tilde{u}_R$ **and** $\tilde{d}_R$

$$\begin{pmatrix} M^2_{\tilde{u}_L} \\ M^2_{\tilde{d}_L} \\ M^2_{\tilde{e}_R} \\ M^2_{\tilde{u}_R} \\ M^2_{\tilde{d}_R} \end{pmatrix} = \begin{pmatrix} 1 & c_{\tilde{u}_L} & \delta_{\tilde{u}_L} & -\frac{1}{5} & -1 \\ 1 & c_{\tilde{d}_L} & \delta_{\tilde{d}_L} & -\frac{1}{5} & -1 \\ 1 & c_{\tilde{e}_R} & \delta_{\tilde{e}_R} & -\frac{6}{5} & -1 \\ 1 & c_{\tilde{u}_R} & \delta_{\tilde{u}_R} & \frac{4}{5} & -1 \\ 1 & c_{\tilde{d}_R} & \delta_{\tilde{d}_R} & -\frac{2}{5} & -2 \end{pmatrix} \begin{pmatrix} m^2_{\mathbf{27}} \\ M^2_{1/2} \\ \cos 2\beta \\ K \\ D_N \end{pmatrix}$$

- Note that $D = \left(Q_d^N v_d^2 + Q_u^N v_u^2 + Q_s^N s^2\right)$
- If able to measure s^2 one can determine K'
  - $S(t_G) = -m^2_{H'} + m^2_{\overline{H}'}$
  - $S'(t_G) = 4m^2_{H'} - 4m^2_{\overline{H}'}$

Sum Rules

From the solution of the 1-loop RGEs, we obtain the following sum rules:

Sum rules for $SU(5)$ and $SO(10)$

$$M^2_{\tilde{u}_L} + M^2_{\tilde{d}_L} - M^2_{\tilde{u}_R} - M^2_{\tilde{e}_R} = C_3 + 2C_2 - \frac{25}{18}C_1 = 5.0 M^2_{1/2}\ (GeV)^2$$

$$\frac{1}{2}\left(M^2_{\tilde{u}_L} + M^2_{\tilde{d}_L}\right) + M^2_{\tilde{d}_R} - M^2_{\tilde{e}_R} - \frac{1}{2}\left(M^2_{\tilde{e}_L} + M^2_{\tilde{\nu}_L}\right) = 2C_3 - \frac{10}{9}C_1 = 8.1 M^2_{1/2}\ (GeV)^2$$

Sum rules for the E_6SSM

$$M^2_{\tilde{u}_L} + M^2_{\tilde{d}_L} - M^2_{\tilde{u}_R} - M^2_{\tilde{e}_R} = C_3^{E_6} + 2C_2^{E_6} - \frac{25}{18}C_1^{E_6} - \frac{3}{4}C_1' = 2.8 M^2_{1/2}\ (GeV)^2$$

$$\frac{1}{2}\left(M^2_{\tilde{u}_L} + M^2_{\tilde{d}_L}\right) + M^2_{\tilde{d}_R} - M^2_{\tilde{e}_R} - \frac{1}{2}\left(M^2_{\tilde{e}_L} + M^2_{\tilde{\nu}_L}\right) = 2C_3^{E_6} - \frac{10}{9}C_1^{E_6} - \frac{3}{4}C_1' = 4.4 M^2_{1/2}\ (GeV)^2$$

- Values for $Q = 1\ TeV$

Higgs and Third Generation Sfermion Soft Masses: *1-Loop RGE*

Third Generation and Higgs Soft Masses RGE

$$16\pi^2 \frac{dm^2_{\tilde{Q}_3}}{dt} = X_t + X_b - \frac{32}{3} g_3^2 M_3^2 - 6 g_2^2 M_2^2 - \frac{2}{15} g_1^2 M_1^2 + \frac{1}{5} g_1^2 S$$

$$16\pi^2 \frac{dm^2_{\tilde{t}_R}}{dt} = 2X_t - \frac{32}{3} g_3^2 M_3^2 - \frac{32}{15} g_1^2 M_1^2 - \frac{4}{5} g_1^2 S$$

$$16\pi^2 \frac{dm^2_{\tilde{b}_R}}{dt} = 2X_b - \frac{32}{3} g_3^2 M_3^2 - \frac{8}{15} g_1^2 M_1^2 + \frac{2}{5} g_1^2 S$$

$$16\pi^2 \frac{dm^2_{\tilde{L}_3}}{dt} = X_\tau - 6 g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2 - \frac{3}{5} g_1^2 S$$

$$16\pi^2 \frac{dm^2_{\tilde{\tau}_R}}{dt} = 2X_\tau - \frac{24}{5} g_1^2 M_1^2 + \frac{6}{5} g_1^2 S$$

$$16\pi^2 \frac{dm^2_{\tilde{N}_3}}{dt} = 2X_\nu$$

$$16\pi^2 \frac{dm^2_{H_d}}{dt} = 3X_b + X_\tau - 6 g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2 - \frac{3}{5} g_1^2 S$$

$$16\pi^2 \frac{dm^2_{H_u}}{dt} = 3X_t + X_\tau - 6 g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2 + \frac{3}{5} g_1^2 S$$

- $X_t = 2y_t^2 \left(m^2_{H_u} + m^2_{\tilde{Q}_3} + m^2_{\tilde{t}_R} + A_t^2 \right)$
- $X_b = 2y_b^2 \left(m^2_{H_d} + m^2_{\tilde{Q}_3} + m^2_{\tilde{b}_R} + A_b^2 \right)$
- $X_\tau = 2y_\tau^2 \left(m^2_{H_d} + m^2_{\tilde{L}_3} + m^2_{\tilde{\tau}_R} + A_\tau^2 \right)$
- $X_\nu = 2y_\nu^2 \left(m^2_{H_u} + m^2_{\tilde{L}_3} + m^2_{\tilde{N}_3} + A_\nu^2 \right)$

- m_φ^2 depend on the trilinear A_i and Yukawa y_i couplings
- Not possible to solve analytically
- **Use the first and second generation inputs to reduce the parameter space**
  - Scan over different regions of the parameter space by choosing an "illustrative" set of measurable masses (GeV)

Slepton Mass	Set 1	Set 2	Set 3
$M_{\tilde{u}_L}$	1550.210	1951.322	3550.2
$M_{\tilde{d}_L}$	1552.080	1952.868	3551.0
$M_{\tilde{e}_R}$	700.0	1430.0	2700.0
$M_{\tilde{u}_R}$	1500.0	1898.0	3500.0
$M_{\tilde{d}_R}$	1550.0	1600.0	3600.0

- Scan over the parameter space
- Ensure vacuum stability
  - Charge and Colour Breaking Minima and Unbounded from below conditions [Casas, Lleyda and Munoz, 9507294]

$SU(5)$ Constraints

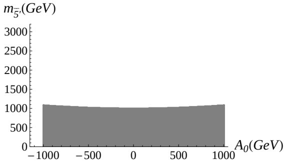
- From the first two generations:

Input Parameter	Set 1	Set 2	Set 3
$m_{\overline{5}}$ (GeV)	781.7	893.7	2856.6
m_{10} (GeV)	654.8	1385.0	2690.5
$M_{1/2}$ (GeV)	655.8	647.3	1129.3
$\tan\beta$	6.1	8.0	4.6
K (GeV)^2	3.413 $\times 10^3$	-52.679 $\times 10^3$	113.83 $\times 10^3$
$M_{\tilde{e}_L}$ (GeV)	915.3	967.2	2819.6
$M_{\tilde{\nu}_L}$ (GeV)	912.0	964.0	2818.5

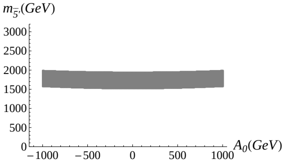
- All ingredients for Yukawa couplings
- Recall $K(t) = \frac{1}{2b_1} S(t_G) \left(1 - \frac{\alpha_1(t)}{\alpha_1(t_G)}\right)$
  - $S(t_G) = m^2_{5'} - m^2_{\overline{5}'}$
- Consider universal trilinear couplings at t_G, A_0
- Two unknowns left, A_0 and one Higgs mass, say $m_{\overline{5}'}$

$\left(A_0, m_{\overline{5}'}\right)$-Plane Scan

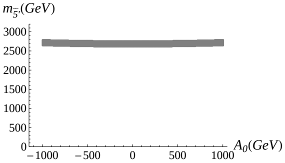
- Scan over the $\left(A_0, m_{\overline{5}'}\right)$-plane
  - $-1000\text{GeV} \leq A_0 \leq 1000\text{GeV}$
  - $10\text{GeV} \leq m_{\overline{5}'} \leq 5000\text{GeV}$
- Apply CCB, UFB and EW constraints



(a) Set 1



(b) Set 2



(c) Set 3

- A significant region of the parameter space is excluded

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$SO(10)$ Constraints

- Recall the consistency relation $K(t) = \frac{-4g_{10}^2 D}{2b_1}\left(1 - \frac{\alpha_1(t)}{\alpha_1(t_G)}\right)$
  - Results in a constraint on the $\widetilde{d}_R$ mass

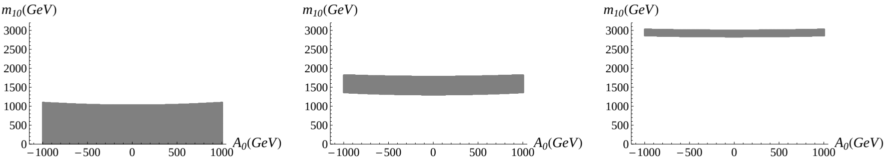
Input Parameter	Set 1	Set 2	Set 3
$M_{\widetilde{d}_R}$ $SU(5)$	1550.0	1600.0	3600.0
$M_{\widetilde{d}_R}$ $SO(10)$	1518.0	1565.5	3830.2

Input Parameter	Set 1	Set 2	Set 3
m_{16} (GeV)	669.9	1268.9	2811.6
$g_{10}^2 D$ $(\mathrm{GeV})^2$	-19.971 $\times 10^3$	308.263 $\times 10^3$	-666.100 $\times 10^3$
$m_{\widetilde{N}_3}(t_G)$ (GeV)	590.6	1775.2	2138.8
$M_{\tilde{e}_L}$ (GeV)	860.0	909.0	3108.1
$M_{\tilde{\nu}_L}$ (GeV)	856.3	905.5	3107.2

- $m_{\widetilde{N}_3}^2(t_G) = m_{16}^2 + 5g_{10}^2 D$
- $M_{1/2}$, $\tan\beta$ and K remain the same as for $SU(5)$

(A_0, m_{10})-Plane Scan

- We are left with two unknowns, A_0 and the common Higgs mass m_{10}
- same procedure as for $SU(5)$



(a) Set 1 (b) Set 2 (c) Set 3

- RH sneutrinos in the running from $Q \sim 10^{12}$ GeV to Q_{GUT}:
- m_{10} scale slightly different than $m_{\overline{5}'}$ for $SU(5)$
  - Mainly due to the influence of $M_{\tilde{d}_R}$
  - Contribution of $M_{\tilde{N}_3}$ is very tiny

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Physical Mass Predictions

As a consequence of the **Goldstone Theorem**, when spontaneous symmetry breaking occurs:

- $n_{phy\ Higgs} = n_{real\ DOF} - n_{Goldstones}$
- $SU(2)_L \otimes U(1)_Y \to U(1)_Q$
  - 3 Goldstones

- SM 1 Higgs doublet $\to$ 4 real DOF
  - $4-3=1$ physical Higgs mass eigenstate
- 2 Higgs doublet models $\to$ 8 real DOF
  - $8-3=5$ physical Higgs mass eigenstates:
    - $h^0,\ H^0,\ H^\pm,\ A^0$

$$m_{A^0}^2 = \frac{2b}{\sin 2\beta}, \qquad m_{H^\pm}^2 = m_W^2 + m_{A^0}^2$$

$$m_{h^0,\ H^0}^2 = \frac{1}{2}\left\{ m_Z^2 + m_{A^0}^2 \mp \left[\left(m_Z^2 + m_{A^0}^2 \right)^2 - 4 m_{A^0}^2 m_Z^2 \cos^2 2\beta \right]^{\frac{1}{2}} \right\}$$

$$\Delta m_{h^0}^2 = \frac{3}{4\pi^2}\frac{m_t^4}{v^2}\left[\log\frac{m_{\tilde{t}}^2}{m_t^2} + \frac{(A_t - \mu\cot\beta)^2}{m_{\tilde{t}}^2}\left(1 - \frac{(A_t-\mu\cot\beta)^2}{12 m_{\tilde{t}}^2} \right) \right]$$

$$m_{\tilde{t}_1,\ \tilde{t}_2}^2 = \frac{1}{2}\left[\left(m_{\tilde{Q}_3}^2 + m_{\tilde{t}_R}^2 + 2m_t^2 + \Delta_{u_L} + \Delta_{u_R} \right) \mp \sqrt{ \left(m_{\tilde{Q}_3}^2 - m_{\tilde{t}_R}^2 + \Delta_{u_L} - \Delta_{u_R} \right)^2 + 4 m_t^2 (A_t - \mu\cot\beta)^2 } \right]$$

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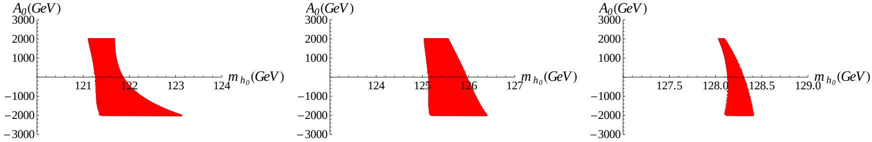
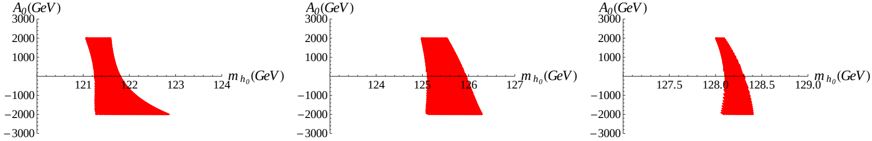
m_{h^0} vs A_0



(a) Set 1 (b) Set 2 (c) Set 3

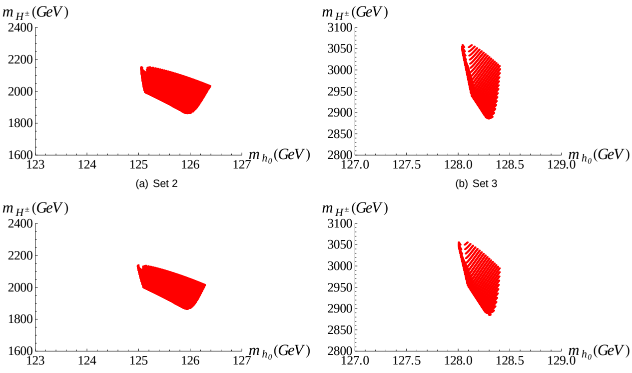
Figure: $SU(5)$



(a) Set 1 (b) Set 2 (c) Set 3

Figure: $SO(10)$

m_{h^0} vs $m_{H^\pm}$



(a) Set 2 (b) Set 3
(c) Set 2 (d) Set 3

Figure: (a) and (b) $\longrightarrow SU(5)$. (c) and (d) $\longrightarrow SO(10)$

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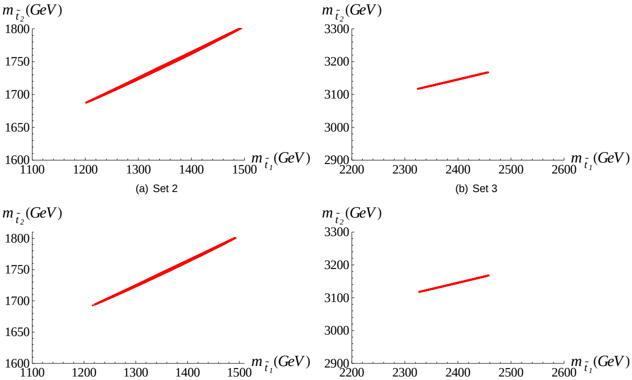
$m_{\tilde{t}_1}$ vs $m_{\tilde{t}_2}$



(a) Set 2 (b) Set 3 (c) Set 2 (d) Set 3

Figure: (a) and (b) $\longrightarrow SU(5)$. (c) and (d) $\longrightarrow SO(10)$

$m_{\tilde{b}_1}$ vs $m_{\tilde{b}_2}$

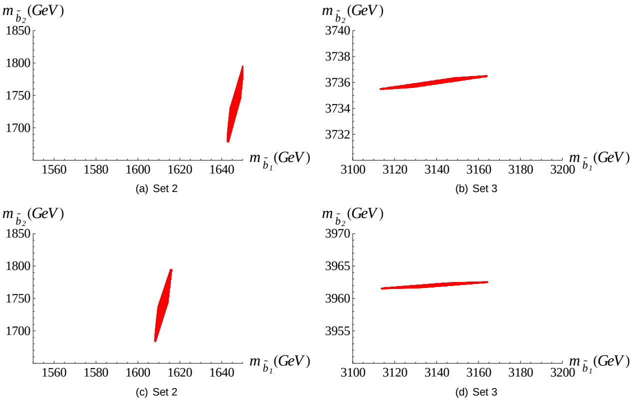


Figure: (a) and (b) $\longrightarrow SU(5)$. (c) and (d) $\longrightarrow SO(10)$

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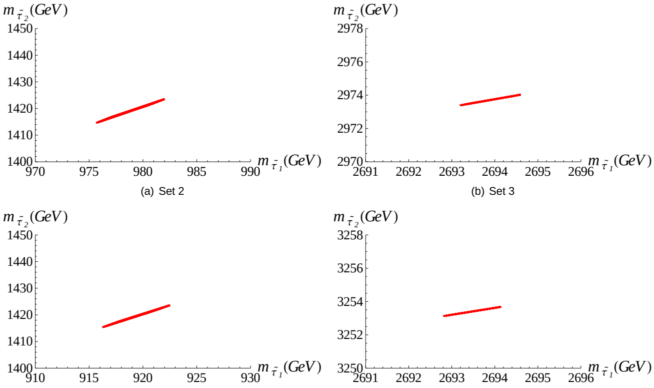
$m_{\tilde{\tau}_1}$ vs $m_{\tilde{\tau}_2}$



(a) Set 2 (b) Set 3

(c) Set 2 (d) Set 3

Figure: (a) and (b) $\longrightarrow SU(5)$. (c) and (d) $\longrightarrow SO(10)$

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Fine tuning m_{h^0}

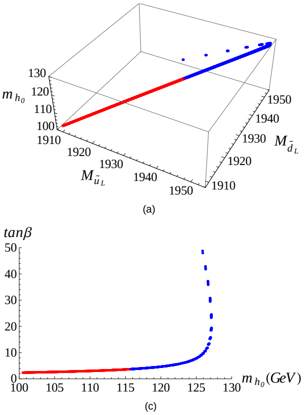


Figure: Set 2 $SO(10)$

Conclusions

- Overview of the SM interactions
- Discussed motivations for BSM physics
- Motivations for Grand Unification
- Overview of standard GUT representations
- Studied the first and second generation sfermion mass spectrum with GUT constraints
- Third generation analysis constrained by the first and second

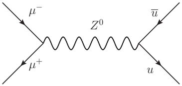
QCD BACKUP

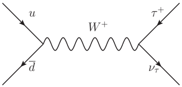
- Local gauge invariance $\mathscr{L}_{QCD} = \mathscr{L}'_{QCD}$: define $\mathbf{A}_\mu = T^a A^a_\mu$ and $\mathbf{U} = \exp(ig_3\alpha_a(x)T^a)$
  - $\psi_i \to \psi'_i = \mathbf{U}\psi_i$
  - $\mathbf{A}_\mu \to \mathbf{A}'_\mu = \mathbf{U}\mathbf{A}_\mu\mathbf{U}^{-1} - \frac{i}{g_3}\partial_\mu\mathbf{U}\mathbf{U}^{-1}$
  - $\mathbf{U}$ are 3×3 complex unitary matrices, $\mathbf{U}\mathbf{U}^\dagger = 1$, and $\det(\mathbf{U}) = 1 \longrightarrow$ form a $SU(3)$ group
    - T^a are the $SU(3)_C$ generators and quarks placed in $SU(3)_C$ triplets whereas leptons are singlets
    - Generic $SU(N)$ group has N^2-1 generators $\longrightarrow SU(3)_C$ has $3^2-1=8$ generators $\longrightarrow$ **8 gluons**
    - $\left[T^a, T^b\right] = if^{abc}T^c \longrightarrow$ Non-abelian or non-comutative algebra
    - $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_3 f^{abc}A^b_\mu A^c_\nu$
    - Allows interactions between gauge fields (gluons) as opposed to QED eg: $-g_3 f^{abc}\partial_\mu A^a_\nu A^{\mu b}A^{\nu c}$
- **QCD is a non-abelian $\mathbf{SU(3)}$ gauge theory**

Electroweak Unification BACKUP

Electroweak Lagrangian

$$
\begin{aligned}
\mathscr{L}_{EW} &= \mathscr{L}_{scalar} + \frac{1}{4}\left(f_{\mu\nu}f^{\mu\nu} + F^k_{\mu\nu}F^{k\mu\nu}\right) + \mathscr{L}_{matter} \\
F^k_{\mu\nu} &= \partial_\mu W^k_\nu - \partial_\nu W^k_\mu + g\varepsilon^{ijk}W^i_\mu W^j_\nu \\
f_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \\
i,j,k &= 1,\ldots,3
\end{aligned}
$$



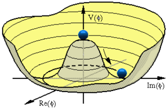


- Imposing local gauge invariance leads to a $\mathbf{SU(2)_L \otimes U(1)_Y}$ **gauge theory**
- Left-handed quarks and leptons placed in $SU(2)_L$ doublets $\longrightarrow$ weakly interacting
- Right-handed are $SU(2)_L$ singlets $\longrightarrow$ non weakly interacting
- 1 $U(1)_Y$ generator designated as **weak hypercharge** Y $\longrightarrow$ 1 gauge boson B_μ
- $2^2 - 1 = 3$ $SU(2)_L$ generators $\sigma^1,\ \sigma^2, \sigma^3$ (Pauli matrices) $\longrightarrow$ 3 gauge bosons $W_\mu^{1,2,3}$
- Experiment tells us that weak bosons are massive! Is the theory actually gauge invariant?

Electroweak Symmetry Breaking and the Higgs Mechanism

- $SU(2)_L \otimes U(1)_Y$ prediction of massless gauge bosons is not seen experimentally
- Mass terms of the form $M_A^2 A_\mu A^\nu$ forbidden by gauge invariance
- Explicit mass terms for fermions of the form $\mathscr{L}_{mass} = -m\left(\overline{\psi}_L \psi_R + \overline{\psi}_R \psi_L\right)$ also violate gauge invariance
  - ψ_L and ψ_R transform differently under $SU(2)_L$
- **Problem of mass generation solved by the presence of a scalar filed ϕ, the Higgs field**
  - Expected to have a non-zero value in the *vacuum* state (min energy configuration of the Universe)
  - Higgs particle present in the vacuum in contrast with all other fundamental particles

$$\begin{aligned}
\mathscr{L}_{scalar} &= \left(D^\mu \phi\right)^\dagger \left(D_\mu \phi\right) - V(\phi), \quad \text{with} \quad \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad SU(2)_L \text{ doublet} \\
D_\mu &= \partial_\mu + \frac{ig'}{2} B_\mu Y + \frac{ig}{2} \sigma^k W_\mu^k \\
V(\phi) &= -\mu^2 \phi^\dagger \phi + \lambda \left(\phi^\dagger \phi\right)^2 \quad \text{minimization} \rightarrow |\phi|^2 = \frac{\mu^2}{2|\lambda|} \equiv \frac{v^2}{2} \\
\langle \phi \rangle_0 &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad \text{(vacuum)}
\end{aligned}$$



- Y and σ^k don't leave the vacuum invariant
- Linear combination $Q = \frac{1}{2}\left(\sigma^3 + Y\right)$ does $\longrightarrow$ **Electric Charge**
- Electroweak symmetry is broken surviving a remnant one, QED
  - $SU(2)_L \otimes U(1)_Y \rightarrow U(1)_Q$

Mass generation:

- Consider radial perturbations around the vacuum, $h(x)$, and redefine it: $\phi = \begin{pmatrix} 0 \\ \frac{h+v}{\sqrt{2}} \end{pmatrix}$
- Plug it in $\mathscr{L}_{EW}$ and look for quadratic terms

$$(D^\mu \phi)^\dagger (D_\mu \phi) \longrightarrow \frac{1}{4}(gv)^2 \left(W^1_\mu W^{1\mu} + W^2_\mu W^{2\mu}\right) + \frac{1}{4}v^2 \left(gW^3_\mu - g' B_\mu\right)^2$$

  - 1st term gives the mass of the W^+ and W^- bosons, $M_W = \frac{1}{2}gv \sim 81$ GeV
  - 2nd term need to be diagonalized
    - Eigenvalues $M_1 = 0$ and $M_2 = \frac{1}{2}v\sqrt{g^2 + g'^2}$
    - M_1 is the mass(less) of the photon and $M_2 = M_Z \sim 91 GeV$

$$V(\phi) \longrightarrow \frac{1}{2}\left(2\mu^2\right) h^* h$$

  - Identify the Higgs boson mass $m_{h^0} = 2\mu^2$

$$\mathscr{L}_{matter} \longrightarrow y_e L_L \phi e_R + \cdots + \mathscr{L}_{int} = \underbrace{-\frac{1}{\sqrt{2}}(y_e v)}_{\text{electron mass}} \overline{e}_L e_R + \cdots$$

  - Identify the Fermion masses $m_f = y_f \frac{v}{\sqrt{2}}$
  - $y_f \longrightarrow$ Yukawa couplings: **strength of interaction with the Higgs field**

E_6 embedding of G_{SM}: *The E_6SSM **27** representation*

We consider as E_6 SUSY GUTs the exceptional supersymmetric model E_6SSM

[King, Moretti and Nevzorov, 0510419, 0701064] [Athron, King, Miller, Moretti and Nevzorov, 0904.2169]

- Extended $G_{SM} \otimes U(1)_N$ at the low scale
- The extra $U(1)_N$ breaks close to the EW scale by the vev of an Higgs type singlet

Maximal subalgebra of E_6

$$E_6 \to SO(10) \otimes U(1)_\psi$$

Branching rule for 27

$$\mathbf{27} \to \left(\mathbf{1}; \frac{4}{2\sqrt{6}}\right) \oplus \left(\mathbf{10}; \frac{-2}{2\sqrt{6}}\right) \oplus \left(\mathbf{16}; \frac{1}{2\sqrt{6}}\right)$$

$SO(10) \to SU(5) \otimes U(1)_\chi$

$$\mathbf{1} \to \left(\mathbf{1}; \frac{1}{2\sqrt{10}}\right)$$

$$\mathbf{10} \to \left(\mathbf{5}; \frac{2}{2\sqrt{10}}\right) \oplus \left(\overline{\mathbf{5}}; \frac{-2}{2\sqrt{10}}\right)$$

$$\mathbf{16} \to \left(\mathbf{10}; \frac{-1}{2\sqrt{10}}\right) \oplus \left(\overline{\mathbf{5}}; \frac{3}{2\sqrt{10}}\right) \oplus \left(\mathbf{1}; \frac{-5}{2\sqrt{10}}\right)$$

Branching of a 27-plet with normalized $\sqrt{40}Q_N$

$$\mathbf{27} \to \mathbf{10}_1 \oplus \overline{\mathbf{5}}_2 \oplus \overline{\mathbf{5}}_{-3} \oplus \mathbf{5}_{-2} \oplus \mathbf{1}_5 \oplus \mathbf{1}_0$$

- To preserve unification needs two extra $SU(2)$ doublets H' and $\overline{H'}$ from incomplete $\mathbf{27}'$ and $\overline{\mathbf{27}'}$
  - New doublet-25-plet splitting

$E_6 \longrightarrow SU(5) \otimes U(1)_N \longrightarrow G_{SM} \otimes U(1)_N$

We can then identify the E_6SSM matter as

Ordinary squarks and sleptons

$\mathbf{10}_1 \to (\mathbf{3},\mathbf{2})_{(\frac{1}{6},\ 1)} \oplus (\overline{\mathbf{3}},\mathbf{1})_{(-\frac{2}{3},\ 1)} \oplus (\mathbf{1},\mathbf{1})_{(1,\ 1)} = Q_L \oplus \tilde{u}_R^* \oplus \tilde{e}_R^*$

$\overline{\mathbf{5}}_2 \to (\mathbf{1},\mathbf{2})_{(-\frac{1}{2},\ 2)} \oplus (\overline{\mathbf{3}},\mathbf{1})_{(\frac{1}{3},\ 2)} = L \oplus \tilde{d}_R^*$

$\mathbf{1}_0 \to (\mathbf{1},\mathbf{1})_{(0,\ 0)} = \tilde{N}_R$

Higgs and exotics

$\overline{\mathbf{5}}_{-3} \to (\mathbf{1},\mathbf{2})_{(-\frac{1}{2},\ -3)} \oplus (\overline{\mathbf{3}},\mathbf{1})_{(\frac{1}{3},\ -3)} = H_1 \oplus \overline{D}$

$\mathbf{5}_{-2} \to (\mathbf{1},\mathbf{2})_{(\frac{1}{2},\ -2)} \oplus (\mathbf{3},\mathbf{1})_{(-\frac{1}{3},\ -2)} = H_2 \oplus D$

$\mathbf{1}_5 \to (\mathbf{1},\mathbf{1})_{(0,\ 5)} = S$

- Extra $U(1)_N$ predicts a Z' boson by its breaking at the soft SUSY scale
- $\tilde{N}_R$ does not participate in gauge interactions $\Longrightarrow$ gain mass at some intermediate high scale (10^{11-14} GeV)
- Predicts exotic quarks D and $\overline{D}$
- Unify ordinary matter, exotic matter and Higgs in a spinor representation