Introduction to Cosmology

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LECTURE 6 - Thermal history of the Universe I

In the next three lectures we will take a closer look at the thermodynamical evolution of the Universe as it expands from an initially hot and dense state. The notion of local thermal equilibrium is extremely important in determining how the temperature, energy density and entropy density of the Universe evolve with the Hubble expansion, so we will begin by reviewing some of the basic aspects of systems of both relativistic and non-relativistic particles in thermal equilibrium. On the other hand, departures from thermal equilibrium will allow some species to acquire a significant cosmological abundance. This is behind the origin of the Cosmic Microwave Background and Big Bang nucleosynthesis, which we will analyze in more detail later on.

Review of equilibrium thermodynamics

The perfect black body form of the CMB is the best evidence we have for local thermal equilibrium in the early Universe. In general, thermal equilibrium is the natural state for which a system of interacting particles evolves. By the time the CMB was "emitted" at photon decoupling, 379 000 years had passed since the initial singularity, which means that the Universe had more than enough time to reach this state. We expect that in the very early Universe most particles were also in thermal equilibrium with photons, so it is important to recall the basic properties of particle distributions in thermal equilibrium.

A system of particles in *kinetic equilibrium* has a phase space occupancy f given by the familiar Bose-Einstein or Fermi-Dirac distributions at temperature T:

$$f(\mathbf{p}) = \frac{1}{e^{\frac{E-\mu}{T}} \pm 1} \ , \tag{1}$$

where $E = |\mathbf{p}|^2 + m^2$ is the energy of the particles, μ the chemical potential and the $+$ sign corresponds to fermions while the $-$ to bosons. Furthermore, if a species is in *chemical equilibrium*, its chemical potential is related to the chemical potentials of the species it interacts with. For example, if a species A interacts with species B, C and D via scattering processes of the form:

$$A + B \longleftrightarrow C + D \ , \tag{2}$$

then chemical equilibrium implies $\mu_A + \mu_B = \mu_C + \mu_D$. Local thermal equilibrium is achieved for species which are both in kinetic and chemical equilibrium.

The phase space distribution allows one to compute the associated number density n, energy density ρ and pressure p for a dilute and weakly-interacting gas of particles with g internal degrees of freedom:

$$\begin{aligned} n &= g \int \frac{d^3p}{(2\pi)^3} f(\mathbf{p}) \ , \\ \rho &= g \int \frac{d^3p}{(2\pi)^3} E(\mathbf{p}) f(\mathbf{p}) \ , \\ p &= g \int \frac{d^3p}{(2\pi)^3} \frac{|\mathbf{p}|^2}{3E(\mathbf{p})} f(\mathbf{p}) \ . \end{aligned} \tag{3}$$

Note that the expression for the pressure agrees with our previous analysis of the energy-momentum tensor, $p = n\langle\gamma m v^2\rangle/3$, with the factor 3 associated with the assumed isotropy of the momentum distribution. Also, the number of internal degrees of freedom g corresponds to the number of spin states or polarizations of the particle. For example, an electron has two spin states $\pm 1/2$ and similarly a photon has two possible polarizations, so that $g_e = g_\gamma = 2$.

Let us now compute the above expressions in two asymptotic limits - relativistic and non-relativistic particles, which will be sufficient for our discussion of how the different particle species evolve in an expanding universe. We will consider the case $|\mu| \ll T$ and neglect all chemical potentials, since all evidence indicates that this is a good approximation [1].

(a) Relativistic species

For $T \gg m$, the Bose-Einstein and Fermi-Dirac distributions reduce to:

$$f(y) = \frac{1}{e^y \pm 1} \ , \tag{4}$$

where we have defined $y = |\mathbf{p}|/T$. This implies for the particle number density that:

$$\begin{aligned} n &= g\int_0^{+\infty} \frac{4\pi|\mathbf{p}|^2 d|\mathbf{p}|}{(2\pi)^3}\frac{1}{e^y \pm 1} \\ &= \frac{g}{2\pi^2}T^3\int_0^{+\infty}\frac{y^2}{e^y \pm 1}dy \ . \end{aligned} \tag{5}$$

It is then useful to use the following results:

$$\begin{aligned} \frac{1}{e^x+1} &= \frac{1}{e^x-1} - \frac{2}{e^{2x}-1} \ , \\ \int_0^{+\infty}\frac{y^n}{e^y-1}dy &= \zeta(n+1)\Gamma(n+1) \ , \end{aligned} \tag{6}$$

where $\zeta(z)$ is the Riemann Zeta-function. For bosons, it is then straightforward to obtain:

$$\begin{aligned} n_b &= \frac{g}{2\pi^2}T^3\zeta(3)(2!) \\ &= \frac{g}{\pi^2}\zeta(3)T^3 \ . \end{aligned} \tag{7}$$

Similarly, for fermions we have:

$$\begin{aligned} n_f &= \frac{g}{2\pi^2}T^3\left(\int_0^{+\infty}\frac{y^2}{e^y-1}dy - 2\int_0^{+\infty}\frac{y^2}{e^{2y}-1}dy\right) \\ &= \frac{g}{2\pi^2}T^3\left(\int_0^{+\infty}\frac{y^2}{e^y-1}dy - \frac{1}{4}\int_0^{+\infty}\frac{z^2}{e^z-1}dz\right) \\ &= \frac{g}{\pi^2}T^3\left(\zeta(3)(2!) - \frac{1}{4}\zeta(3)(2!)\right) \\ &= \frac{3}{4}\frac{g}{\pi^2}\zeta(3)T^3 \ , \end{aligned} \tag{8}$$

implying $n_f = (3/4)n_b$. We can perform similar calculations for the energy density to obtain:

$$\begin{aligned} \rho_b &= \frac{\pi^2}{30}gT^4 \ , \\ \rho_f &= \frac{7}{8}\frac{\pi^2}{30}gT^4 \ . \end{aligned} \tag{9}$$

Finally, for both bosons and fermions, in the relativistic limit $E \sim |\mathbf{p}|$, so that from Eq. (3) we have $p = \rho/3$, as expected.

(b) Non-relativistic species

For $T \ll m$, the exponential factor dominates the denominator in both the Bose-Einstein and Fermi-Dirac distributions in Eq. (1), so that the bosonic or fermionic nature of the particles becomes indistinguishable. Furthermore, we have:

$$E = (|\mathbf{p}|^2 + m^2)^{1/2} = m\left(1 + \frac{|\mathbf{p}|^2}{m^2}\right)^{1/2} \simeq m + \frac{|\mathbf{p}|^2}{2m} \ . \tag{10}$$

Defining $x = |\mathbf{p}|/\sqrt{2mT}$, we have for the number density:

$$n \simeq \frac{g}{2\pi^2} e^{-m/T} (2mT)^{3/2} \int_0^{+\infty} x^2 e^{-x^2} dx. \tag{11}$$

We may then use the following result:

$$\int_0^{+\infty} x^n e^{-x^2} dx = \frac{1}{2}\Gamma\left(\frac{1+n}{2}\right) \ , \tag{12}$$

and, taking $n = 2$ with $\Gamma(3/2) = \sqrt{\pi}/2$, we obtain:

$$n \simeq g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T} \ , \tag{13}$$

which gives the Boltzmann distribution. From Eq. (10) it easy to see that to leading order $\rho = mn$ in this case. To obtain the associated pressure, note that to leading order $|\mathbf{p}|^2/E \simeq |\mathbf{p}|^2/m$, so that:

$$\begin{aligned} p &\simeq \frac{g}{2\pi^2} e^{-m/T} \frac{(2mT)^{5/2}}{3m} \int_0^{+\infty} x^4 e^{-x^2} dx \\ &= \frac{g}{2\pi^2} e^{-m/T} \frac{(2mT)^{5/2}}{3m} \frac{3\sqrt{\pi}}{8} \\ &= g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T} T \\ &= nT \ , \end{aligned} \tag{14}$$

where we have used $\Gamma(5/2) = 3\sqrt{\pi}/4$. Notice that restoring the missing Boltzmann constant k_B this corresponds to the familar result for a non-relativistic perfect gas, $p = nk_BT$. Since $T \ll m$, we have $p \ll \rho$ and the pressure may be neglected for a gas of non-relativistic particles, as we had anticipated.

Energy and entropy density

Let T denote the temperature of the photon bath in the early universe. If there are other relativistic species in the early Universe, the total energy density of radiation is given by:

$$\rho_r = \frac{\pi^2}{30} g_*(T) T^4 \ , \tag{15}$$

where $g_*(T)$ corresponds to the effective number of relativistic degrees of freedom present in the universe at the temperature T, including both bosons and fermions. This may receive contributions from two types of species:

1. *Thermal bath*: relativistic species in thermal equilibrium with the photons $T_i = T \gg m_i$:

$$g_*^{th}(T) = \sum_{bosons} g_i + \frac{7}{8} \sum_{fermions} g_i \ . \tag{16}$$

2. *Decoupled species*: relativistic species that are not in thermal equilibrium with the photons, $T \neq T_i \gg m_i$:

$$g_*^D(T) = \sum_{bosons} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{fermions} g_i \left(\frac{T_i}{T}\right)^4 . \tag{17}$$

The full number of relativistic degrees of freedom is thus given by $g_*(T) = g_*^{th}(T) + g_*^D(T)$. This number remains roughly constant away from particle mass thresholds $T \sim m_i$, and as we had seen previously for $T \gtrsim 1$ TeV all the Standard Model degrees of freedom are relativistic and in equilibrium and $g_* = 106.75$. For $T < 1$ MeV, the only relativistic species are the photons and the three neutrinos, and as we will see below $g_* \simeq 3.36$. We illustrate the variation of $g_*(T)$ in the Standard Model in the figure below.

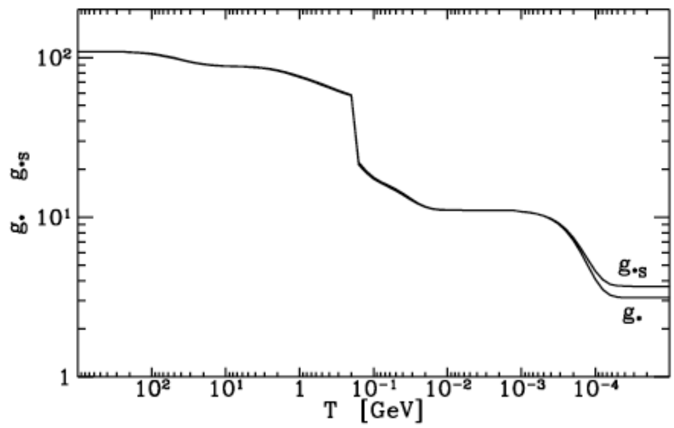

Figure 1: Variation of the number of relativistic degrees of freedom, g_* and g_{*S}, with temperature according to the Standard Model of particle physics [1].

A very important quantity is the entropy of the Universe. In our earlier discussion in the context of newtonian cosmology, we considered that there was no entropy variation (i.e. heat transfer) per comoving volume in an expanding Universe, and we obtained the evolution of the energy density, $\dot{\rho} = -3H(\rho+p)$, in agreement with the result of energy-momentum conservation in general relativity for a perfect fluid. This means that energy-momentum conservation implies an isentropic expansion in thermal equilibrium. Since the entropy is constant, it is a useful quantity to compute. To do this, recall that the first law of thermodynamics for zero chemical potential can be written as:

$$TdS = dE + pdV . \tag{18}$$

Using the fact that both the energy and the entropy are extensive quantities, i.e.

$$\frac{\partial E}{\partial V} = \frac{E}{V} , \qquad \frac{\partial S}{\partial V} = \frac{S}{V} , \tag{19}$$

we may write the first law in the form:

$$\begin{aligned} T\frac{\partial S}{\partial V}dV + T\frac{\partial S}{\partial T}dT - pdV &= \frac{\partial E}{\partial V}dV + \frac{\partial E}{\partial T}dT \\ \left(T\frac{S}{V} - p\right)dV + T\frac{\partial S}{\partial T}dT &= \frac{E}{V}dV + \frac{\partial E}{\partial T}dT , \end{aligned} \tag{20}$$

where we have taken $S = S(V,T)$ and $E = E(V,T)$. Equating the terms in dV and dT, this yields:

$$\begin{aligned} \frac{\partial E}{\partial T} &= T\frac{\partial S}{\partial T} , \\ \frac{E}{V} &= T\frac{S}{V} - p . \end{aligned} \tag{21}$$

From the second equality, we find for the entropy:

$$S = \frac{E + pV}{T} \ . \tag{22}$$

We may define the entropy density $s = S/V$, which is thus given by:

$$s = \frac{\rho + p}{T} \ . \tag{23}$$

For a relativistic species, we have $p_i = \rho_i/3$, and hence

$$s_i = \frac{4}{3}\frac{\rho_i}{T_i} \ . \tag{24}$$

The total entropy density of radiation in the early Universe is given by a sum over all relativistic species:

$$s = \frac{2\pi^2}{45} g_{*S}(T) T^3 \ , \tag{25}$$

where $g_{*S}(T) = g_{*S}^{th}(T) + g_{*S}^{D}(T)$ is the effective number of relativistic degrees of freedom contributing to the entropy. Note that for species in thermal equilibrium $g_{*S}^{th}(T) = g_*^{th}(T)$. However, given that $s_i \propto T_i^3$, for decoupled species we find:

$$g_{*S}^{D}(T) = \sum_{bosons} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{fermions} g_i \left(\frac{T_i}{T}\right)^3 \neq g_*^D(T) \ . \tag{26}$$

This difference is apparent in figure 1 at low temperatures as neutrinos decouple from the photon bath as we will see below. Entropy conservation implies that $S = a^3 s$ remains constant as the universe expands, which implies:

$$g_{*S} T^3 a^3 = const. \tag{27}$$

Away from particle mass thresholds, g_{*S} is approximately constant and $T \propto a^{-1}$ as expected. Also, since $s \propto a^{-3}$, this allows one to define the number of particles of a given species in a comoving volume:

$$N_i = a^3 n_i = \frac{n_i}{s} \ , \tag{28}$$

where we have redefined the scale factor to absorb the constant factors. If no particles are being created or destroyed in a comoving volume, this quantity remains constant, and we have:

$$N_i = \begin{cases} \frac{45\zeta(3)}{2\pi^4} \frac{g_i}{g_{*S}} \ , & T \gg m_i \\ \frac{45}{4\sqrt{2}\pi} \frac{g_i}{g_{*S}} \left(\frac{m_i}{T}\right)^{3/2} e^{-m_i/T} \ , & T \ll m_i \end{cases} \ . \tag{29}$$

An important consequence of this is that, in the absence of interactions that produce or destroy baryon number, the baryon-to-entropy ratio is conserved:

$$\eta_S = \frac{n_B}{s} = \frac{n_b - n_{\bar{b}}}{s} = const. \tag{30}$$

where b and $\bar{b}$ denote baryons and antibaryons, respectively. Notice that, on the other hand, the baryon-to-photon ration n_B/n_γ only remains constant away from particle mass thresholds when g_{*S} is constant.

Finally, note that g_{*S} accounts for particles becoming non-relativistic and “disappearing”' from the thermal bath, since their energy density and entropy is exponentially suppressed. When this happens, their entropy is transferred into the species that remain in the plasma, which makes the radiation temperature decrease more slowly than $T \propto a^{-1}$.

Decoupling and freeze-out

In the previous discussion we have distinguished between particle species which are in thermal equilibrium with the photons and those which are decoupled from it. To understand how a species decouples from the plasma we need to include interactions between the different particles. This is described by the Boltzmann equation, which gives the general evolution of the particle number density:

$$\dot{n}_i + 3Hn_i = -\langle \sigma_i v_i \rangle \left[n_i^2 - (n_i^{EQ})^2 \right] \ . \tag{31}$$

The left hand side of the equation simply yields the effect of Hubble expansion, which in the absence of particle creation or destruction gives $n_i \propto a^{-3}$. The *collision term* on the right hand side, on the other hand, includes the possibility that particles are created and/or destroyed via processes with a total cross section σ_i, with v_i denoting the interaction velocity of the particles. Also, n_i^{EQ} denotes the equilibrium distribution of the particle species, and it is easy to see that any departure from thermal equilibrium makes the system evolve towards thermal equilibrium once again. For example, if there is a deficit of particles, $n_i < n_i^{EQ}$, the right hand side gives a positive contribution to $\dot{n}_i$ and makes the number density grow until equilibrium is attained. Similarly, if n_i is above its equilibrium value, more particles will be destroyed until equilibrium is achieved.

Although we will not have time in this course to look at a full derivation or solutions of the Boltzmann equation, it will be useful to note that the evolution of the number density is a competition between the Hubble expansion and the collision term. This may be quantified by comparing the interaction rate $\Gamma_i = n_i \langle \sigma_i |v|_i \rangle$ with the Hubble rate. We then have that:

- For $\Gamma_i \gtrsim H$, the collision term dominates and the system will evolve and remain in thermal equilibrium;
- For $\Gamma_i \lesssim H$, the collision term cannot compensate for the Hubble expansion and the system departs from thermal equilibrium.

There is, of course, a smooth transition between these two regimes, but this criterion provides a simple and sufficiently good approximation to analyze the evolution of the number density. The physical interpretation of this criterion is not difficult. For $\Gamma_i > H$, particles are being created and destroyed within a Hubble time and equilibrium is maintained. If, on the other hand, $\Gamma_i < H$, interactions can no longer keep up with Hubble expansion and virtually no particles are created or destroyed within a Hubble time. This implies that the abundance of the species i will remain constant after *decoupling*, which is known as *freeze-out*.

Scattering processes are typically mediated by gauge bosons, such as photons, gluons or the weak bosons, $W^\pm$ and Z^0. The scattering cross sections depend on whether the gauge bosons are relativistic or non-relativistic at decoupling. Defining the generalized structure constant $\alpha = g_A^2/(4\pi)$ for the gauge coupling associated with a generic gauge boson A, we have:

- For a massive gauge boson, $m_A \gtrsim T$, the cross section is typically $\sigma \sim G_A^2 T^2$, where the generalized Fermi constant $G_A = \alpha/m_A^2$.
- For a massless gauge boson, $m_A \ll T$, the cross section is $\sigma \sim \alpha^2/T^2$.

Suppose then that we have a relativistic species which is maintained in thermal equilibrium by scattering processes mediated by gauge bosons in one of the above situations. Since $n_i \sim T^3$ up to $\mathcal{O}(1)$ factors and $v \sim 1$, $\Gamma_i \sim \sigma T^3$. As we have seen earlier, when the Universe is dominated by radiation, we have:

$$H = \frac{\pi}{\sqrt{90}} g_*^{1/2} \frac{T^2}{M_P} \ . \tag{32}$$

This implies that up to constant factors

$$\frac{\Gamma_i}{H} \sim \begin{cases} G_A^2 M_P T^3 \ , & m_A \gtrsim T \\ \alpha^2 \frac{M_P}{T} \ , & m_A \ll T \end{cases} \ . \tag{33}$$

This means that the particle can be in thermal equilibrium at high temperature, $m_A \ll T \ll M_P$, and decouple later on, once the temperature drops below $T_D \sim (G_A^2 M_P)^{-1/3} \sim \alpha^{-2/3}(m_A/M_P)^{1/3} m_A \lesssim m_A$.

If a particle decouples when relativistic, $T_D \gg m_i$, its energy will simply be redshifted by expansion, $E(t) = E(t_D)a(t_D)/a(t)$. Since it is no longer interacting with the plasma, its phase space occupancy $f(\mathbf{p})$ remains unchanged, and in particular $E/T = const$. This means that its temperature will also redshift with the scale factor:

$$T(t) = T_D \frac{a(t_D)}{a(t)} \ , \qquad m_i \ll T_D \ . \tag{34}$$

This implies, in particular, that its temperature is not affected by subsequent changes in g_{*S}. If, on the other hand, a particle is non-relativistic at the time of decoupling, $T_D \lesssim m_i$, its momentum will redshift with expansion and consequently its kinetic energy scales as $E \propto |\mathbf{p}|^2 \propto a^{-2}$. Since $n \propto a^{-3}$ due to the absence of particle production or destruction for any decoupled species, $f(\mathbf{p})$ is maintained after decoupling, and in this case we require $(E-\mu)/T = const.$, yielding:

$$\begin{aligned} T(t) &= T_D \left(\frac{a(t_D)}{a(t)}\right)^2 \ , \\ \mu(t) &= m + (\mu_D - m)T(t)/T_D \ , \qquad m_i \gtrsim T_D \ . \end{aligned} \tag{35}$$

Note that the variation of the chemical potential ensures that $n \propto a^{-3}$, which is formally required even though we will neglect chemical potentials in most of our discussion, as mentioned earlier.

It is important to emphasize that, despite the different scaling behaviour of the temperature for relativistic and non-relativistic species after decoupling, in both cases the equilibrium distribution is maintained.

Neutrino decoupling

Let us now look at a particular example - the decoupling of neutrinos. Neutrinos are light particles with masses $m_\nu \lesssim 1$ eV and that are kept in thermal equilibrium with electrons and positrons via weak interactions of the form:

$$\begin{aligned} e^- + \nu_e &\longleftrightarrow e^- + \nu_e \\ e^- + e^+ &\longleftrightarrow \nu_e + \bar{\nu}_e \ . \end{aligned} \tag{36}$$

The cross section for these interactions is $\sigma = G_F^2 T^2$, where $G_F = 10^{-5}/m_p^2 \simeq 1.17 \times 10^{-5}$ GeV^{-2} is Fermi's constant and $m_p \simeq 1$ GeV is the proton mass. Both neutrinos and electrons/positrons are fermions, so that:

$$n_{e,\nu_e} = \frac{3\zeta(3)}{4\pi^2} g_{e,\nu_e} T^3 \simeq 0.1 g_{e,\nu_e} T^3 \ , \tag{37}$$

with $g_e = 2$ and $g_{\nu_e} = 1$ since neutrinos are left-handed particles. This gives a total interaction rate:

$$\Gamma_\nu = 0.1(g_e + g_{\nu_e})T^3 \times G_F^2 T^2 \simeq 0.3 G_F^2 T^5 \ , \tag{38}$$

where we have taken $v \sim 1$ for both species. The number of relativistic degrees of freedom for $T \gtrsim 1$ MeV includes photons, electrons, positrons and three neutrinos in the thermal bath, yielding $g_{*S} = 10.75$. Using Eq. (32) for the Hubble rate in the radiation-dominated era, we find:

$$\frac{\Gamma_\nu}{H} \simeq \left(\frac{T}{2\ \mathrm{MeV}}\right)^3 \ , \tag{39}$$

so that neutrinos decouple from the thermal bath at $T_D \sim 2$ MeV. Note that solving the full Boltzmann equation one obtains $T_D \sim 1$ MeV, which gives an idea of how good this criterion is.

Shortly after neutrinos decouple, the temperature drops below the electron mass, $m_e = 0.511$ MeV, and the entropy in $e^\pm$ is transferred to the photons, but not to the decoupled neutrinos. We then have:

$$g_{*S} = \begin{cases} 2 + \frac{7}{8} \times 4 = \frac{11}{2} & , \ T \gtrsim m_e \\ 2 & , \ T < m_e \end{cases} \ , \tag{40}$$

where we have neglected neutrinos and other decoupled species. Since, in equilibrium, $g_{*S}(aT)^3$ remains constant, we find that aT increases after $T < m_e$ by a factor $(11/4)^{1/3}$, while aT_ν remains the same. This implies that the present temperature of neutrinos is given by:

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_0 \simeq 1.95 \text{ K} . \tag{41}$$

Note that the decrease in the number of relativistic degrees of freedom does not lead to an actual increase in the temperature, it just makes it decrease more slowly than a^{-1}. This means that, besides the CMB, we expect a background of relic neutrinos from the Big Bang, the CνB, which is slightly colder than the relic photons. Unfortunately, since neutrinos interact so weakly with matter, they are very difficult to detect and it is not certain that one might be able to detect the neutrino background directly in the future. There is, however, compelling indirect evidence for its existence, since their contribution to the entropy and energy density affects BBN and CMB anisotropies. In fact, this can be used to determine the number of neutrino species from cosmological measurements, and the most recent results from the Planck satellite yield $N_\nu = 3.30 \pm 0.27$, consistently with the known three species, ν_e, ν_μ and ν_τ [2].

Having computed the present temperature of neutrinos, we may also compute the present values of g_* and g_{*S}:

$$\begin{aligned} g_*(T_0) &= 2 + \frac{7}{8} \times 2 \times 3 \times \left(\frac{4}{11}\right)^{4/3} = 3.36 \ , \\ g_{*S}(T_0) &= 2 + \frac{7}{8} \times 2 \times 3 \times \left(\frac{4}{11}\right) = 3.91 \ , \end{aligned} \tag{42}$$

from which we can see the explicit difference between the two effective numbers of relativistic degrees of freedom when there are relativistic species decoupled from the photons. We can also use this to compute the present abundance of radiation:

$$\begin{aligned} \rho_r(T_0) &= \frac{\pi^2}{30} g_*(T_0) T_0^4 \simeq 8 \times 10^{-34} \text{ gcm}^{-3} \ , \\ \Omega_r h^2 &\simeq 4 \times 10^{-5} \ , \end{aligned} \tag{43}$$

so that the contribution from relativistic degrees of freedom to the present energy density in the Universe is negligible, as we had discussed earlier in the course.

Problem 6

Suppose there is an additional species of massive neutrinos contributing to the dark matter density in the Universe. Assume additionally that these neutrinos interact with electrons and positrons in the same way as ordinary neutrinos, such that they decouple at $T_D \sim 1$ MeV $> m_\nu$ and that they are non-relativistic today. Hence show that their present abundance is

$$\Omega_\nu h^2 \simeq \frac{m_\nu}{91 \text{ eV}} . \tag{44}$$

Use this to obtain an upper bound for the mass of these neutrinos. [Note that, for a massive fermion, $g = 2$.]

References

[1] E. W. Kolb and M. S. Turner, *The early universe* (Westview Press, 1990).

[2] P. A. R. Ade *et al.* [Planck Collaboration], arXiv:1303.5076 [astro-ph.CO].