Gravitational radiation from shock wave collisions in higher dimensions

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Aveiro University & i3n

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IV Black Hole Workshop

In collaboration with Flávio Coelho, Carlos Herdeiro & Carmen Rebelo

Based on: JHEP07(2011)121 [arXiv:1105.2298] & work in progress

Transplanckian collisions @ speed of light: Motivation

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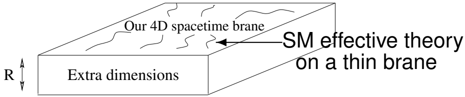

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N. Arkani-Hamed et al. hep-th/9803315 (ADD)

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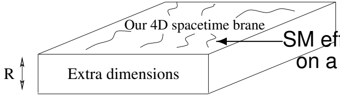

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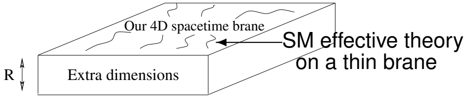

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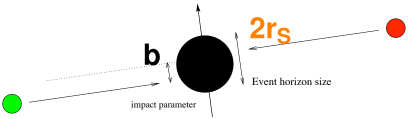

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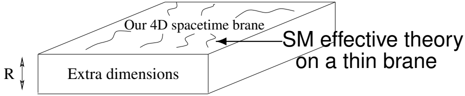

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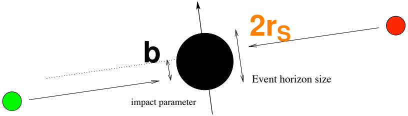

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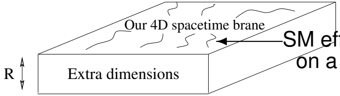

SM effective theory on a thin brane

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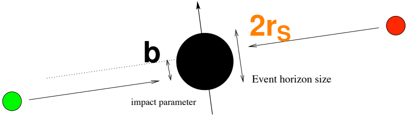

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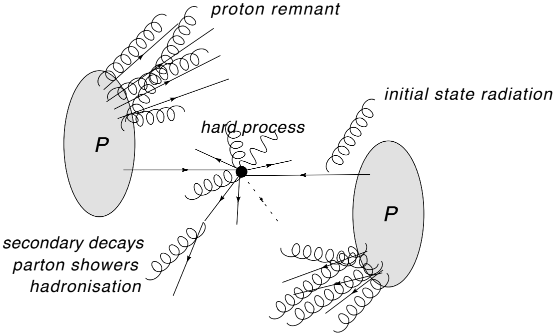

$v/c > 0.999$@*LHC*

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First bounds rely on **bad** knowledge of gravitational radiation

CMS collaboration arXiv:1012.3357

ATLAS-CONF-2011-065

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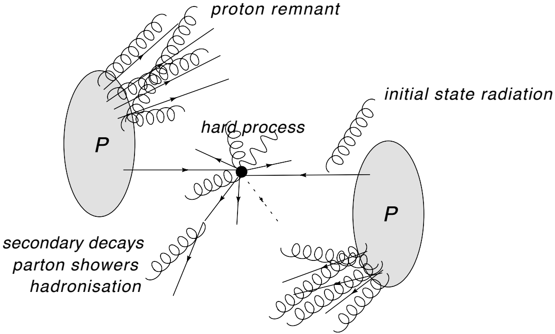

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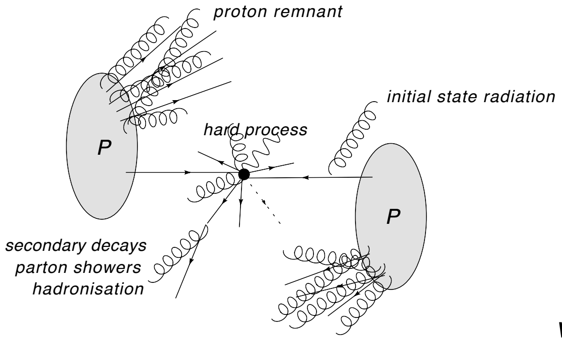

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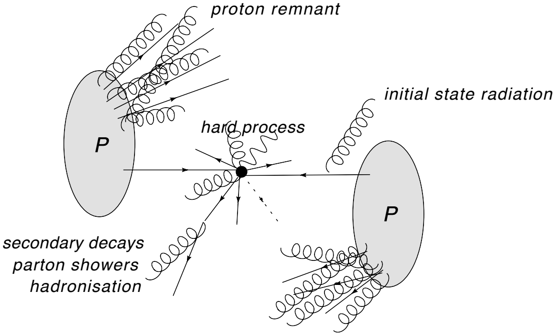

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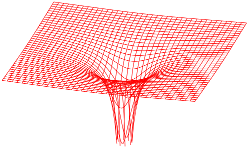

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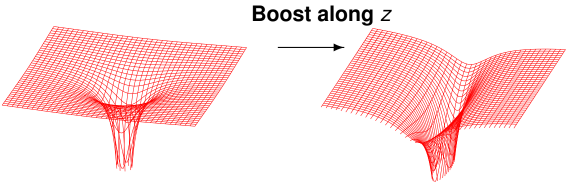

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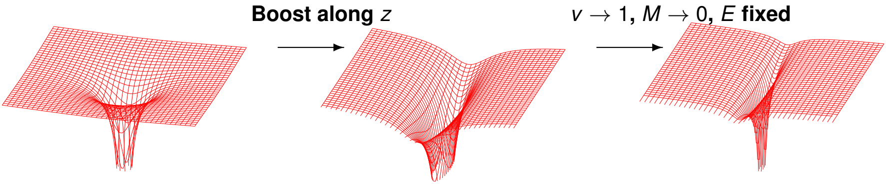

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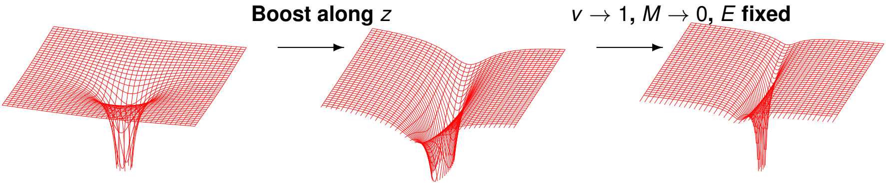

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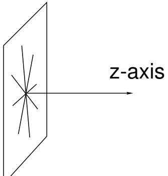

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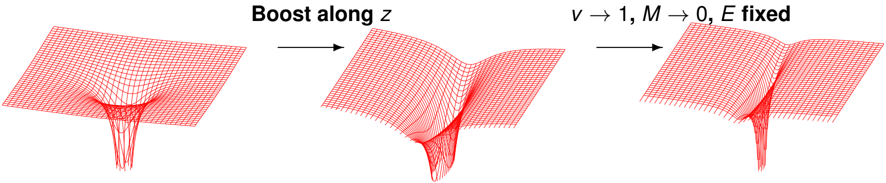

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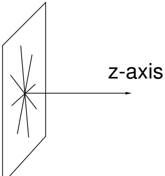

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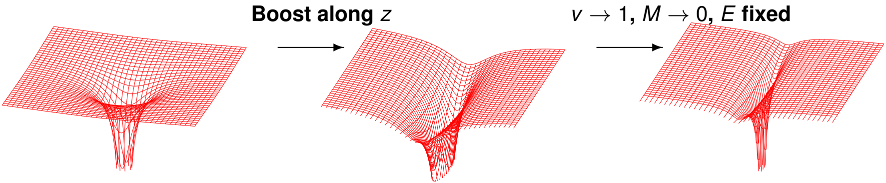

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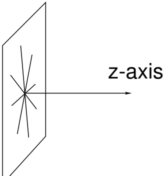

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Basic properties of a single shock wave

- Solution of Einstein's equations, **point source** $P^\mu = E\, n^\mu$

$$T^{\mu\nu} = E\delta(u)\delta^{(D-2)}(x^i)n^\mu n^\nu \quad , \quad n^\mu n_\mu = 0$$

- On the **shock** we have a **profile**

$$\Phi(\rho) = \begin{cases} -2\ln(\rho)\,, & D=4 \\ \dfrac{2}{(D-4)\rho^{D-4}}\,, & D>4 \end{cases} .$$

- **Riemann** tensor singular on the shock
- Null **geodesics** and tangent vectors are discontinuous
- **No difference** if we smear & quantum corrections small

S. B. Giddings and V. S. Rychkov, hep-th/0409131

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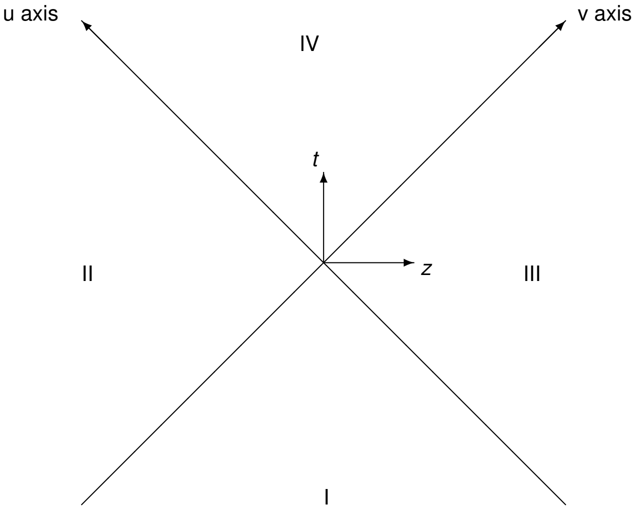

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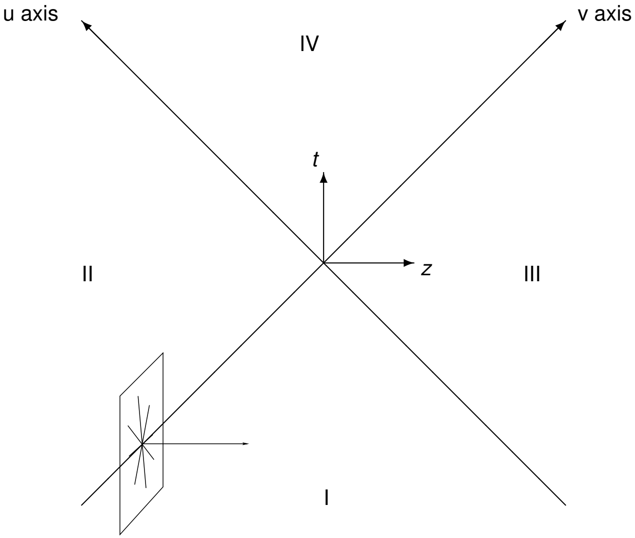

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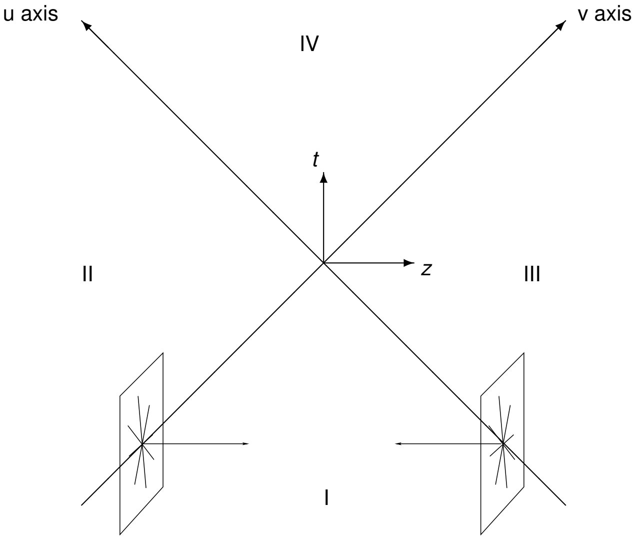

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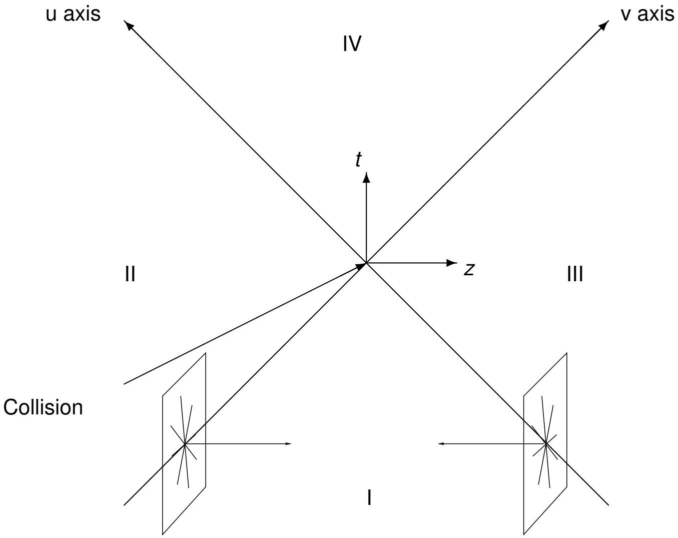

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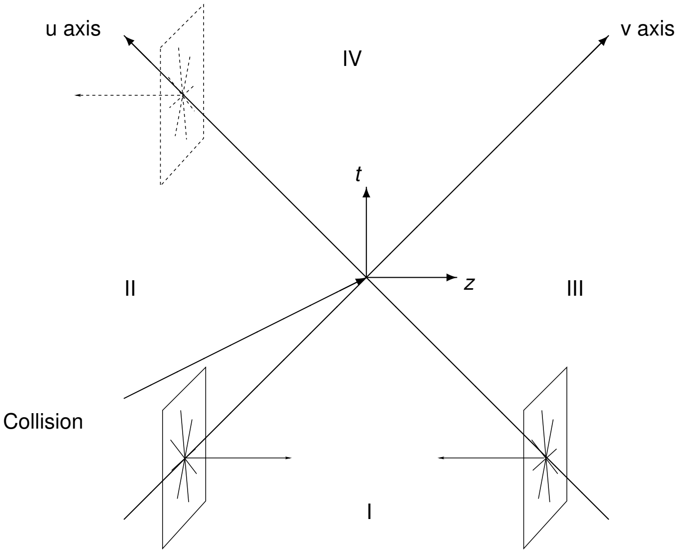

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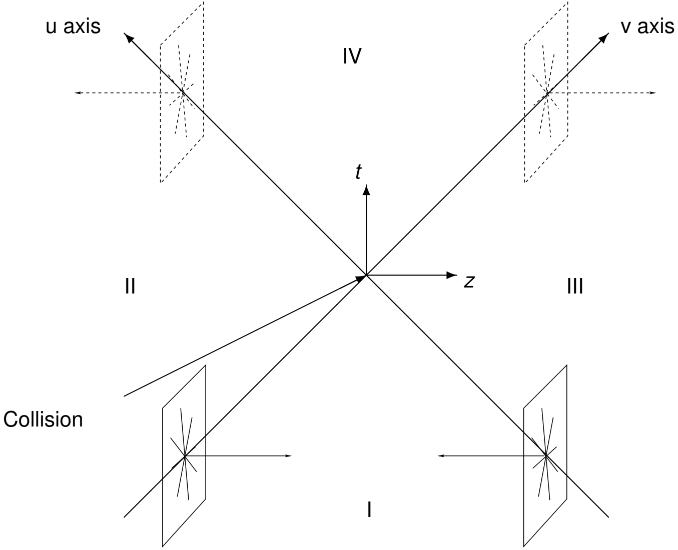
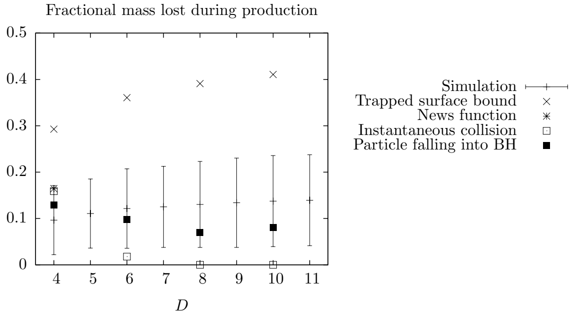


Constraints due to the apparent horizon

- Apparent horizon area ⇒ lower bound on $M_{trapped}$



Frost, Gaunt, MOPS, Casals, Dolan, Parker, Webber arXiv:0904.0979

D'Eath and Payne, PRD Volume 46, Number 2, 658, 675 and 694

Cardoso, Berti and Cavaglia hep-ph/0505125

Berti, Cavaglia and Gualtieri hep-th/0309203

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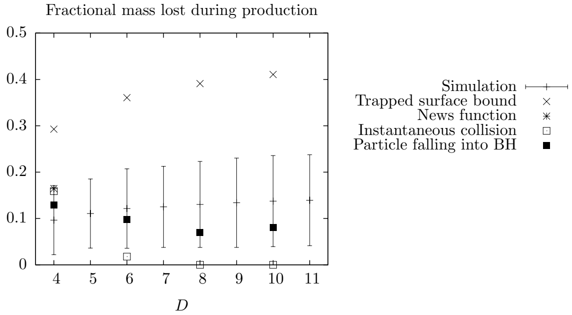

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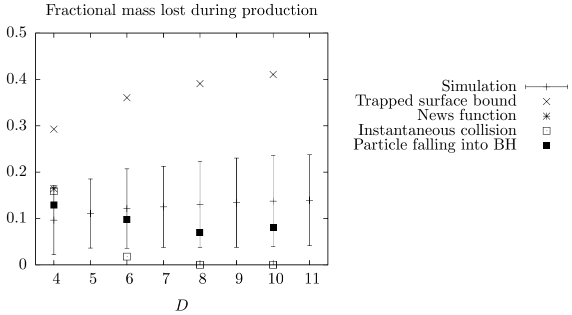

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   - The first order calculation & radiation extraction
3. Conclusions and Outlook

Strong shock vs weak shock

D'Eath and Payne found a trick $\Rightarrow$ **perturbative approach**

1. Perform a large boost along $+z$ with velocity $\beta \equiv \tanh \alpha$
2. Energy parameter of right/left moving shock $\kappa \to e^{\pm\alpha}\kappa$
3. Exact boundary conditions on $u = 0^+$ (strong shock ν)

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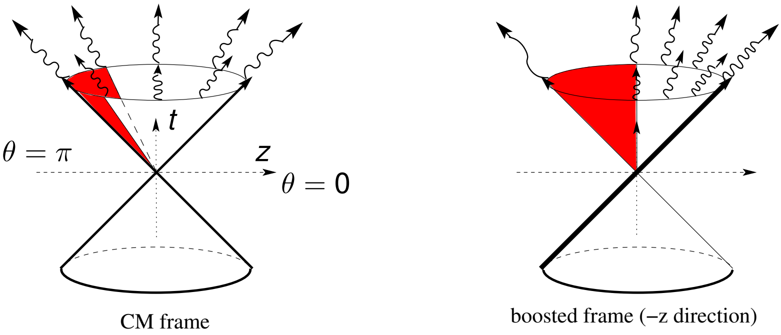
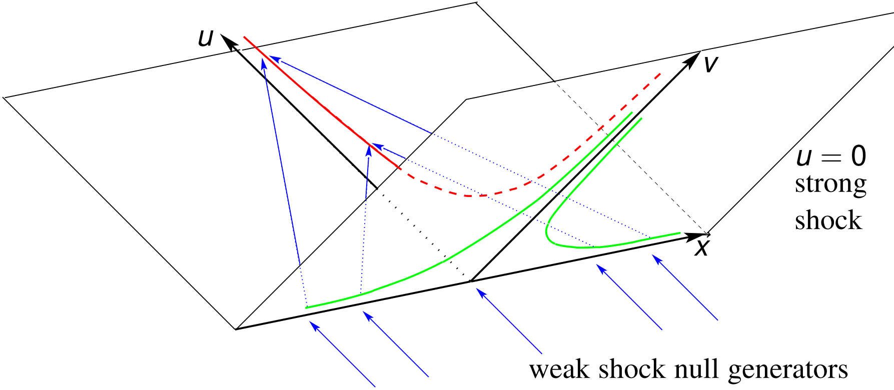
Validity & Physical interpretation

Note: $h_{\mu\nu}^{(i)} \sim \left[\frac{1}{\rho^{D-2}}(\sqrt{2}v - \Phi)\theta(\sqrt{2}v - \Phi)\right]^i$



Future light cone of the collision

$$u = 0\ , \qquad v = \frac{\Phi(\rho)}{\sqrt{2}}\ ,$$



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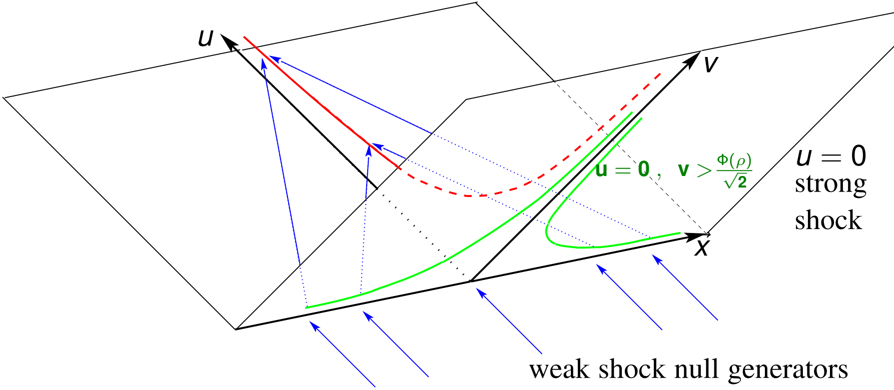

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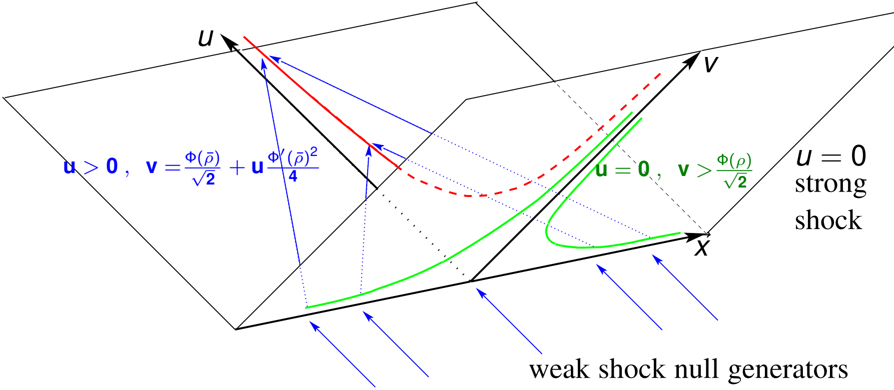

Perturbative expansion

- Assume perturbative ansatz

$$g_{\mu\nu} = \nu^{\frac{2}{D-3}} \left[\eta_{\mu\nu} + \sum_{i=1}^{\infty} \left(\frac{\lambda}{\nu} \right)^i h^{(i)}_{\mu\nu} \right]$$

- Fix gauge order by order (de Donder)

$$x^\mu \to x^{N\mu} = x^\mu + \sum_{i=1}^{\infty} \left(\frac{\lambda}{\nu} \right)^i \xi^{(i)\mu}$$

- Obtain **decoupled sets of wave equations with source**

$$\Box h^{(i)N}_{\mu\nu} = T^{(i-1)}_{\mu\nu} \,.$$

Note: The source is zero for $i = 1$!

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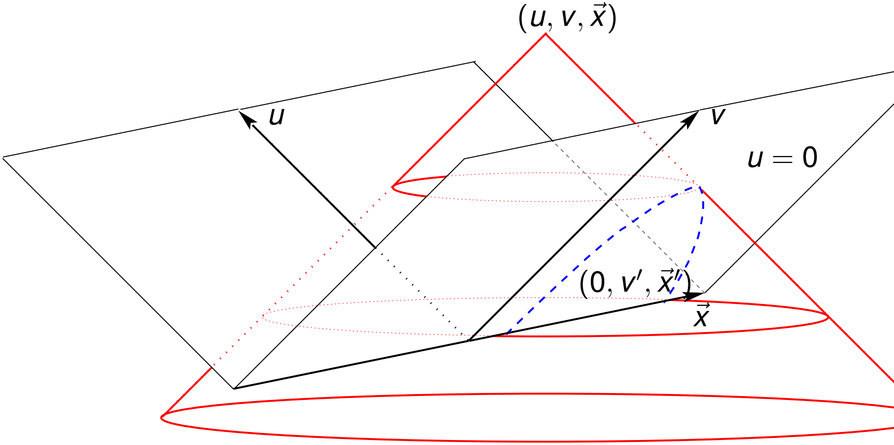
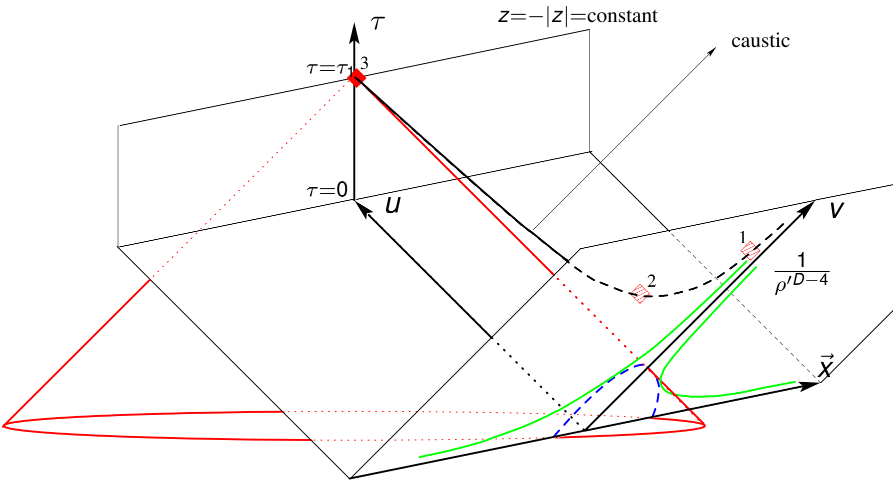
Causal structure of the background & formal solution



$$h_{\mu\nu}^{(i)N}(y) = F.P. \int_{u'>0} d^D y' \, G(y, y') \left[T_{\mu\nu}^{(i-1)}(y') + 2\delta(u')\partial_{v'} h_{\mu\nu}^{(i)N}(y') \right]$$

Outline

Integration limits and ray analysis 1



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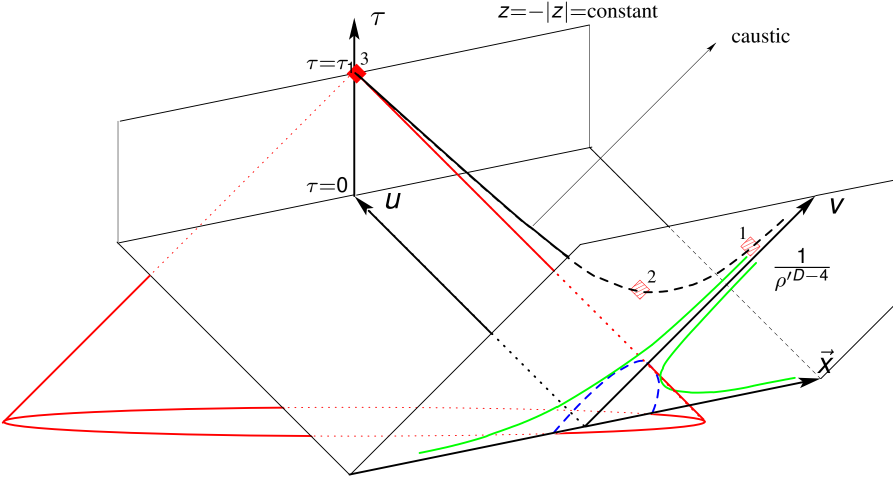

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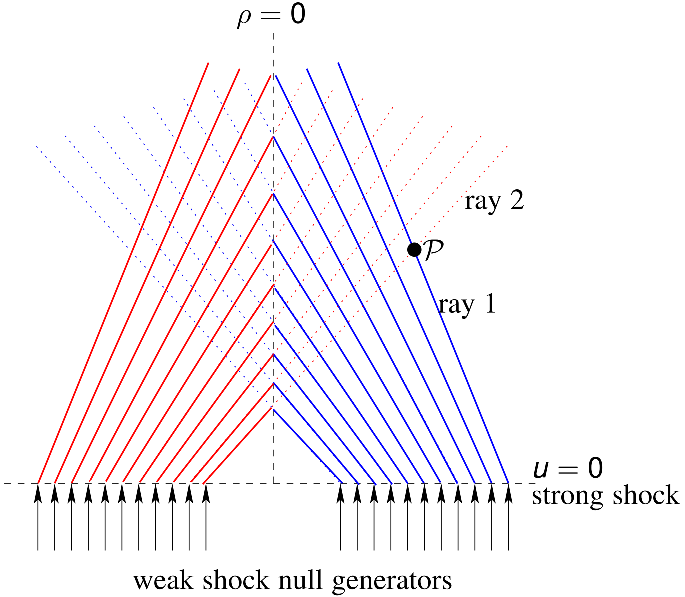
Ray analysis off axis



The first order result – radiation extraction

- **Radiative** components $h_{ij} = \delta_{ij} D(u, v, \rho) + \Delta_{ij}(x) E(u, v, \rho)$
- At first order ($i = 1$) only $E(u, v, \rho)$
- In our paper $\rightarrow$ Landau-Lifshitz pseudo-tensor
  Yoshino and Shibata, arXiv:0907.2760
- Recently shown equivalent to news function in higher D
  Tanabe, Kinoshita and Shiromizu, arXiv:1104.0303

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E_{\text{radiated}} &= \int dt \int_{S^{D-2}} \frac{dEnergy}{dSdt} \\
&\simeq \frac{\Omega_{D-3}}{32\pi G_D} \lim_{\hat{\theta}\to 0, r\to\infty} \left(r^2 \rho^{D-4} \int h^{ij}{}_{,v} h_{ij,v} dt \right) \\
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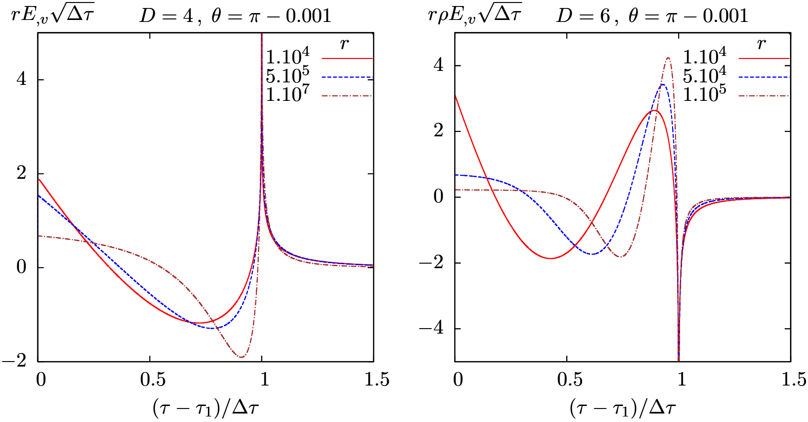
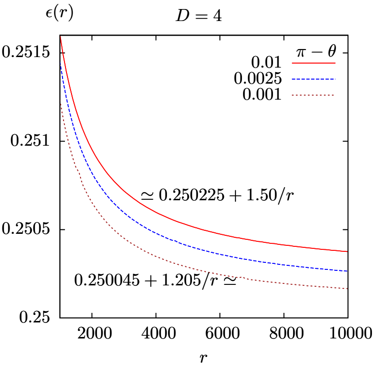
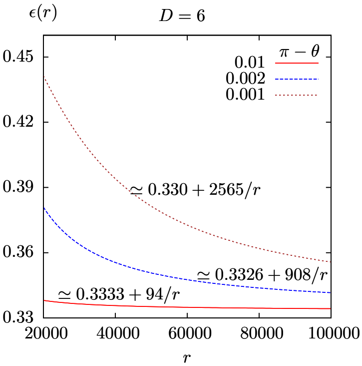
Reduced Wave forms



Extracting the axis limit





D	4	5	6	7	8	9	10
AH bound (%)	29.3	33.5	36.1	37.9	39.3	40.4	41.2
First order(%)	25.0	–	33.3	–	37.5	–	40.0

1. Aichelburg-Sexl shock waves
   - Definition & Physical interpretation
   - Superposition and apparent horizons
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   - Perturbative set up to determine future development
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Thanks for your attention! Questions?

BACKUP

The Standard Model – Particle content

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"Low" energy degrees of freedom (after symmetry breaking):

- 1 Higgs particle ($s = 0$),
- 3 families of leptons and 3 of quarks ($s = 1/2$),
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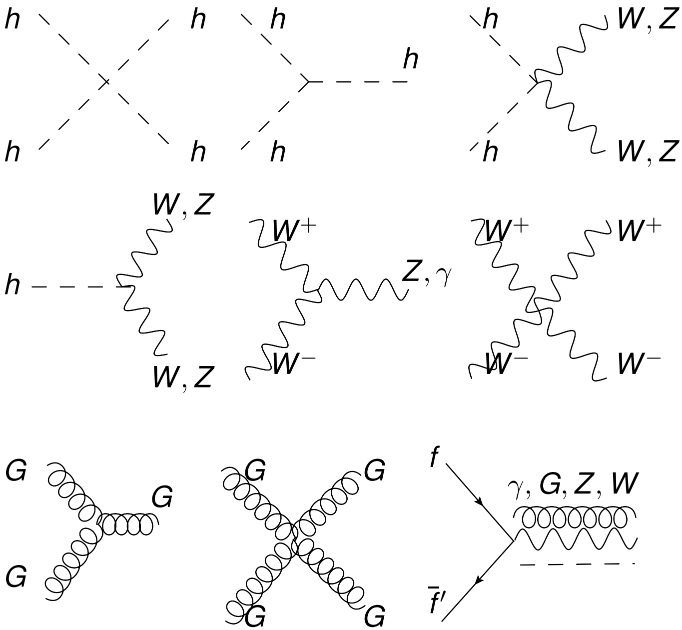

The hierarchy problem: SM vs Gravity

The action for **gravity coupled to matter** is

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Operator type	Couplings	at $E \sim 1\ \mathrm{TeV}$
$T_{\alpha\beta} h^{\alpha\beta}$	E/M_4	10^{-16}
SM Interactions	$\sim e, g_{QCD}, \frac{m_H}{v}, \frac{v}{E}, \frac{m_f}{v}$	$O(10^{-6}) - O(1)$

Solving the hierarchy problem with Extra Dimensions

$$\mathbf{M_4 \sim 10^{16} M_{EW}}$$

Hierarchy due to taking the scale for new physics from gravity (mesoscopic) rather than the electroweak scale (microscopic).

The ADD solution: Assume $\mathbf{M_{EW}}$ is more fundamental.

N. Arkani-Hamed et al. hep-th/9803315 (ADD)

- Assume our space time is 4+n dimensional

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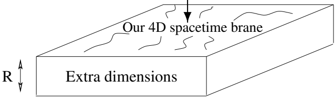

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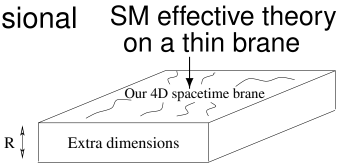

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1. Predicts **deviations from Newtonian gravity** as we approach short distances.
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- **SM on a 4D brane of thickness L $\lesssim (1\text{TeV})^{-1} \sim 10^{-13}\mu$m**
  To avoid bounds from Electroweak precision and fast proton decay.
  Quarks and leptons may have to be on sub-branes for $L \lesssim (1\text{TeV})^{-1}$.
- **All SM particles propagating on a single brane.**
  Good approximation if process occurs at large scales compared to L.

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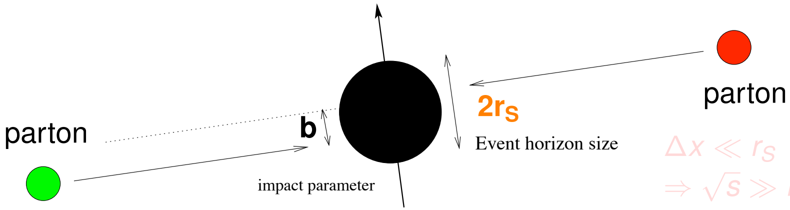

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S. B. Giddings and S. D. Thomas, hep-ph/0106219

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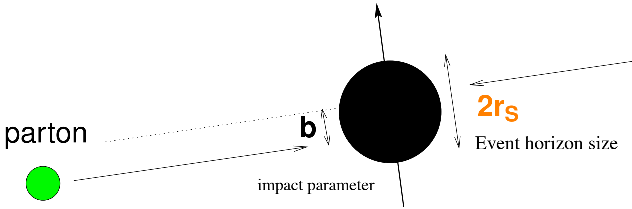

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Evidence for classical BH in transplanckian scattering

- Numerical relativity in 4 and higher dimensions

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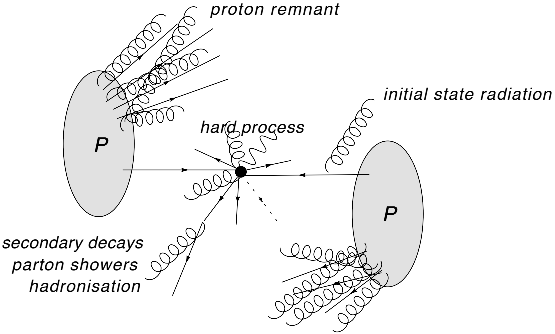

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CMS collaboration arXiv:1012.3357

ATLAS-CONF-2011-065

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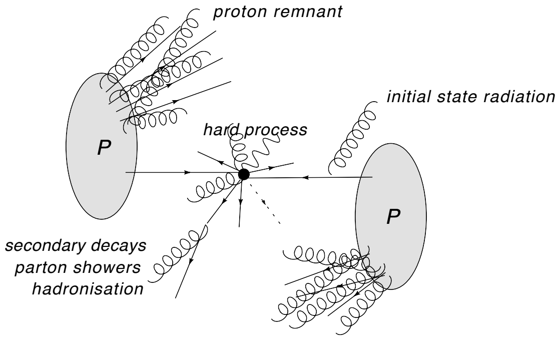

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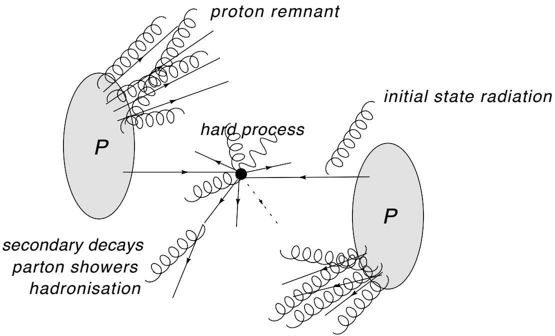

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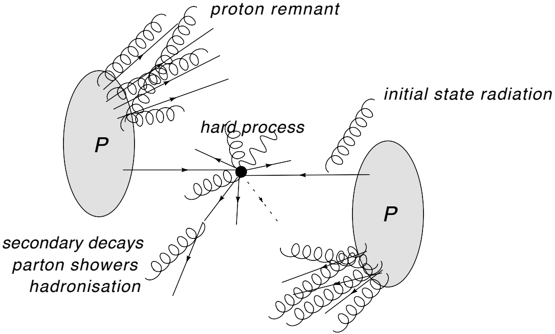

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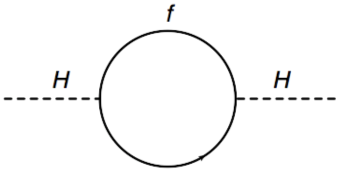

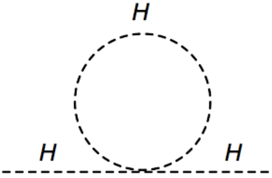

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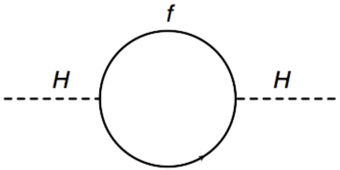

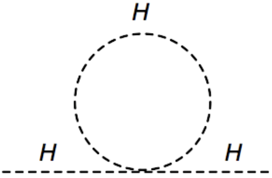

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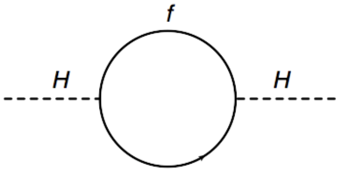

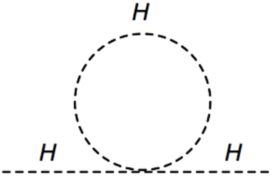

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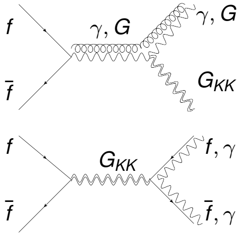

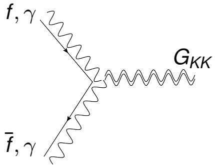

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Also quantum gravity approximations indicate small corrections:

T. Banks and W. Fischler, hep-th/9906038

S. N. Solodukhin, hep-ph/0201248

S. D. H. Hsu, hep-ph/0203154

Transient period

- During formation we should have an asymmetric BH with electric and gravitational multipole moments.

  →**Distorted geometry.**

- The time for loss of multipoles is r_S (natural units).
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