# Radiating Collapse to Black Holes in 5D

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# Objectives and Previous Results

- **Construct exact solutions of radiating collapse in 5D**
  - Prove existence and stability of matched solutions
  - Analyse gravitational wave emission
- **Previous results in 4D with cylindrical symmetry**
  - Spatially homogeneous collapsing spacetimes matched to Einstein-Rosen waves *(Mena, Tod, PRD 2004)*
- **Previous results in higher dim with no radiation**
  - Dust, Perfect fluids inhomogeneous collapse to BH and NS *(Ghosh, Beecham, PRD 2001, Goswami, Joshi, PRD 2004)*
  - Generalised FLRW and LTB with $\Lambda \neq 0$ matched to Kottler *(Mena, Natário, Tod, AHP 2010)*

# Matching Problem

- To find a surface $\sigma$ which is the matching boundary between two spacetimes $(M^+, g^+)$ and $(M^-, g^-)$
- **Matching Conditions between** $(M^\pm, g^\pm)$
  - Equality of first and second fundamental forms on surface $\sigma$

$$
\begin{aligned}
h^+_{\alpha\beta} &= h^-_{\alpha\beta} \equiv g^-_{ij} e_\alpha^{-i} e_\beta^{-j} \\
K^+_{\alpha\beta} &= K^-_{\alpha\beta} \equiv n^-_i e_\alpha^{-j} \nabla^-_j e_\beta^{-i}
\end{aligned}
$$

  where $\vec{e}_\alpha$ are generators of $\sigma$.

  - If matching fluids with vacuum then $p$ vanishes at boundary
  - If dust fluid then boundary is ruled by geodesics

# Radiating Collapse: the exterior

**Bizoń-Chmaj-Schmidt metric** (PRL, 2005)

$$ds^{2+} = -Ae^{-2\delta}dt^2 + \frac{1}{A}dr^2 + \frac{r^2}{4}e^{2B}(\sigma_1^2 + \sigma_2^2) + \frac{r^2}{4}e^{-4B}\sigma_3^2$$

where $A, \delta$ and $B$ are functions of $t$ and $r$. The one-forms $\sigma_i$ are

$$\begin{aligned}
\sigma_1 &= \cos\psi d\theta + \sin\theta \sin\psi d\phi \\
\sigma_2 &= \sin\psi d\theta - \sin\theta \cos\psi d\phi \\
\sigma_3 &= d\psi + \cos\theta d\phi
\end{aligned}$$

where $\theta, \psi, \phi$ are Euler angles on $S^3$.

- For $B = 0$ ones gets the 5D Schwarzschild solution.
- For $B \neq 0$ pure gravitational waves with radial symmetry

# Radiating Collapse: the exterior

- **The $(4+1)$-dimensional vacuum EFEs give**

$$
\begin{aligned}
\frac{\partial A}{\partial r} &= -\frac{2A}{r} + \frac{8e^{-2B} - 2e^{-8B}}{3r} - 2r\left(\frac{e^{2\delta}}{A}\left(\frac{\partial B}{\partial t}\right)^2 + A\left(\frac{\partial B}{\partial r}\right)^2\right)\\
\frac{\partial A}{\partial t} &= -4rA\frac{\partial B}{\partial t}\frac{\partial B}{\partial r}\\
\frac{\partial \delta}{\partial r} &= -2r\left(\frac{e^{2\delta}}{A^2}\left(\frac{\partial B}{\partial t}\right)^2 + \left(\frac{\partial B}{\partial r}\right)^2\right)
\end{aligned}
$$

- **Together with the quasi-linear wave equation for $B$**

$$
\frac{\partial}{\partial t}\left(\frac{e^{\delta}r^3}{A}\frac{\partial B}{\partial t}\right) - \frac{\partial}{\partial r}\left(\frac{Ar^3}{e^{\delta}}\frac{\partial B}{\partial r}\right) + \frac{4r(e^{-2B} - e^{-8B})}{3e^{\delta}} = 0
$$

- **Needs appropriate data at matching boundary $\sigma$**

# Numerical and Analytical Stability

(*Bizon et al, PRL, 2005; Dafermos, Holzegel, ATMP, 2006*)

Linearise around 5D Sch to get linear wave eqn for perturbation $\delta B$

$$\frac{\partial^2(\delta B)}{\partial t^2} - \frac{1}{r^3} A_0 \frac{\partial}{\partial r}\left(r^3 A_0 \frac{\partial(\delta B)}{\partial r}\right) + \frac{8A_0}{r^2}\delta B = 0$$

where $A_0 = 1 - 1/r^2$.

- Use tortoise coord and $\delta B = u(x)e^{-ikt}$ to get Schrödinger eqn
- Study quasinormal modes, plot least damped dominating mode
- Show local convergence to the Schwarschild solution

Nonlinear stability of 5D Sch: Let $(h_{ij}, K_{ij})$ be data evolving to 5D Sch. Then, for smooth triaxial Bianchi IX data sufficiently close to $(h_{ij}, K_{ij})$ in a suitable norm, the solution will stay close to 5D Sch.

# Radiating Collapse: the interior

**Eguchi-Hanson metric** (PLB, 1978)

$$h_{EH} = \left(1 - \frac{a^4}{\rho^4}\right)^{-1} d\rho^2 + \frac{\rho^2}{4}(\sigma_1^2 + \sigma_2^2) + \frac{\rho^2}{4}\left(1 - \frac{a^4}{\rho^4}\right)\sigma_3^2$$

with $a$ constant. Metric is self-dual to the Euclidean EFEs.

**The generalised FLRW metric** built on this is

$$ds^{2-} = -dt^2 + R^2(t)h_{EH}$$

with the Einstein equations for a dust source reducing to

$$\mu R^4 = \mu_0, \qquad \dot{R}^2 = \frac{\kappa\mu_0}{6R^2}$$

- We show that this metric is matched to exterior radiating metric.

# The Matching

- Match surfaces parametrized by $\sigma^+ : \{t = t(\tau), r = r(\tau)\}$
$$\sigma^- : \{T = \tau, \quad \rho = \rho_0\}$$

- The matching conditions give $t(\tau), r(\tau)$ and $A, B, \nabla_n B$ in terms of the interior data at $\sigma$, e.g.

$$B \overset{\sigma}{=} -\frac{1}{6}\ln\left(1 - \frac{a^4}{\rho^4}\right)$$

$$\nabla_n B \overset{\sigma}{=} -\frac{2a^4}{3R\rho^5}\left(1 - \frac{a^4}{\rho^4}\right)^{1/2}$$

- Data close to 5D Schwarzschild for small $B$ and $\nabla_n B$

$$B \overset{\sigma}{=} O\left(\frac{a^4}{\rho^4}\right), \qquad \nabla_n B \overset{\sigma}{=} \frac{1}{rR} O\left(\frac{a^4}{\rho^4}\right)$$

- Can control $\nabla_n B$ by controlling $R$ through $\mu_0$ in the Friedman eq

# The Matching

**Theorem:** Interior gives consistent data for exterior at comoving TL surface. Local existence of radiating exterior is then guaranteed. Boundary data can be chosen close to data for 5D Schwarzschild.

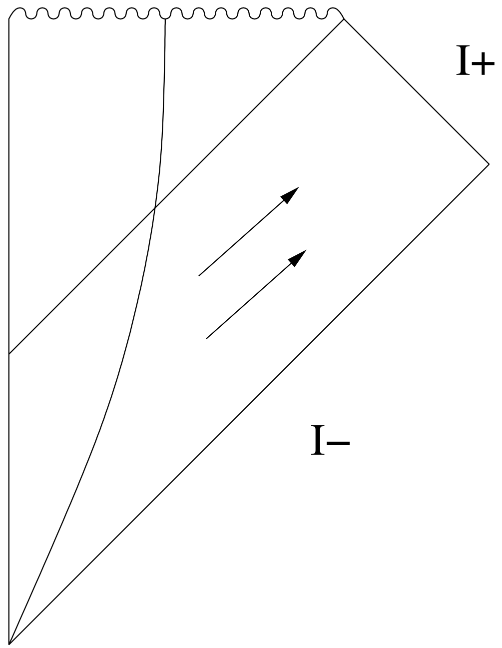


Figure 1: Diagram for radiating collapse to a black hole in 5D.

# Conclusions and the Future

- We have proved local existence of classes of exact models of radiating gravitational collapse to BHs in 5D.

- Study the emission of gravitational radiation.
- Analyse global existence of solutions.
- Attempt at models of collapse to 5D rotating BHs?

# Superenergy Tensor and Radiation

(*Bel, 1958; Senovilla, CGQ, 2000; Garcia-Parrado, CGQ, 2008*)

- **Bel-Robinson tensor** $T^{abcd}$: analog of EMG energy-tensor
- **Dimensions**: square of energy density, also called superenergy

$$T^{abcd} \equiv C_a{}^m{}_d{}^n C_{bncm} + C_a{}^m{}_b{}^n C_{cmdn} - \frac{1}{2}\, g_{bd} C_a{}^{mnp} C_{cmnp} - \frac{1}{2} g_{ac} C_b{}^{mnp} C_{dmnp} + \frac{1}{8} g_{ac} g_{bd}\, C_{mnpq} C^{mnpq}$$

where $C_{aedf}$ is the Weyl tensor. $T^{abcd}$ in 5D is totally symmetric but not trace-free.

- **Superenergy density** seen by observer with $u^a$ $(u^a u_a = -1)$ is $\epsilon = T^{abcd} u_a u_b u_c u_d$ and the corresponding Poynting vector is

$$P^a = -(\delta^a{}_b + u^a u_b) T^{bcde} u_c u_d u_e.$$

# Superenergy jump across boundary

(with A. Garcia-Parrado and J. Senovilla, in progress)

- **Jump of superenergy** through matching boundary $\sigma$:

$$T^+_{abcd}|_\sigma - T^-_{abcd}|_\sigma \equiv [T_{abcd}]_\sigma = c_{aedf}[C_b{}^e{}_d{}^f] + [C_{aecf}]c_b{}^e{}_d{}^f + c_{aedf}[C_b{}^e{}_c{}^f] + [C_{aedf}]c_b{}^e{}_c{}^f - \frac{1}{4}h_{ab}h_{cd}c_{bgfh}[C^{bgfh}]$$

where

$$c_{aedf} = \frac{1}{2}(C^+_{aedf}|_\sigma + C^-_{aedf}|_\sigma)$$

- **Superenergy of jump**: Time variations in the discontinuity of Weyl tensor caused by gravitational radiation through $\sigma$

$$T\{C^+ - C^-\}_{abcd}$$

- To calculate this one needs second derivatives of metric at $\sigma$
- **Work in progress!**