Introduction to Cosmology

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LECTURE 7 - Thermal history of the Universe II

In this lecture we will continue our study of the thermal history of the universe, analyzing two important aspects of Big Bang cosmology where departures from thermal equilibrium play a crucial role. We will begin by studying generic models for the creation of a baryon asymmetry in our universe and then describe the synthesis of light nuclear elements in the early universe.

Baryogenesis

The existence of anti-matter has been known for some time. The original theoretical proposal goes back to 1928 and the British physicist Paul A. M. Dirac, who in developing his generalization of the Schrödinger equation for relativistic spin-1/2 fermions, such as the electron, realized that his description required the existence of anti-electrons - particles with the same spin and mass as the electron but opposite charge. The positron was discovered four years later by Carl Anderson, and today we know that all particles have an associated anti-particle, although neutral particles as the photon are their own anti-particles.

This poses a cosmological problem, however, since we know that particles and anti-particles quickly annihilate each other, in processes such as $e^+e^- \rightarrow \gamma\gamma$. If these annihilations are efficient in the early universe and there are equal amounts of matter and anti-matter in the universe, there would be very few matter particles in the universe, which would contain mostly radiation. We know, however, that we are made essentially of particles as protons, neutrons and electrons and that is very little anti-matter in our astrophysical vicinity. For example, measurements of cosmic rays yield a flux of anti-protons about 10^4 times smaller than the corresponding proton flux.

The overabundance of matter over anti-matter could be a local observation, with other regions of the Universe having an opposite overabundance and giving an overall symmetric Universe. However, this would imply that at the boundary between regions with opposite overabundances annihilations of matter and anti-matter particles would lead to an enormous flux of radiation that has not been observed. It is thus widely accepted that at least the observable part of our Universe must have developed an excess of particles over anti-particles at some point in the cosmological history.

Of particular importance is the overabundance of baryons, essentially protons and neutrons, over anti-baryons, a crucial aspect for the synthesis of light nuclear elements in the early Universe as we mentioned previously and discuss in more detail below. From the light element abundances and measurements of the CMB anisotropy spectrum, we can infer the present baryon-to-entropy ratio to be:

$$\eta_s = \frac{n_B}{s} = \frac{n_b - n_{\bar{b}}}{s} \simeq 10^{-10} , \tag{1}$$

a quantity that remains constant if there are no processes producing more baryons than anti-baryons. Baryon number is actually an apparently accidental global symmetry of the Standard Model (SM). Protons and neutrons have baryon number $B = +1$ while their anti-particles carry a $B = -1$ baryonic charge, corresponding to $B = +1/3$ for the three elementary quarks that constitute them and $B = -1/3$ for the associated anti-quarks. All perturbative processes in the SM conserve this charge, although such a symmetry was not an initial ingredient of the theory. The same is true for lepton number, which is $L = +1$ for electrons and neutrinos and the opposite for their anti-particles.

Annihilations of baryons and anti-baryons are not completely efficient, and in a baryon-symmetric universe we could still find small freeze-out abundances $n_b/s \sim n_{\bar{b}}/s \sim 10^{-21}$, which is however too small compared to the observed asymmetry.

This implies that there must be some mechanism occurring in the early universe that produces an overabundance of baryons compared to anti-baryons and which is generically known as baryogenesis. A strict requirement is that baryogenesis takes place before Big Bang nucleosynthesis, for $T \gtrsim 10$ MeV, so that the initial conditions for the production of light nuclear elements are in place. Several models have been proposed in the literature, typically based on extensions of the SM, and although quite different in detail, all these models must satisfy the three conditions first described by Andrei Sakharov in 1967 for the production of a baryon asymmetry [1]:

1. **B-violation**

   The requirement of interactions that do not conserve baryon number B is quite obvious, or otherwise all processes would produce the same number of baryons and anti-baryons.

   The prototypical example of a B-violating process is the decay of heavy gauge or Higgs bosons that arise in extensions of the SM that unify the three known gauge interactions - strong, weak and electromagnetic. Generically, these grand unified theories (GUT) are based on gauge symmetries that enhance the SM gauge group $SU_c(3) \times SU(2)_W \times U(1)_Y$, and that are spontaneously broken at high energy scales $M_{GUT} \sim 10^{16}$ GeV via a mechanism similar to the electroweak Higgs mechanism. This leaves the SM gauge group as the only exact symmetry at low energies but explains the apparent unification of the associated gauge couplings at energies $E > M_{GUT}$. In the process, some of the original Higgs and gauge bosons acquire large masses, and typically exhibit B-violating interactions that allow them to decay into quarks and leptons. For example, for the simplest GUT with $SU(5)$ gauge group we have:

   $$X \to qq, \qquad X \to \bar{q}\bar{l} \ , \tag{2}$$

   where X represents a generic gauge or Higgs boson and q, l denote the SM quarks and leptons, respectively. The first decay channel gives a $B = +2/3$ and $L = 0$ final state, while the second yields $B = -1/3$ and $L = -1$. This means that there is no consistent assignments of a baryon or lepton number to the boson X and that both charges are not conserved in these decays. Notice that $B - L = +2/3$ for both decays, so that this combination is still a symmetry of the underlying GUT, which is however not always the case.

2. **Departure from thermal equilibrium**

   The second of Sakharov's condition is also easy to understand in a cosmological constant, given our discussion in the previous lecture. Let us consider a generic process $X_0 \to Y_0 + Z_B$, with X_0 representing a generic initial state with vanishing baryon number, Y_0 denoting all particles in the final state also with vanishing baryonic charge and Z_B corresponding to all produced particles with an overall B-charge. If this process is in thermal equilibrium, than its rate must be equal to the rate of the inverse process:

   $$\Gamma[X_0 \to Y_0 + Z_B] = \Gamma[Y_0 + Z_B \to X_0] \ , \tag{3}$$

   and not net baryon asymmetry can arise from this since B is produced at the same rate that it is destroyed. We thus need a departure from thermal equilibrium that suppresses the inverse processes for a viable baryogenesis mechanism.

   In the GUT example above, departures from thermal equilibrium may occur when the X-boson is non-relativistic, $m_X \gtrsim T$, at the time of decay, $t \sim 1/\Gamma$. In this case, the X boson may decay into relativistic quarks and leptons with typical energies and momenta $\sim T$, while there is not enough energy for the latter to annihilate back into an X boson. In practice, there are always some particles in the thermal bath with sufficiently large energies and momenta, but these correspond to the tails of the statistical thermal distribution, so that the rate of the inverse processes, $qq \to X$ or $\bar{q}\bar{l} \to X$ is Boltzmann-suppressed by a factor $e^{-m_X/T}$.

3. **C and CP violation**

The third Sakharov condition is most subtle requirement for a baryogenesis mechanism. Let us again consider the generic process $X_0 \to Y_0 + Z_B$. If charge conjugation C is a symmetry of the interactions, then we expect the rate of the charge-conjugated process involving the anti-particles, $\bar{X}_0 \to \bar{Y}_0 + \bar{Z}_B$, to be the same:

$$\Gamma[X_0 \to Y_0 + Z_B] = \Gamma[\bar{X}_0 \to \bar{Y}_0 + \bar{Z}_B] \ , \tag{4}$$

Again this means that a baryon excess B is being produced at the same rate as the opposite excess $-B$, with no net baryon asymmetry resulting.

Even if C is violated in some process, this may not be sufficient, since the convolution of charge conjugation and a parity transformation, CP must also be violated. It is a well established fact that SM fermions, i.e. quarks and leptons, come in opposite parity states with different charges under the SM gauge group. The intrinsic parity of a particle corresponds to the way its wavefunction transforms under spatial inversions $\mathbf{x} \to -\mathbf{x}$, and spin-1/2 fermions can be left- or right-handed. In the SM, for example, left-handed fermions come in doublet representations of the weak isospin gauge group $SU(2)_W$, while right-handed fermions correspond to singlet representations with no weak isospin.

To illustrate the need for CP violation, consider a scenario where our X boson can decay into either left-handed or right-handed quarks, $X \to q_L q_L$ and $X \to q_R q_R$. Under C, $q_L \to \bar{q}_L$, while under CP we have $q_L \to \bar{q}_R$, where $\bar{q}_R$ is the anti-particle of q_R, which is left-handed. Similarly, we have $q_R \to \bar{q}_R$ under C and $q_R \to \bar{q}_L$ under CP. In this case, violation of C ensures that

$$\Gamma[X \to q_L q_L] \neq \Gamma[\bar{X} \to \bar{q}_L \bar{q}_L] \ , \tag{5}$$

but if CP is conserved, we have:

$$\begin{aligned} \Gamma[X \to q_L q_L] &= \Gamma[\bar{X} \to \bar{q}_R \bar{q}_R] \ , \\ \Gamma[X \to q_R q_R] &= \Gamma[\bar{X} \to \bar{q}_L \bar{q}_L] \ , \end{aligned} \tag{6}$$

and taking into account both decays, we find:

$$\Gamma[X \to q_L q_L] + \Gamma[X \to q_R q_R] = \Gamma[\bar{X} \to \bar{q}_R \bar{q}_R] + \Gamma[\bar{X} \to \bar{q}_L \bar{q}_L] \ . \tag{7}$$

Then, if we have equal numbers of X particles and $\bar{X}$ anti-particles initially, we will end up with the same number of quarks and anti-quarks, even though an asymmetry between left-handed and right-handed particles may be produced. Note that in this example both decay channels have $B = +2/3$ in the final state, and we need additional decay channels in order to satisfy the first of Sakharov's conditions and have violation of baryon number conservation.

Having enumerated the Sakharov conditions for a successful baryogenesis, let us now look at the GUT baryogenesis model in more detail. The relevant interactions in this model correspond to the decays described above, annihilations of X and $\bar{X}$ bosons and the corresponding inverse processes. Annihilations processes are 'self-quenching', since the rates are proportional to n_X and naturally become suppressed as the universe expands and n_X is diluted. We can then discard annihilations to a first approximation, so that the Boltzmann equation for the evolution of n_X is given by:

$$\dot{n}_X + 3Hn_X = -\Gamma_D (n_X - n_X^{EQ}) \ , \tag{8}$$

where Γ_D denotes the decay width of the X bosons. This is slightly different from the case we have seen in the previous lecture where the collision term corresponded to $2 \to 2$ scattering processes and was then proportional to $n_X^2 - n_X^{EQ2}$.

Let us consider a simple toy model with two possible B-violating decay channels such that:

$$\begin{aligned} \Gamma[X \to B = +1] &= \frac{1}{2}(1+\epsilon)\Gamma_D \ , \\ \Gamma[X \to B = -1] &= \frac{1}{2}(1-\epsilon)\Gamma_D \ , \end{aligned} \tag{9}$$

where ϵ quantifies the amount of C- and CP-violation. Then, the Boltzmann equation for the number densities of baryons and anti-baryons is given by:

$$\begin{aligned} \dot{n}_b + 3Hn_b &= -\frac{1}{2}(1+\epsilon)\Gamma_D(n_X - n_X^{EQ}) \ , \\ \dot{n}_{\bar{b}} + 3Hn_{\bar{b}} &= -\frac{1}{2}(1-\epsilon)\Gamma_D(n_X - n_X^{EQ}) \ , \end{aligned} \tag{10}$$

so that subtracting these equations we get for $n_B = n_b - n_{\bar{b}}$:

$$\dot{n}_B + 3Hn_B = \epsilon\Gamma_D(n_X - n_X^{EQ}) \ . \tag{11}$$

We thus see that $\epsilon > 0$ and departures from thermal equilibrium are crucial in obtaining a non-vanishing baryon asymmetry. For a generic heavy gauge or Higgs boson, we have:

$$\Gamma_D \simeq \alpha m_X \begin{cases} \frac{m_X}{T} & , \ T \gtrsim m_X \\ 1 & , \ T \lesssim m_X \end{cases} \ , \tag{12}$$

where $\alpha = g^2/4\pi$ corresponds to the relevant coupling constant and the suppression factor at high-temperatures is due to time dilation, taking into account the Lorentz boost factor $\gamma = E/m_X \sim T/m_X$ for $T \gtrsim m_X$, while $\gamma \sim 1$ for $T \lesssim m_X$. This implies that very few bosons decay at high temperatures $T \gg m_X$, and for an initial population in local thermal equilibrium one expects $n_X \sim n_{\bar{X}} \sim n_\gamma$. The crucial aspect is whether decays and inverse decays are effective at the time the bosons become non-relativistic, $T \sim m_X$, since at this point their number density must decrease exponentially in order to preserve their equilibrium distribution. Recalling that baryogenesis must occur in the radiation era before nucleosynthesis takes place, with $H \simeq 0.33\sqrt{g_*}T^2/M_P$, we have for $T = m_X$:

$$K \equiv \frac{\Gamma_D}{H}\bigg|_{T=m_X} \sim \frac{\alpha M_P}{0.33\sqrt{g_*}m_X} \tag{13}$$

If $K \ll 1$, decays are ineffective at this stage, and one can expect a significant departure from thermal equilibrium and, hence an overabundance of X and $\bar{X}$ bosons relative to their equilibrium distribution. At the time the bosons effectively decay, when $\Gamma_D \sim H$ or equivalently $T \sim \sqrt{K}m_X \ll m_X$, we have $n_X \sim n_\gamma \gg n_X^{EQ}$ yielding a net baryon number:

$$n_B \simeq \epsilon\Gamma_D n_X \Delta t \simeq \epsilon n_\gamma \ , \tag{14}$$

where we used $\Delta t \sim \Gamma_D^{-1}$ for the effective decay time. Hence, as $s \sim g_* n_\gamma$, the baryon-to-entropy ratio resulting from the out-of-equilibrium decay of the heavy bosons is:

$$\eta_s \simeq \frac{\epsilon}{g_*} \ . \tag{15}$$

For $g_* \simeq 10^2 - 10^3$ typical of GUT models with a large number of relativistic degrees of freedom, we see that only a tiny amount of C and CP violation is required, with $\epsilon \sim 10^{-8} - 10^{-7}$.

In the opposite limit, $K \gg 1$, decays and inverse decays are in equilibrium for $T \sim m_X$, and n_X will track its equilibrium distribution closely, so that no significant departures from thermal equilibrium will occur and no baryon asymmetry can develop. In the regime $K \simeq 1$ a sizeable asymmetry may still be produced, although in this case a detailed analysis of the Boltzmann equation is required. Note that $K \lesssim 1$ requires heavy bosons, as from Eq. (13):

$$m_X \gtrsim \frac{\alpha}{0.33 g_*} M_P \sim \left(\frac{\alpha}{0.01}\right) 10^{16} \text{ GeV} \ . \tag{16}$$

For a GUT gauge boson $\alpha = \alpha_{GUT} \simeq 1/45$, while for a Higgs boson the coupling can be much smaller so that $K \ll 1$ for $m_X \simeq M_{GUT}$. Note also that in our discussion we have neglected the effects of $2 \to 2$ scatterings of the baryons and anti-baryons, which also contain in general a C- and CP-violating contribution and may damp the produced asymmetry. However, this only becomes significant for $K \gg 1$ and may in general be neglected in the most relevant case $K \ll 1$.

The GUT baryogenesis mechanism is actually disfavoured, since it requires large temperatures in the early Universe, $T \gtrsim M_{GUT}$, in order to have an initial population of gauge or Higgs bosons in local thermal equilibrium, in particular after inflation as we will discuss later on in this course. Nevertheless, this model illustrates the basic properties of baryogenesis models and the Sakharov conditions, and alternative GUT mechanisms based on non-thermal or quasi-thermal distributions of heavy bosons have been proposed in the literature.

Although, as we have mentioned earlier, B is conserved by all perturbative processes in the SM, there are actually B-violating non-perturbative processes known as sphalerons in the electroweak theory. They correspond to tunneling processes between different vacuum states due to the non-trivial vacuum structure of gauge theories, and as such are typically exponentially suppressed. However, at high temperatures these processes become classically allowed and may occur in thermal equilibrium for 100 GeV $\lesssim T \lesssim 10^{13}$ GeV. These processes conserve the combination $B - L$ while violating $B + L$, the main problem being the too small amount of CP-violation in the SM, corresponding roughly to $\epsilon \sim 10^{-20}$. If other sources of CP-violation can be found in extensions of the SM, such as supersymmetric theories, the electroweak phase transition when the Higgs field acquires a non-zero vacuum expectation value may provide the necessary out-of-equilibrium condition if it is strongly first-order. The addition of heavy right-handed neutrinos to explain the smallness of the (left-handed) SM neutrino masses via the *see-saw* mechanism also leads to successful models of *leptogenesis*, as similarly to the model we described above they can decay out of equilibrium and produce a net lepton number L, which electroweak sphalerons may later convert into a B-asymmetry since only $B - L$ is conserved. This leads to an important general issue, since sphalerons may actually wash out any baryon asymmetry produced at high temperatures, $T \gg 10^{13}$ GeV, so that either L or $B - L$ asymmetries lead to more robust mechanisms. There are several other models of baryogenesis in the market, many of which unfortunately cannot be tested directly in the laboratory with the present technology, not even the powerful LHC, as it “only” reaches energies $\sim 10^3$ GeV that are much below the GUT scale. These models are too many to be presented in these lectures (see e.g. [2] for a pedagogical review of baryogenesis mechanisms), but an important scenario to mention is the Affleck-Dine mechanism [3], in which a homogeneous scalar field acquires a non-zero baryon number, a feature that we will explore in Problem 7.

Big Bang nucleosynthesis

As we have briefly discussed before, the synthesis of the light nuclear elements and the predictions for their cosmological abundances is one of the great successes of the Hot Big Bang model, yielding tight constraints on any modifications of the cosmological evolution at temperatures around $T \lesssim 10$ MeV. While heavy elements can be generated in the interior of stars via nuclear fusion processes, the lighter elements such as deuterium (D), ^{3}He, ^{4}He and ^{7}Li cannot be produced in this way, with the corresponding ratios approaching zero values in young stars. The observed abundances have thus to be present already in the primordial gas.

For a generic non-relativistic nuclear species with Z protons and $A - Z$ neutrons, where A is the mass number, the number density in thermal equilibrium is given by:

$$n_A = g_A \left(\frac{m_A T}{2\pi}\right)^{3/2} \exp\left(\frac{\mu_A - m_A}{T}\right) . \tag{17}$$

Thermal equilibrium requires the nuclear reactions that produce such a nucleus from Z protons and $A - Z$ neutrons to be faster than the Hubble rate, as we have seen before, and in this case the species is in both kinetic and chemical equilibrium. The chemical potential of the species is then related to the proton and neutron chemical potentials via:

$$\mu_A = Z\mu_p + (A - Z)\mu_n . \tag{18}$$

We can use this fact to express the exponential factor in terms of the proton and neutron number densities:

$$\exp\left(\mu_A/T\right) = \exp\left(\frac{Z\mu_p + (A - Z)\mu_n}{T}\right) = n_p^Z n_n^{A-Z} 2^{-A} \left(\frac{2\pi}{m_N T}\right)^{\frac{3A}{2}} \exp\left(\frac{Zm_p + (A - Z)m_n}{T}\right) , \tag{19}$$

where we have used that $m_n - m_p \simeq 1.29$ MeV $\ll m_p$, defining $m_N = m_P$ in the pre-factor, and that $g_p = g_n = 2$. Now, the binding energy of the nuclear species is defined as:

$$B_A = Zm_p + (A-Z)m_n - m_A \ , \tag{20}$$

and replacing Eqs. (19) and (20) into Eq. (21), taking into account that $m_A \simeq Am_N$ for small binding energies, we obtain to leading order:

$$n_A = g_A A^{3/2} 2^{-A} \left(\frac{2\pi}{m_N T}\right)^{3(A-1)/2} n_p^Z n_n^{A-Z} \exp\left(\frac{B_A}{T}\right) \ . \tag{21}$$

To have an idea of the values involved, the smallest binding energy is for deuterium, with $B_2 = 2.22$ MeV ($g_2 = 3$) and for ^{4}He we have $B_4 \simeq 28.3$ MeV ($g_4 = 1$), which are well below the nuclear masses $m_A \simeq A$ GeV.

Since all particle number densities decrease as a^{-3} if the particle number per comoving volume is conserved, It is convenient to define the mass fraction of each nuclear species as:

$$\begin{aligned} X_A &= \frac{n_A A}{n_B} \\ &= g_A \left[\zeta(3)^{A-1} \pi^{\frac{1-A}{3}} 2^{\frac{3A-5}{2}}\right] A^{5/2} \left(\frac{T}{m_N}\right)^{3(A-1)/2} \eta^{A-1} X_p^Z X_n^{A-Z} \exp\left(\frac{B_A}{T}\right) \ , \end{aligned} \tag{22}$$

where we have defined the baryon-to-photon ratio $\eta = n_B/n_\gamma \simeq 7\eta_S$, with $n_\gamma = (2\zeta(3)/\pi^2)T^3$. For example, this yields:

$$\begin{aligned} X_2 &= 16.3 \left(\frac{T}{m_N}\right)^{3/2} \eta e^{B_2/T} X_n X_p \ , \\ X_3 &= 57.4 \left(\frac{T}{m_N}\right)^{3} \eta^2 e^{B_3/T} X_n X_p^2 \ , \\ X_4 &= 113 \left(\frac{T}{m_N}\right)^{9/2} \eta^2 e^{B_4/T} X_n^2 X_p^2 \ , \\ X_{12} &= 3.2 \times 10^5 \left(\frac{T}{m_N}\right)^{33/2} \eta^{11} e^{B_{12}/T} X_n^6 X_p^6 \ . \end{aligned} \tag{23}$$

At large temperatures, $T \gg 1$ MeV, neutrons and protons are kept in thermal equilibrium by weak interactions:

$$\begin{aligned} n &\longleftrightarrow p + e^- + \bar{\nu}_e \ , \\ \nu_e + n &\longleftrightarrow p + e^- \\ e^+ + n &\longleftrightarrow p + \bar{\nu}_e \end{aligned} \tag{24}$$

This sets the chemical equilibrium condition $\mu_n + \mu_\nu = \mu_p + \mu_e$, giving:

$$\frac{X_n}{X_p} = \frac{n_n}{n_p} = \exp\left(-\frac{Q}{T} + \frac{\mu_e - \mu_\nu}{T}\right) \ , \tag{25}$$

where $Q = m_n - m_p = 1.29$ MeV. From the charge neutrality of the Universe, we expect $\mu_e/T \sim n_e/n_\gamma = n_p/n_\gamma \sim \eta \ll 1$, so that we usually may neglect the chemical potential for the electron. This should hold for the neutrinos as well, although the relic neutrino background has yet to be detected, but in general we expect that $X_n/X_p \simeq e^{-Q/T}$ to hold. Since all baryons must be either in the form of free nucleons or in bound states, we have:

$$\sum_i X_i = X_p + X_n + X_2 + X_3 + X_4 + X_{12} + \ldots = 1 \ , \tag{26}$$

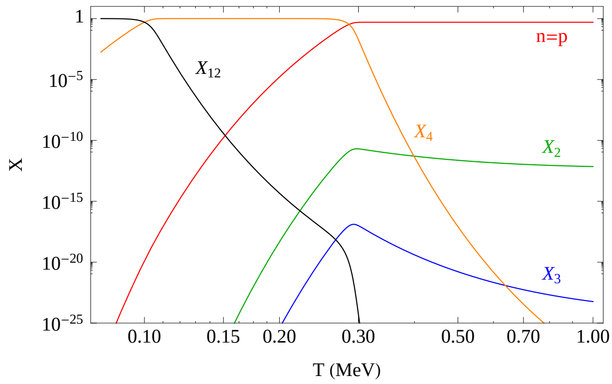

Figure 1: Evolution of the mass fractions in thermal equilibrium for a system of neutrons, protons, D, ^{3}He, ^{4}He and ^{12}C as a function of the temperature. For simplicity, we take $X_n = X_p$ neglecting the neutron-proton mass difference.

and we can use this to determine the thermal abundances of each nuclear species. In Figure 1 we illustrate this for a simplified case where we take $X_n = X_p$ and consider only the abundances of D, ^{3}He, ^{4}He and ^{12}C.

As one can easily see in this figure, at large temperatures $T > 1$ MeV neutrons and protons are basically free and around 0.3 MeV the thermal mass fractions of light elements begin to rise. This may be surprising since the average binding energy per nucleon varies between 1 and 8 MeV, but from Eq. (23) we see that heavier elements are suppressed by larger powers of $\eta \ll 1$, which explains why the temperature needs to drop somewhat below 1 MeV for their relative abundances to increase. The figure also shows that in thermal equilibrium heavier elements will eventually be the dominant form of baryonic matter, and in the example this corresponds to carbon.

Hence, if thermal equilibrium was the whole story we would end up with very little Hydrogen or Helium around, which is not what is observed in the present Universe. However, as we have seen in several examples before, departures from thermal equilibrium are crucial in understanding the evolution of the Universe, and nuclear abundances are not an exception.

As we have discussed before, weak interactions freeze out around $T = T_F \simeq 1$ MeV, leading to neutrino decoupling from the plasma and later on $T \lesssim m_e/3$ to annihilation of electrons and positrons that transfer their entropy to photons, raising the photon temperature with respect to the neutrino temperature. The weak interactions that convert neutrons into protons and vice-versa also freeze out around this time, $t \simeq 1$ sec, yielding:

$$\left.\frac{n_n}{n_p}\right|_F \simeq e^{-Q/T_F} \simeq \frac{1}{6} \ . \tag{27}$$

After this the neutron-to-proton ratio does not really remain constant since some neutrons can still decay, $n_n = n_n(T_F)e^{-t/\tau_n}$, where $\tau_n \simeq 886$ sec. This will decrease the neutron-to-proton ratio by the time nuclesynthesis actually takes place, for $t \simeq 100$ sec, yielding:

$$\left.\frac{n_n}{n_p}\right|_{NUC} \simeq \frac{1}{7} \ . \tag{28}$$

As we have seen above, for $T \simeq 0.3$ MeV the thermal equilibrium mass fraction of ^{4}He begins to increase very quickly and to approach unity. Howeverm this assumes that the reactions that produce ^{4}He are in thermal equilibrium. These reactions are mainly:

$$\begin{aligned} \mathrm{D}+\mathrm{D} &\longrightarrow {}^3\mathrm{He}+n \ , \\ {}^3\mathrm{He}+\mathrm{D} &\longrightarrow {}^4\mathrm{He}+p \ , \end{aligned} \tag{29}$$

$$\begin{aligned} \mathrm{D}+\mathrm{D} &\longrightarrow {}^3\mathrm{H}+p\ , \\ {}^3\mathrm{H}+\mathrm{D} &\longrightarrow {}^4\mathrm{He}+n\ , \end{aligned} \tag{30}$$

and

$$\mathrm{D}+\mathrm{D} \longrightarrow {}^4\mathrm{He}+\gamma\ . \tag{31}$$

Hence, the rate at which ^{4}He is produced depends on the number densities of the lighter elements, which in thermal equilibrium are very small as we have seen above. Furthermore, Coulomb-barrier suppression becomes more significant as the temperature drops, and one needs two protons to tunnel through this repulsive barrier to form a ^{4}He nucleus. This barrier is given approximately by:

$$\langle\sigma v\rangle \propto \exp\left(-2\bar{A}^{1/3}(Z_1Z_2)^{2/3}(T/1\ \mathrm{MeV})^{-1/3}\right)\ , \tag{32}$$

where $\bar{A} = A_1A_2/(A_1+A_2)$ for the two nuclei involved in the reaction. For these reasons, around $T \simeq 0.5$ MeV the mass fraction of ^{4}He drops below its equilibrium value, as the nuclear reactions cannot keep up with expansion. However, the abundances of the lighter elements are actually starting to exceed their thermal equilibrium values and around $T = T_{NUC} \simeq 0.1$ MeV they become sufficiently large to resume the production of ^{4}He. At this stage, essentially all available neutrons are quickly bound into helium nuclei, yielding:

$$X_4 = \frac{4n_4}{n_B} \simeq \frac{4\times(n_n/2)}{n_p+n_n} = \frac{2n_n}{n_n+n_p} = \frac{2(n_n/n_p)}{1+(n_n/n_p)}\ . \tag{33}$$

Using Eq. (28), this gives:

$$X_4 = \frac{2/7}{1+1/7} = \frac{1}{4} \simeq 0.25\ , \tag{34}$$

which is in remarkable agreement with the observational value for the Helium mass fraction [4],

$$X_4 = 0.249 \pm 0.009\ . \tag{35}$$

The abundances of the lighter elements can also be estimated by computing the time at which the relevant reactions freeze out and these species decouple, yielding $X_2 \sim X_3 \sim 10^{-5}$, also in agreement with the observational values. As we discussed in the first lecture, some ^{7}Li is also produced, with a mass fraction $X_7 \sim 10^{-10}$, but the prediction exceeds the observational value by a factor $3-4$, which is known as the 'lithium problem'. This points towards a modification of the cosmological evolution that modifies the lithium abundance but not the mass fractions of the lighter elements, which poses a difficult theoretical challenge that may require beyond the SM (BSM) physical processes.

Although in thermal equilibrium we would expect elements heavier than helium to dominate the baryonic abundances at low temperatures due to their larger binding energies, the departures from thermal equilibrium described above prevent this from occurring. In particular, the 'light element bottleneck' that delays nucleosynthesis down to $T \sim 0.1$ MeV ($t \simeq 100$ sec) and the increasing Coulomb-barrier suppression prevent the formation of heavier nuclei, along with the absence of stable elements with mass numbers 5 and 8.

In stellar cores, the nuclear density is sufficiently large to enable the triple-alpha (^{4}He nuclei) reaction that can overcome these mass gaps and produce an excited state of carbon-12:

$$\begin{aligned} {}^4\mathrm{He}+{}^4\mathrm{He} &\longrightarrow {}^8\mathrm{Be}+\gamma\ , \\ {}^4\mathrm{He}+{}^8\mathrm{Be} &\longrightarrow {}^{12}\mathrm{C}^* \longrightarrow {}^{12}\mathrm{C}+\gamma\ . \end{aligned} \tag{36}$$

The produced ^{1}2C can react further to produce oxygen and nytrogen isotopes in the so-called CNO cycle, while heavier elements are produced in supernovae explosions at the end of stellar lifetimes.

The successful predictions of Big Bang Nucleosynthesis (BBN) pose tight constraints on the cosmological parameters and consequently on BSM scenarios that may change them. In particular, the light element abundances are sensitive to three important parameters:

1. **Neutron half-life**

   Neutrons decay through weak interactions of the form in Eq. (24), and all relevant weak interaction rates depend on this parameter:

   $$\Gamma_W \sim G_F^2 T^5 \propto \frac{T^5}{\tau_n} \ . \tag{37}$$

   These rates determine the temperature at which neutrons and protons decouple and $\Gamma_W \sim H \propto T^2$ implies $T_F \propto \tau_n^{1/3}$. A larger neutron half-life thus implies an earlier freeze-out and a larger value of $n_n/n_p(T_F)$, increasing the prediction for the ^{4}He abundance.

2. **Number of relativistic species**

   The freeze out of weak interactions is also determined by the value of g_*, since $H \propto g_*^{1/2} T^2$ and so $T_F \propto g_*^{1/6}$. The existence of additional massless (or very light) particles in BSM theories could then increase the ^{4}He abundance. In the SM, as we have seen previously, we expect $g_* = 5.5 + (7/4)N_\nu$, where N_ν is the number of neutrino species. The current bounds on the helium-4 abundance then give $N_\nu = 3.24 \pm 1.2$ [4], which is in very good agreement with the existence of only three SM neutrinos but may accommodate an additional degree of freedom.

3. **Baryon-to-photon ratio**

   As we have seen above, the nuclear equilibrium distributions are determined by different powers of η. This means that for larger values of η the abundances of the light elements that lead to ^{4}He formation grow earlier, yielding larger values of $n_n/n_p(T_F)$ and hence increase the helium-4 abundance. The dependence of the helium-4 prediction on η is actually much smaller than on the other two parameters, but the amounts of D and ^{3}He left after nuclear reactions freeze out do depend strongly on the baryon-to-photon ratio, decreasing as η^{-n}, with $n \sim 1-2$. BBN thus constrains $\eta = (5-7) \times 10^{-10}$ giving the value that has to produced at $T \gg 1$ MeV by a baryogenesis mechanism as described above.

Problem 7

Consider the action for a complex scalar field ϕ, with spin 0, in Minkowski space:

$$S = \int d^4x \left[-(\partial_\mu \phi)(\partial^\mu \phi^*) - V(\phi)\right] \ . \tag{38}$$

where $\phi^*(x)$ is the complex conjugate field and $V(\phi)$ an arbitrary potential.

(a) By varying this action with respect to the field or its complex conjugate, show that the associated field equations can be written as:

$$\partial_\mu \partial^\mu \phi - \frac{\partial V}{\partial \phi^*} \phi = 0 \ . \tag{39}$$

(b) Find the condition that the potential function $V(\phi)$ must satisfy for the 4-vector current:

$$j^\mu = \phi^* \partial^\mu \phi - \phi \partial^\mu \phi^* \tag{40}$$

to be conserved, i.e. $\partial_\mu j^\mu = 0$.

(c) Show that under this condition the charge $Q = \int d^3x j^0$ is conserved, i.e. $dQ/dt = 0$ and that the action is invariant under the transformation $\phi \to e^{i\alpha}\phi$ for an arbitrary constant parameter α.

(d) By writing the field in the polar form $\phi = \rho e^{i\theta}$, where $\rho = |\phi|$, verify that a non-zero charge corresponds to a non-zero angular velocity.

In extensions of the Standard Model of particle physics, there are complex scalar fields for which the associated charge Q is baryon number - the same charge carried by fundamental quarks. If the scalar potential contains terms that do not satisfy the condition found above and acquires an angular velocity in the early universe, a non-zero baryon number can be produced. These fields may subsequently decay into quarks and produce the observed baryon asymmetry. This is called the Affleck-Dine mechanism.

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