Introduction to Cosmology

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LECTURE 8 - Thermal history of the Universe III

In this lecture we will finish our study of the thermal history of the universe by discussing the important process of recombination of protons and electrons into Hydrogen atoms and the subsequent decoupling of photons that leads to the cosmic background of microwave radiation that we observe today. We will also compute the thermal abundance of weakly interacting massive particles that are expected to form most of the dark matter in the universe.

Cosmic Microwave background

The Cosmic Microwave Background (CMB) is, as we discussed in previous lectures, one of the most important probes of the Hot Big Bang model and the physics of the very early universe. The emission of the CMB is a three-stage process occurring after the synthesis of light nuclear elements, and is intrinsically related to the formation of neutral atoms once the temperature of the universe drops sufficiently to ensure their stability. As neutral atoms do not interact significantly with photons, the universe becomes transparent and radiation propagates freely afterwards, simply redshifting with expansion to yield the microwave radiation that presently fills the whole sky. Let us then analyze each of these three stages in more detail:

(a) Recombination

As we have seen previously, for temperature below $T = 0.1$ MeV the baryons in the universe are mainly in the form of free protons and ^{4}He nuclei, with the latter representing about 25% of the total baryon mass fraction, with traces of other light elements such as deuterium and ^{3}He. Free protons remain in thermal equilibrium with electrons and photons via electromagnetic scattering process of the form:

$$p + e^- \quad \longleftrightarrow \quad H + \gamma \ , \tag{1}$$

yielding a small abundance of neutral Hydrogen atoms in equilibrium with free protons, electrons and photons. In chemical equilibrium we have $\mu_p + \mu_e = \mu_H$, noting that $\mu_\gamma = 0$ since the number of photons is not conserved in electromagnetic processes. Also, charge neutrality of the universe implies that the number of electrons and protons must be equal, i.e. $n_e = n_p$.

Once the temperature of the universe drops below the mass of the electron, $m_e = 0.511$ MeV, both protons, electrons and Hydrogen atoms are non-relativistic, so that we have in equilibrium:

$$\frac{n_H}{n_e n_p} = \frac{g_H}{g_p g_e}\left(\frac{2\pi m_H}{m_p m_e T}\right)^{3/2} \exp\left(-\frac{m_H - m_p - m_e}{T}\right) \tag{2}$$

The binding energy of Hydrogen, $B_H = m_p + m_e - m_H = 13.6$ eV, is much smaller than the proton mass, so that we may take $m_H \simeq m_p$, and using $g_H = 4$, $g_p = g_e = 2$, this simplifies to yield:

$$\frac{n_H}{n_e n_p} = \left(\frac{2\pi}{m_e T}\right)^{3/2} e^{B_H/T} \ . \tag{3}$$

Although Helium represents a significant fraction of the total baryon mass in the universe, their number density represents less than 10% of the total baryon number density, since they are four times as massive as free protons. It is thus not a bad approximation to ignore the Helium component in our discussion, so that:

$$\begin{aligned} n_B \simeq n_p + n_H &= \eta n_\gamma \\ &= \frac{2\zeta(3)}{\pi^2}\eta T^3 \ . \end{aligned} \tag{4}$$

The fractional ionization of the universe is given by:

$$X_e = \frac{n_e}{n_B} = \frac{n_p}{n_B} \ . \tag{5}$$

Substituting this into Eq. (3), we the obtain for the fractional ionization in thermal equilibrium:

$$\frac{1-X_e}{X_e^2} = \frac{n_H n_B}{n_p n_e} = \frac{2\zeta(3)}{\pi^2}\eta\left(\frac{2\pi T}{m_e}\right)^{3/2} e^{B_H/T} \ , \tag{6}$$

which is known as the Saha equation for the equilibrium ionization fraction. We can solve this for $X_e(T)$, which is illustrated in the figure below.

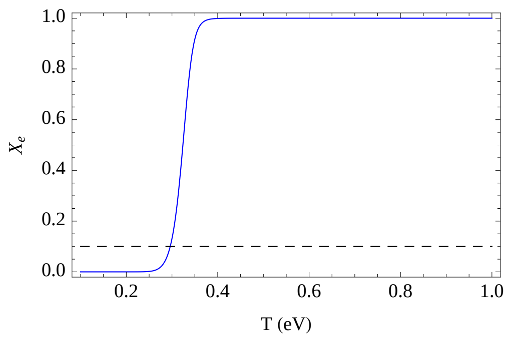

Figure 1: The equilibrium ionization fraction as a function of the temperature of the Universe for $\eta = 6 \times 10^{-10}$, emphasizing the point at which recombination is complete, $X_e = 0.1$.

One can also invert this numerically or iteratively to obtain the temperature at which recombination of protons and electrons into neutral atoms is complete. This is defined as the temperature below which $X_e < 0.1$, i.e. when 90% of the electrons have combined with protons to form Hydrogen, and one finds:

$$T_{rec} \simeq 0.3 \text{ eV} \simeq 3600 \text{ K} \ . \tag{7}$$

Notice that this is below the binding energy of Hydrogen as one could naively expect mainly due to the smallness of the baryon-to-entropy ratio. Recalling that away from mass thresholds, $T \propto a^{-1} \propto (1+z)$ and comparing with the observed temperature of the CMB today, $T_0 \simeq 2.73$ K, we find for the redshift of the recombination epoch:

$$1 + z_{rec} = \frac{T_{rec}}{T_0} \simeq 1320. \tag{8}$$

Recalling that matter-radiation equality occurred at $z_{eq} \sim 3500$, we conclude that recombination takes place already in the matter-dominated era, where $a(t) = (t/t_0)^{2/3}$, yielding:

$$t_{rec} = \frac{t_0}{(1+z_{rec})^{3/2}} \sim 288\ 000 \text{ yrs.} \tag{9}$$

(b) Photon decoupling

While protons and electrons are equilibrium, electrons are also kept in equilibrium with photons via Thomson scattering processes:

$$e^- + \gamma \longleftrightarrow e^- + \gamma \ , \tag{10}$$

with interaction rate given by:

$$\Gamma_{th} = n_e \sigma_{th} = n_e \frac{8\pi\alpha^2}{3m_e^2} \ , \tag{11}$$

where σ_{th} is the scattering cross section, $\alpha \simeq 1/137$ is the fine-structure constant characterizing electromagnetic interactions and we have used that for photons $v = c = 1$ in natural units. Equilibrium will thus be maintained while $\Gamma_{th} > H$, i.e. while there are enough free electrons for photons to scatter off. In the matter era, we have:

$$H = H_0\sqrt{\Omega_{m0}}\left(\frac{T}{T_0}\right)^{3/2} \ , \tag{12}$$

and we can equate this with Eq. (11) to find the temperature at which photons and electrons decouple. Writing $n_e = X_e n_B = X_e \eta 2\zeta(3) T^3/\pi^2$, we then find:

$$X_e(T_D) T_D^{3/2} = \frac{\pi^2}{2\zeta(3)} \frac{H_0\sqrt{\Omega_{m0}}}{\eta \sigma_{th} T_0^{3/2}} \tag{13}$$

We can solve this equation numerically for the decoupling temperature, yielding $T_D \simeq 0.27$ eV, which is slightly below the value at which recombination takes place. Although the temperature difference is small, the ionization fraction decreases significantly from recombination to decoupling, giving $X_e(T_D) \simeq 9 \times 10^{-3}$, showing that a large degree of neutrality in the plasma has to be achieved for the universe to become transparent. Proceeding as before, we find for the redshift and time of photon decoupling:

$$\begin{aligned} z_{CMB} &\simeq 1100 \ , \\ t_{CMB} &\simeq 380\,000 \text{ yrs} \ . \end{aligned} \tag{14}$$

This is called the surface of last scattering, since afterwards photons free stream across the universe without any significant interactions with electrons or any other particles. The temperature of the radiation redshifts with expansion as $T \propto a^{-1}$ and yields the blackbody spectrum observed today in the microwave region. The surface of last scattering has a non-vanishing thickness, as the process begins with recombination at $z_{rec} \sim 1320$ and ends with photon decoupling at $z_{CMB} \sim 1100$.

(c) Residual ionization

Previously we have assumed equilibrium abundances for electrons, protons and Hydrogen atoms in discussing the period of recombination. However, as we have seen in previous cases, this only holds if interactions are sufficiently fast compared to the Hubble rate. The thermally averaged cross section for the process in Eq. (1) is given by:

$$\langle \sigma_n |v| \rangle = \frac{4\pi^2\alpha}{m_e^2} \frac{B_H/n}{(3m_e T)^{1/2}} \ , \tag{15}$$

where the binding energy of the n-th excited level of the Hydrogen atom is $E_n = -B_H/n^2$. For simplicity, let us take only the ground state $n = 1$, for which:

$$\langle \sigma_{rec} |v| \rangle = \frac{4\pi^2\alpha}{m_e^2} \frac{B_H}{(3m_e T)^{1/2}} \ . \tag{16}$$

Comparing $\Gamma_{rec} = n_e\langle\sigma_{rec}|v|\rangle$to the Hubble rate during the matter era yields:

$$X_e(T_F)T_F^{3/2} = \frac{\sqrt{3}}{8\zeta(3)\alpha}\frac{H_0\sqrt{\Omega_{m0}}m_e^{5/2}}{\eta T_0^{3/2}B_H} \tag{17}$$

Solving this numerically, we find a freeze-out temperature $T_F = 0.25$ eV and a residual ionization $X_e(T_F) = 2\times10^{-3}$ that will remain constant until the first stars form, as we discuss below.

Hence, in practice the emission of the CMB consisted of three stages, occurring at slightly different redshifts, starting with electron-proton recombination, photon decoupling and the freeze out of a residual ionization in the plasma. The estimates that we computed above can be made more precise by solving the relevant Bolztmann equations for the different species involved and fitting to simple analytical expressions, so that one finds [1]:

$$\begin{aligned} 1+z_{dec} &\simeq 1100(\Omega_0/\Omega_{B0})^{0.018} \simeq 1100-1200\ , \\ 1+z_{rec} &\simeq 1380(\Omega_{B0}h^2)^{0.023} \simeq 1240-1380\ , \\ 1+z_F &\simeq \frac{1180}{1+0.021\log(\Omega_0/\Omega_{B0})} \simeq 1080-1180\ . \end{aligned} \tag{18}$$

After the Universe became transparent a period known as the “dark ages” began, with no significant electromagnetic radiation in frequencies besides those in the range covered by the CMB at any given time. However, as small perturbations about the overall homogeneous and isotropic background began to grow, overdense regions underwent gravitational collapse to form structure, giving rise to the first stars, quasars and galaxies. As it travelled through the intergalactic medium, the light from these first stars began to reionize the neutral Hydrogen and also Helium atoms, a period known as reionization. Although observational evidence for the beginning of this process is scarce and our knowledge of the dynamics of structure formation is limited, reionization of the intergalactic plasma should be completed at a redshift of $z \simeq 6.5$, about 10^9 years after the Big Bang.

Thermal abundance of WIMPs

Our discussion of the main events in the thermal history of the universe would not be complete without discussing the possibility of a thermal abundance of dark matter particles. As we have discussed in previous lectures, there is plenty indirect evidence for the existence of an additional component of non-relativistic/pressureless matter that does not interact significantly with ordinary atoms or radiation, so that its existence is inferred solely from gravitational processes. This implies that dark matter must consist of new particles with no electromagnetic or strong interactions, although they may interact via weak processes such as those mediated by the Z^0 and $W^\pm$ bosons. Neutrinos are an example of particles with only such interactions but, as computed in Problem 6, they cannot account for the full dark matter abundance with the current limits on their mass lying below 1 eV. Although slightly heavier additional neutrino species could account for the overall fraction, light dark matter particles could attain large relativistic velocities that would prevent them from clumping together and lead to the large scale structure that we observe today.

The conventional dark matter candidates are heavy particles with feeble interactions with the same or smaller strength as weak interactions in the Standard Model. Several different candidates can be found in extensions of the Standard Model such as supersymmetric theories and scenarios with compact/warped extra-dimensions, but generically these are known as Weakly Interacting Massive Particles, or WIMPs. These particles could be produced in the early universe by several different mechanisms, but the simplest one is the creation of a thermal abundance via the decoupling and freeze out mechanism that we have discussed so far. Let us then exploit this mechanism in more detail.

Let us suppose there is a single WIMP, χ, accounting for all the dark matter in the universe, with mass m_χ and g_χ internal degrees of freedom. In the early universe, during the radiation-dominated era, χ is kept in thermal equilibrium with Standard Model particles via some weakly interacting process with cross section σ, which we will take as an arbitrary parameter for now. These interactions will freeze out once the interaction rate falls below the

Hubble rate, $\Gamma_\chi = n_\chi \langle \sigma v \rangle \lesssim H$. Let us assume that χ is non-relativistic when interactions freeze out, so that:

$$n_\chi(T_F) = g_\chi \left(\frac{m_\chi T_F}{2\pi} \right)^{3/2} e^{-m_\chi/T_F} \ , \tag{19}$$

where T_F is the temperature of the thermal bath at freeze out. Using $H \simeq (\pi/\sqrt{90})\sqrt{g_*}T^2/M_P$ in the radiation era, where we recall that $M_P = (8\pi G)^{-1/2} \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass, we find the freeze out condition $\Gamma_\chi = H$ can be written as:

$$x_F^{1/2} e^{-x_F} = \frac{\pi}{\sqrt{90}} \frac{(2\pi)^{3/2}}{g_\chi} \frac{\sqrt{g_{*F}}}{m_\chi M_P \langle \sigma v \rangle} \ , \tag{20}$$

where we defined the dimensionless variable $x = m_\chi/T$, so that $x_F = m_\chi/T_F$, and g_{*F} is the number of relativistic degrees of freedom at the time of freeze out. Notice that the cross section has units of area, which in the natural units that we have been using in this course corresponds to an inverse mass squared, so that the right hand side of this condition is dimensionless as expected.

After freeze out occurs, interactions become rare, so that the total number of WIMPs is conserved, $N_\chi = n_\chi/s$. We may then use the conservation of the particle number to compute the relic abundance of WIMPs today:

$$\begin{aligned} \Omega_{\chi 0} &= \frac{\rho_{\chi 0}}{\rho_{c0}} \\ &= \frac{m_\chi n_{\chi 0}}{3H_0^2 M_P^2} \\ &= \frac{m_\chi s_0 N_\chi}{3H_0^2 M_P^2} \\ &= \frac{m_\chi n_\chi(T_F)}{3H_0^2 M_P^2} \frac{s_0}{s_F} \ , \end{aligned} \tag{21}$$

where s_0 and s_F denote the entropy density today and at the time of freeze-out, respectively, so that $s_0/s_F = (g_{*0}/g_{*F})(T_0/T_F)^3$. Note that, for simplicity, we will neglect the effects of other decoupled species, as is presently the case of neutrinos for example, such that we will not distinguish between g_* and g_{*S}. Using $n_\chi(T_F) = H(T_F)/\langle \sigma v \rangle$, we then find after some algebra:

$$\Omega_{\chi 0} = \frac{\pi g_{*0}}{3\sqrt{90}} \frac{T_0^3}{M_P^3} \frac{H_0^{-2}}{\langle \sigma v \rangle} g_{*F}^{-1/2} x_F \ . \tag{22}$$

Using that $T_0 \simeq 2.4 \times 10^{-13}$ GeV, $H_0 \simeq 10^{-42} h^2$ GeV and $g_{*0} = 3.91$, this yields:

$$\Omega_{\chi 0} h^2 = \frac{4 \times 10^{-10}\ \mathrm{GeV}^{-2}}{\langle \sigma v \rangle} g_{*F}^{-1/2} x_F \ . \tag{23}$$

We may then solve this for $\langle \sigma v \rangle$ and substitute into Eq. (20) to determine the freeze our ration x_F. Taking $g_\chi = 2$ for concreteness, this yields:

$$x_F^{3/2} e^{-x_F} = (\Omega_{\chi 0} h^2) g_{*F} \frac{6.4 \times 10^9\ \mathrm{GeV}^2}{m_\chi M_P} \ . \tag{24}$$

This equation can be solved numerically for different values of the WIMP mass, and taking $g_{*F} \sim 10$ and $\Omega_{\chi 0} h^2 \simeq 0.1$ [2] we obtain $x_F \sim 20 - 30$ for $m_\chi = 0.1 - 10^3$ GeV. This then implies:

$$\Omega_{\chi 0} h^2 = \frac{3.2 \times 10^{-9}\ \mathrm{GeV}^{-2}}{\langle \sigma v \rangle} \sqrt{\frac{10}{g_{*F}}} \left(\frac{x_F}{25} \right) \ . \tag{25}$$

From this we can deduce that the typical value of the cross section for WIMP interactions that yields a WIMP relic density compared to the observed dark matter abundance is $\langle \sigma v \rangle \sim 10^{-8}$ GeV^{-2}. Converting this to conventional units, this corresponds to $\langle \sigma v \rangle \sim 0.1c$ pb, where c is the speed of light and 1b $= 10^{-24}$ cm^2. This value for the average WIMP cross section is quite surprising, since it corresponds to the typical values of processes occurring just above the electroweak scale, and which are currently being probed at the Large Hadron Collider. To see this more explicitly, we can write on dimensional grounds $\langle \sigma v \rangle = \alpha^2/M^2$, where α is an effective coupling and M the typical energy scale of the process. This gives:

$$\Omega_{\chi 0} h^2 \simeq 0.1 \left(\frac{10}{g_{*F}}\right)^{1/2} \left(\frac{x_F}{25}\right) \left(\frac{0.2}{\alpha}\right)^2 \left(\frac{M}{1 \text{ TeV}}\right)^2 . \tag{26}$$

Particle physicists expect new physical processes and new particles to arise at scales $M \gtrsim 1$ TeV, such as supersymmetry or extra-dimensions as discussed above. It is thus a remarkable coincidence that processes around this scale give the correct relic abundance for WIMPs to account for the dark matter in our universe, which is sometimes called the "WIMP miracle". There are hence great expectations of finding evidence for dark matter at the LHC, as even though dark matter particles do not interact via strong or electromagnetic interactions and cannot be detected directly, they may leave unique signatures via the weak processes one expects them to be involved with.

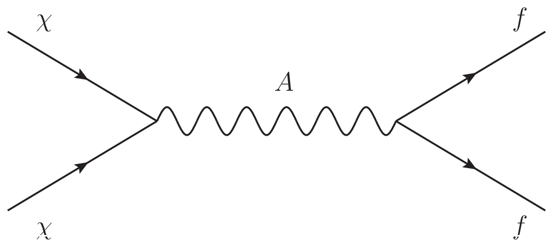

Figure 2: Feynman diagram for a generic scattering process involving WIMPs χ and Standard Model particles, f, mediated by a gauge boson A^μ.

A final interesting observation to make is that scattering processes are typically mediated by gauge bosons. If WIMP interactions with Standard Model particles are mediated by a gauge boson of mass M_A, as illustrated in Figure 2, for non-relativistic WIMPs we have:

$$\langle \sigma v \rangle \sim \alpha^2 \frac{m_\chi^2}{M_A^4} , \tag{27}$$

so that the mass scale defined above is in fact $M = M_A^2/m_\chi$. This then implies that

$$\Omega_{\chi 0} h^2 \propto \frac{1}{\langle \sigma v \rangle} \propto \frac{1}{m_\chi^2} , \tag{28}$$

and one may conclude that the relic abundance diverges as $m_\chi \to 0$, i.e. that light WIMPs could overclose the universe. However, very light dark matter particles could be relativistic at decoupling, for which the above analysis is not valid. Furthermore, as discussed earlier, if dark matter particles have too large velocities they can avoid gravitational collapse and hence structure formation, being hence disfavoured. On the other hand, for heavy WIMPs, the cross section may have the opposite behaviour, $\sigma \propto m_\chi^2$, which yields an upper bound on the WIMP mass $m_\chi < 340$ TeV as obtained in [3].

Problem 8

The interstellar plasma is reionized after recombination by the light of the first stars, and $X_e \to 1$. Compute the largest redshift and temperature at which this may happen without making the universe opaque again, i.e. keeping $\Gamma_{th} < H$ afterwards. You may assume this happens during the matter-dominated era and use $\Omega_{m0} = 0.3$.

References

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[3] K. Griest and M. Kamionkowski, Phys. Rev. Lett. **64**, 615 (1990).