Introduction to Cosmology

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LECTURE 13 - Cosmological perturbation theory II

In this lecture we will conclude the study of cosmological perturbations by discussing the evolution of cold dark matter (CDM) and radiation perturbations in the post-inflationary era, starting from an adiabatic and nearly scale invariant spectrum generated by inflation. We will show that perturbations grow, so that at some stage they become comparable to the average background density and linear theory no longer holds. Although a full non-linear description of the growth of the perturbations typically requires the use of numerical simulations, we will consider a simplified model of non-linear spherical collapse of non-relativistic perturbations that illustrates the basic features of this evolution. We conclude this lecture by summarizing the main results described throughout this course.

Post-inflationary evolution of CDM and radiation perturbations

As we have derived in the previous lecture, CDM and radiation perturbations evolve according to the following coupled differential equations:

$$\delta_c'' + \frac{a'}{a}\delta_c' - \frac{3}{2}\left(\frac{a'}{a}\right)^2 (\Omega_c\delta_c + 2\Omega_r\delta_r) = 0 \ , \tag{1}$$

$$\delta_r'' + \frac{1}{3}k^2\delta_r - \frac{4}{3}\delta_c'' = 0 \ . \tag{2}$$

In particular, recall that on superhorizon scales $\delta_r = (4/3)\delta_c$, according to the adiabatic nature of the perturbations generated during inflation. This means that on superhorizon scales it is sufficient to consider only the CDM perturbations, whereas on subhorizon scales we need to include the effects of the radiation pressure in the evolution of the latter. Recall also that perturbations cross the horizon at conformal time $\tau_H = 2\pi/k = \lambda$ for each Fourier mode. We also see that perturbations will evolve in distinct ways depending on the dominant background fluid, so that we need to consider separately the evolution in the radiation-, matter- and dark energy-dominated eras.

(a) Matter era

Although after inflation the universe becomes dominated by radiation first and only later non-relativistic matter becomes the dominant component, it is useful to begin by studying the evolution of perturbations in the matter era, when $\Omega_c \simeq 1$ and $\Omega_r \simeq 0$. This means that the CDM perturbations evolve according to:

$$\delta_c'' + \frac{a'}{a}\delta_c' - \frac{3}{2}\left(\frac{a'}{a}\right)^2 \delta_c = 0 \ . \tag{3}$$

Since in the matter-dominated era $a \propto t^{2/3} \propto \tau^2$, we have $a'/a = 2/\tau$ and so:

$$\delta_c'' + \frac{2}{\tau}\delta_c' - \frac{6}{\tau^2}\delta_c = 0 \ . \tag{4}$$

It is then easy to check that $\delta_c \propto \tau^n$, with $n = 2$ or $n = -3$, the general solution being given by:

$$\delta_c(k,\tau) = A(k)\left(\frac{\tau}{\tau_{eq}}\right)^2 + B(k)\left(\frac{\tau}{\tau_{eq}}\right)^{-3} , \tag{5}$$

where τ_{eq} denotes the conformal time of matter-radiation equality, where matter-domination effectively begins. We thus see that the solution exhibits a growing mode $\propto \tau^2 \propto a$ and a decaying mode $\propto \tau^{-3} \propto t^{-1}$. This solution is valid for both sub- and superhorizon modes since no k-dependence influences the evolution of CDM perturbations.

Radiation perturbations follow CDM perturbations on superhorizon scales, also exhibiting a growing and a decaying mode. On the other hand, on subhorizon scales we have to take into account the effects of the radiation pressure, which becomes the dominant term driving the evolution:

$$\delta_r'' \simeq -\frac{1}{3}k^2\delta_r , \tag{6}$$

with a general solution:

$$\delta_r(k,\tau) = C(k)\cos\left(\frac{k\tau}{\sqrt{3}}\right) + D(k)\sin\left(\frac{k\tau}{\sqrt{3}}\right) . \tag{7}$$

We can match the amplitude of CDM and radiation perturbations at horizon-crossing, $k\tau_H = 2\pi$ which is roughly $k\tau \simeq \sqrt{3}\pi$, and taking the growing mode solution to be the dominant one:

$$\delta_r(k,\tau) = -\frac{4}{3}A(k)\left(\frac{2\pi}{k\tau_{eq}}\right)^2 \cos\left(\frac{k\tau}{\sqrt{3}}\right) . \tag{8}$$

This shows that once they become smaller than the horizon, perturbations in the photon fluid and other relativistic components such as neutrinos begin to oscillate, as illustrated in Figure 1.

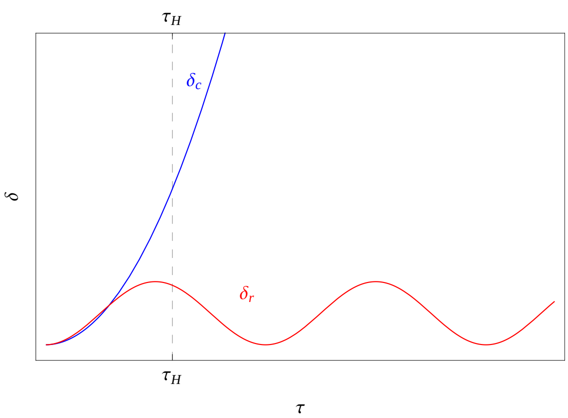

Figure 1: Numerical solution for CDM and radiation perturbations in the matter-dominated era, showing how the former grow on both superhorizon and subhorizon scales, while the latter undergo acoustic oscillations once they become subhorizon.

These *acoustic* oscillations correspond to the propagation of sound waves in the photon fluid due to their non-zero pressure, which does not occur for non-relativistic CDM perturbations. Photon diffusion will actually damp these oscillations, although a detailed study of this process is outside the scope of this course. Since recombination of protons and neutrons occurs in the matter era, we expect that at the time the CMB is emitted different Fourier modes which are already inside the horizon will be in different phases of their oscillation, leading to a damped oscillatory behaviour for the scale distribution of CMB temperature anisotropies, recalling that $\delta_r = 4\delta T/T$ from $\rho_r \propto T^4$. This is illustrated in Figure 2, where we show the CMB temperature anisotropy power spectrum as measured by the Planck satellite. The baryon fluid, although non-relativistic at this stage, is tightly coupled to photons before recombination, so the radiation pressure will also prevent the growth of baryonic structures until last scattering. This also translates into *baryon acoustic oscillations* (BAO), which have been observed in the spectral distribution of galaxies (see e.g. [2]).

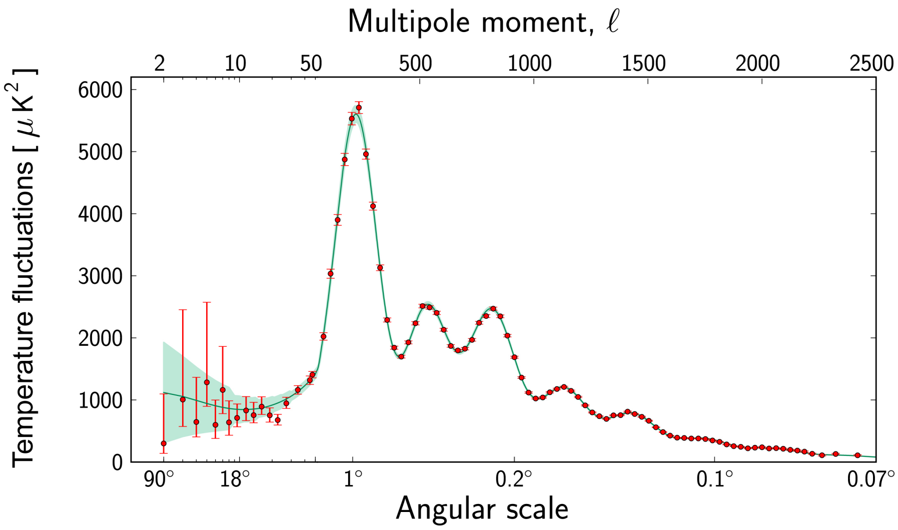

Figure 2: The angular power spectrum of temperature anisotropies in the Cosmic Microwave Background as measured by the Planck satellite [1]. The angular scale in the horizontal axis corresponds to the comoving wavenumber k.

(b) Radiation era

In the radiation era, which precedes matter domination, we have $\Omega_c \simeq 0$ and $\Omega_r \simeq 1$, yielding $a \propto t^{1/2} \propto \tau$. On superhorizon scales, the adiabatic condition relates CDM and radiation perturbations, so that we have for the former:

$$\delta_c'' + \frac{1}{\tau}\delta_c' - \frac{4}{\tau^2}\delta_c = 0 \ . \tag{9}$$

As in the matter era, this exhibits power-law solutions of the form:

$$\delta_c(k,\tau) = A(k)\left(\frac{\tau}{\tau_i}\right)^2 + B(k)\left(\frac{\tau}{\tau_i}\right)^{-2} \ , \tag{10}$$

yielding a growing mode $\propto \tau^2 \propto a^2$ and a decaying mode $\propto \tau^{-2} \propto t^{-1}$. This means that CDM and radiation perturbations always grow as τ^2 while superhorizon independently of whether the dominant component is matter or radiation.

On subhorizon scales, δ_r will exhibit an oscillatory behaviour as in the matter era. Since the conformal time scale for these oscillations is $\sim \sqrt{3}/k \lesssim \tau_H$, once a mode is deep inside the horizon $\tau \gtrsim \tau_H$ the mode is oscillating quickly, and on average we can take $\langle\delta_r\rangle \simeq 0$, giving for the CDM perturbation evolution:

$$\delta_c'' + \frac{1}{\tau}\delta_c' = 0 \ , \tag{11}$$

with solution:

$$\delta_c(k,\tau) = C(k)\log\left(\frac{\tau}{\tau_i}\right) + B(k) \ , \tag{12}$$

meaning that the perturbations have a much slower growth on subhorizon scales in the radiation era. Physically, this means that perturbations can only grow significantly when the background gravitational potential is sufficiently strong to trigger their collapse, which happens both in the matter and radiation eras as long as the photon fluid is also collapsing, i.e. on superhorizon scales.

(c) Dark energy domination

For $\tau \gg \tau_\Lambda$, the cosmological constant or other dynamical form of dark energy takes over and the universe begins expanding in an accelerated way. In this case, we will have $a \propto e^{H_0 t}$, yielding $a'/a = 1/\tau \propto a$, so that the factor

$$\left(\frac{a'}{a}\right)^2 \Omega_c \propto a^{-1} \tag{13}$$

decays quickly as the universe expands. We may hence neglect this term and also Ω_r in the equation for CDM perturbations, which becomes:

$$\delta_c'' + \frac{a'}{a}\delta_c' = 0 \ . \tag{14}$$

Expressing this in terms of cosmic time, we have:

$$\ddot{\delta}_c + 2H_0\dot{\delta}_c = 0 \ , \tag{15}$$

with general solution:

$$\delta_c(k,t) = A(k) + B(k)e^{-2H_0 t} \ , \tag{16}$$

i.e. with a constant mode and a rapidly decaying mode. Hence, the growth of perturbations stops when the cosmological constant takes over, so it seems quite a coincidence that there was sufficient time for large scale structures to form in the universe before Λ-domination. Based on this fact, Steve Weinberg argued in a seminal paper [3] that if the cosmological constant became dominant before CDM perturbations became non-linear and underwent gravitational collapse, no gravitationally bound structures such as galaxies, stars or planets could have formed in the universe. Since this seems to be a necessary condition for the existence of observers such as ourselves, he used this fact to place an upper bound on the value of Λ which is 2-3 orders of magnitude above the present energy density. This gives an *anthropic selection principle* - among the possible different universes we can only live in one with a small cosmological constant. Although this does not fully explain the smallness of the observed value, it certainly improves over the 120 orders of magnitude discrepancy between this and the naive zero-point energy density computed in quantum field theory, $\rho_\Lambda \sim M_P^4$.

Matter power spectrum

In summary, cold dark matter fluctuations grow as τ^2 in the matter era, up to the onset of Λ-domination, and while superhorizon during the radiation era, with no significant growth in the radiation era if they are smaller than the horizon. Perturbations in the radiation fluid also follow the growth of CDM perturbations on superhorizon scales from the adiabaticity of the primordial fluctuations generated during inflation, but undergo acoustic oscillations driven by the radiation pressure once they become subhorizon.

To determine the overall growth of perturbations, we define a *transfer function* as:

$$\delta_k(\tau_0) = T(k)\delta_k(\tau_i) \ , \tag{17}$$

where τ_i denotes the end of inflation/beginning of the radiation era. To compute this for CDM perturbations we have to distinguish small and large scales depending on whether they re-enter the horizon in the radiation- or matter-eras, respectively:

- Small scales ($\tau_H < \tau_{eq}$, $k > k_{eq} \equiv 2\pi/\tau_{eq}$)

  Since these scales re-enter the horizon in the radiation era, they grow from τ_i to τ_H and from τ_{eq} to τ_Λ, with no significant growth from τ_H to τ_{eq}. This gives:

$$T(k)^{small} = \left(\frac{\tau_H}{\tau_i}\right)^2 \left(\frac{\tau_\Lambda}{\tau_{eq}}\right)^2 = C\left(\frac{k}{k_{eq}}\right)^{-2} \ , \quad (18)$$

  where $C = (\tau_\Lambda/\tau_i)^2$ yields the maximum growth for small scales.

- Large scales ($\tau_H > \tau_{eq}$, $k < k_{eq}$)

  These scales only re-enter the horizon in the matter-era, so they always grow until Λ becomes dominant, giving:

$$T(k)^{large} = \left(\frac{\tau_\Lambda}{\tau_i}\right)^2 = C \ . \quad (19)$$

We can then write the full transfer function for CDM perturbations as:

$$T(k) \simeq C \begin{cases} 1 & , \ k < k_{eq} \\ (k/k_{eq})^{-2} & , \ k > k_{eq} \end{cases} . \quad (20)$$

Now, inflation generated a nearly scale invariant spectrum of curvature perturbations, $P_\mathcal{R} \propto k^{-3}$. Curvature and density perturbations are related by the Poisson equation $k^2\Psi_B/a^2 = -4\pi G\delta\rho$, where the gauge invariant Bardeen potential $\Psi_B = \Psi + a^2H(\dot{E} - B/a)$ can be related to the comoving curvature perturbation (see e.g. [4]). Note, in particular, that, in the newtonian gauge, $\Psi_B = \Psi = \Phi$, the latter giving the gravitational potential in the non-relativistic newtonian description. We thus conclude that the primordial power spectrum for CDM perturbations has the following scale dependence:

$$P_i(k) = A_i k^{-3} \times k^4 = A_i k \ . \quad (21)$$

This is the form of the Harrison-Zeldovich spectrum for density perturbations, and deviations from scale invariance yield $P_i(k) \propto k^{n_s}$, which explains why historically one defines the scalar spectral index in the comoving curvature power spectrum as $\Delta^2_\mathcal{R} \propto k^{n_s - 1}$. Applying the transfer function (squared) to this form of the primordial power spectrum, taking $n_s \simeq 1$ to a first approximation, we get:

$$P(k) \propto A_c \begin{cases} (k/k_{eq}) & , \ k < k_{eq} \\ (k/k_{eq})^{-3} & , \ k > k_{eq} \end{cases} , \quad (22)$$

so that matter-radiation equality divides the behaviour of the power spectrum, which exhibits a maximum value for scales that re-entered the horizon during this epoch. One should note that the transfer function has a logarithmic enhancement that we have not taken into account from the small growth of perturbations in the radiation era on subhorizon scales, which we obtained in Eq. (12). Of course the true power spectrum exhibits a smooth transition between the two regimes, but this gives the main features which are observed in the galaxy power spectrum, with power decaying on smaller scales (larger k), as illustrated in Figure 3. Note that this gives a significant evidence for *cold* rather than *hot* dark matter, since for relativistic fluids there is no growth on subhorizon scales due to pressure effects, which would yield a different form for the power spectrum.

The Jeans length and mass

Although CDM has very little interactions with ordinary particles, the overall matter perturbations in the universe involve a baryonic component, which until recombination is tightly coupled and in equilibrium with photons. Let us

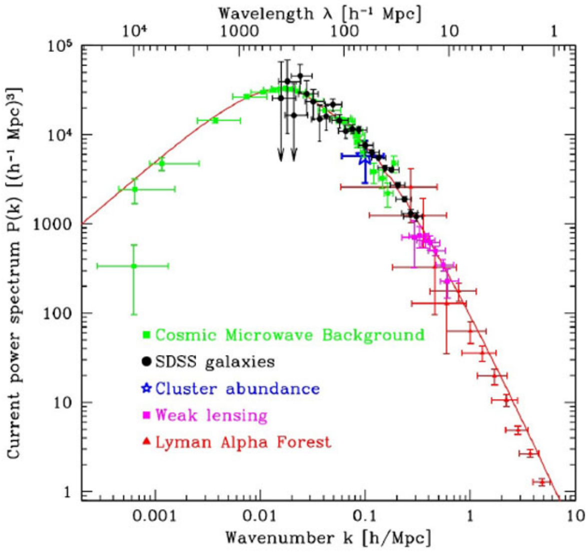

Figure 3: The observed matter power spectrum from observations of the CMB and the distribution of galaxies and clusters [5].

then consider the full non-relativistic matter perturbations in the matter era, including a small but non-vanishing pressure $p_m \ll \rho_m$, such that:

$$w \simeq c_s^2 \ll 1 \ . \tag{23}$$

To include the effects of a possibly significant sound speed and hence the propagation of sound waves in the baryon fluid, let us recall the first order differential equations obtained in the last lecture for the density constrast and velocity, yielding for non-relativistic matter:

$$\begin{aligned} \delta_m' - k^2\theta_m + \frac{1}{2}h' &= 0 \ , \\ \theta_m' + \frac{a'}{a}\theta_m + c_s^2\delta_m &= 0 \ . \end{aligned} \tag{24}$$

Differentiating the first equation with respect to conformal time and using the first order equations to eliminate θ_m, we obtain:

$$\delta_m'' + \frac{a'}{a}\delta_m' + c_s^2k^2\delta_m + \frac{1}{2}\left(h'' + \frac{a'}{a}h'\right) = 0 \ . \tag{25}$$

We may now use the scalar trace equation obtained in the last lecture (see Eq. (13)), which in the matter era yields:

$$h'' + \frac{a'}{a}h' + 3\left(\frac{a'}{a}\right)^2\delta_m = 0 \ , \tag{26}$$

and replacing this into Eq. (25) we obtain:

$$\delta_m'' + \frac{a'}{a}\delta_m' + \left[c_s^2k^2 - \frac{3}{2}\left(\frac{a'}{a}\right)^2\right]\delta_m = 0 \ , \tag{27}$$

which reduces to the CDM perturbation equation, Eq. (1), for vanishing sound speed. From this we see that pressure effects will dominate the evolution of matter perturbations for $c_s^2k^2 \geq (3/2)(a'/a)^2 = 6/\tau^2$. This defines a lower bound on k

$$k > k_J \equiv \frac{\sqrt{6}}{c_s \tau} \ , \tag{28}$$

and a corresponding wavelength known as the *Jeans length*, below which perturbations cannot grow and form structure due to the counteracting effect of the fluid pressure:

$$\lambda_J = \frac{2\pi}{k_J} a = \frac{2\pi}{\sqrt{6}} a \tau c_s \sim c_s d_H \ , \tag{29}$$

so that the Jeans scale is of the order of the sound horizon, which yields the maximum distance that acoustic waves can travel in the matter fluid. Using $a(t) = a_0 (t/t_0)^{2/3}$ in the matter era, one can show that $a\tau = 3t = 2H^{-1}$, and using the Friedmann equation $H^2 = 8\pi G \bar{\rho}_m / 3$, we can express the Jeans length as:

$$\lambda_J(t) = c_s(t) \sqrt{\frac{\pi}{G \bar{\rho}_m(t)}} \ , \tag{30}$$

where the time-dependence was made explicit. In particular, before decoupling and recombination baryons are tighly coupled with photons and so $c_s \sim 1/\sqrt{3}$, so that $\lambda_J \sim d_H$ and no collapse is possible on subhorizon scales. After decoupling the sound speed decreases considerably, $c_s \sim 10^{-5}(T/T_{dec})$, so that $\lambda_J \ll d_H$. Hence, while CDM can grow and collapse on all scales during the matter era, subhorizon baryonic perturbations will only be able to collapse after the emission of the CMB. It is also common to define the *Jeans mass*:

$$M_J = \frac{4\pi}{3} \bar{\rho}_m \lambda_J^3 \ , \tag{31}$$

which thus corresponds to the minimum mass of objects that can undergo gravitational collapse.

Non-linear theory: spherical collapse model

Up to now, we have considered the linearized evolution of perturbations, i.e. we have expanded the Einstein and energy-momentum equations up to linear order in small perturbations. However, we have shown that these perturbations grow during the post-inflationary evolution almost to the present day when the cosmological constant has taken over as the dominant component in the universe. This means that at least for some scales we expect the density contrast to become $\delta_m \sim 1$ at some point in the evolution, so that a linearized approach is no longer valid.

A full non-linear study of the growth and collapse of matter perturbations is generically a complicated problem, only solvable by developing large numerical simulations as we have discussed in our introductory lecture. Here, we will nevertheless explore a simplified model of spherical collapse of non-relativistic matter perturbations. Although spherical symmetry does not provide a fully realistic approach, this gives an exactly solvable model where we can compare the exact results with those of a linearized expansion.

Let us consider a sphere filled with non-relativistic matter of mass M, such that the newtonian equation of motion at radius r gives:

$$\ddot{r} = -\frac{GM}{r^2} \ , \tag{32}$$

which is analogous to what we have considered for a universe filled with with non-relativistic matter in our newtonian cosmology approach. As one can easily check, this equation can be solved exactly and the solution $r(t)$ can be implicitly written in terms of a parameter θ:

$$\begin{aligned} r(\theta) &= A(1 - \cos\theta) \ , \\ t(\theta) &= B(\theta - \sin\theta) \ , \end{aligned} \tag{33}$$

which gives a cycloid curve analogous to what we obtained in problem 2 for a closed (spherical) universe in terms of conformal time, as shown in Figure 4. This solution illustrates a spherical mass distribution that first grows and then recollapses, which models the formation of gravitationally bound structures. Replacing the solution into Eq. (32), we get for the relation between the constants A and B:

$$\frac{A^3}{B^2} = GM \ . \tag{34}$$

The spherical mass distribution corresponds to an overdense region with density:

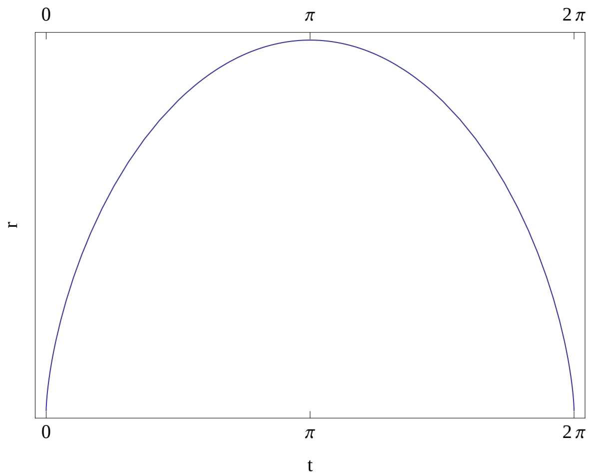

Figure 4: The solution $r(t)$ for the collapse of a non-relativistic matter density fluctuation, setting $GM = 1$ for simplicity.

$$\rho_m = \frac{M}{\frac{4}{3}\pi r^3} \ , \tag{35}$$

while the background density in a matter dominated universe evolves as:

$$\bar{\rho}_m = \frac{3H^2}{8\pi G} = \frac{1}{6\pi G t^2} \ . \tag{36}$$

Using the solution in Eq. (33) with the relation (34), we obtain for the matter density contrast $1 + \delta_m = \rho_m/\bar{\rho}_m$:

$$\delta_m = \frac{9}{2} GM \frac{t^2}{r^3} - 1 = \frac{9}{2} \frac{(\theta - \sin\theta)^2}{(1 - \cos\theta)^3} - 1 \ . \tag{37}$$

This gives the exact solution for the evolution of spherical matter density perturbations, although implicitly in terms of the parameter θ. Expanding to lowest order in this parameter, we get:

$$\delta_m^{linear} \simeq \frac{3}{20}\theta^2 + \ldots \tag{38}$$

We may also expand $t(\theta)$ for small θ and invert, giving $\theta \simeq (6t/B)^{1/3}$, which then yields:

$$\delta_m^{linear} \simeq \frac{3}{20}\left(\frac{6t}{B}\right)^{2/3} + \ldots \tag{39}$$

We may use this to compare the results of linearized theory with the exact solution at particular points in the evolution of the density perturbation:

1. *Turn around*

   This corresponds to the transition from expansion to collapse, for which $dr/dt = (dr/d\theta)(d\theta/t) = 0$, and it is easy to check that this corresponds to $\theta = \pi$, with $t = \pi B$ At this point, we have:

$$\delta_m = \frac{9\pi^2}{16} - 1 \simeq 4.55 \ , \qquad \delta_m^{linear} \simeq \frac{3}{20}(6\pi)^{2/3} \simeq 1.06 \ , \tag{40}$$

   so that linear theory predicts gravitational collapse begins for $\delta_m \sim 1$, whereas in reality the spherical region is already a few times denser than the background spacetime.

2. *Collapse*

   The density perturbations collapses at $\theta = 2\pi$, giving $r = 0$ and $t = 2\pi B$, where $\delta_m \to \infty$. The linearized theory gives, on the other hand, a finite result:

$$\delta_m \to +\infty \ , \qquad \delta_m^{linear} \simeq \frac{3}{20}(12\pi)^{2/3} \simeq 1.69 \ . \tag{41}$$

3. *Virial radius*

   The virial theorem for gravitational interactions gives $K = -V/2$, or equivalently:

$$\frac{1}{2}\left(\frac{dr}{dt}\right)^2 = \frac{GM}{2r} \ . \tag{42}$$

   With the solution in Eq. (33) one can check that this is valid at $\theta = \pi/2$ and $\theta = 3\pi/2$ during expansion and collapse, respectively. At the virial radius during collapse we have:

$$\delta_m = \frac{9}{2}\left(1 + \frac{3\pi}{2}\right)^2 - 1 \simeq 146 \ , \qquad \delta_m^{linear} \simeq \frac{3}{20}\left(6 + 9\pi\right)^{2/3} \simeq 1.58 \ . \tag{43}$$

   The spherical overdense region *virializes* i.e. reaches an equilibrium configuration at this stage, forming a stable gravitationally bound system. This model then shows that perturbations collapse into stable structures when they become ~ 147 times denser than the background, while linear theory predicts a much smaller value.

Hence, we see that the linearized approach significantly underestimates the magnitude of the density contrast during the collapsing stage, which illustrates how in general we may need more powerful and mainly numerical tools to study the final stages of structure formation in the universe.

Conclusion

To finish this lecture we give a brief overview of the main results that we have studied in this course and put together an outline of the history of the universe according to the standard paradigm.

On large scales, our universe is well-described by a homogeneous and isotropic Friedmann-Robertson-Walker spacetime that is expanding and, according to observations, its spatial sections are basically flat. The expansion rate is determined by the matter and energy content of the universe at any given time, according to the Friedmann equation $H^2 = 8\pi G\rho/3$, and is well described by the dynamics of perfect fluids that undergo an isentropic expansion ($dS = 0$). The standard cosmological expansion is thus well-described by (i) radiation or relativistic particles ($\rho_r \propto a^{-4}$); (ii) non-relativistic matter, including baryons (ordinary matter) and so far unknown weakly interacting dark matter particles ($\rho_m \propto a^{-3}$); and (iii) a cosmological constant that yields accelerated expansion at late times ($\rho_\Lambda \simeq const.$) or a more general postulated dark energy fluid that mimics its properties. Due to their distinct dilution rates , these fluids dominate the energy content at different times and give rise to different epochs in the cosmic history.

While most particles are in local thermal equilibrium at early times, as the universe cools down many of the relevant interactions become too slow to keep up with Hubble expansion, leading to departures from a thermal state and the consequent freeze-out of their cosmological abundances, which determine their present contribution to the energy density. Today, after 13.8 billion years of expansion, the universe is mainly made of unknown components - 68% of dark energy, 27% of dark matter and only about 5% of baryonic particles, according to the recent results of the Planck mission [6].

Despite its large scale homogeneity, the universe that we see today is filled with structure - galaxies, clusters, voids... These gravitationally bound systems correspond to small inhomogeneities in the primordial fluids that grew during the cosmic history, with overdense regions eventually collapsing under their own gravitational weight to form stable structures and underdense regions leaving large voids in the universe.

The cosmic history can be summarized as follows:

1. **Inflation**

   Before the onset of the standard Big Bang cosmology, it is believed that the universe was dominated by an exotic fluid that mimics the effects of a cosmological constant. The resulting accelerated expansion, typically quasi-exponential, makes the universe flat and homogeneous, bringing microscopic scales to superhorizon sizes, so that they appear to never have been in causal contact in the standard cosmology. The standard picture of inflation is described by the slow-roll dynamics of a scalar inflaton field, whose nearly-flat potential dominates the energy density at early times. Inflation ends when the potential steepens and the field rolls to its minimum, reheating the universe as it oscillates about it. Moreover, small quantum fluctuations of the inflaton field are stretched and amplified by expansion, their amplitude freezing once they become larger than the Hubble horizon. This produces a spectrum of nearly scale-invariant and gaussian spectrum of curvature perturbations in the underlying FRW spacetime and consequent adiabatic density perturbations that provide the primordial seeds for the anisotropies in the temperature of the CMB and the origin for the large scale structure.

2. **Radiation era**

   The reheating temperature after inflation depends on the how long the inflaton takes to decay, with a larger mean lifetime leading to lower T_R. In principle, this temperature can take any value from a few MeV to the Planck scale, but it is usually believed that it must not exceed the temperature of grand unification, since the GUT phase transition may produce unwanted relics such as monopoles that are otherwise diluted by inflation. Particle physics processes have only been tested up to energies of about 1 TeV, but it is widely believed that new physical processes should operate above this scale and influence the cosmological history. This includes one or more processes that may generate the observed baryon asymmetry in the universe, satisfying the Sakharov conditions of B-, $C-$ and CP-violation, as well as a departure from thermal equilibrium. It is also believed that most of the dark matter in the universe is made of weakly interacting massive particles (WIMPs), arising in extensions of the Standard Model, whose interactions freeze-out at some point in the radiation era for $T \lesssim m_{WIMP}$ and leave a non-vanishing thermal abundance, although the present dark matter abundance may be produced by alternative mechanisms. Once the universe cools down below ~ 1 TeV, the following events take place:

   (a) *Electroweak phase transition* ($T \sim 100$ GeV): the Higgs scalar field acquires a non-zero vacuum expectation value and gives masses to all fermions and weak gauge bosons, breaking the symmetry between the weak and electromagnetic interactions, with the former becoming weaker and short-ranged.

   (b) *QCD phase transition* ($T \sim 200$ MeV): quarks and gluons carry a colour charge and interact according to the rules of quantum chromodynamics (QCD), which describes the strong interactions. At high temperatures, the strong coupling is suppressed and quarks and gluons are essentially free particles, forming the so-called quark-gluon plasma (QGP), whose properties are currently being measured at the LHC. Below 200 MeV, the strong coupling increases sufficiently due to quantum processes to allow for confinement of quarks and gluons into hadrons, in particular protons and neutrons.

   (c) *Weak interactions freeze-out* ($T \sim 1$ MeV): when weak interactions become slower than Hubble expansion, about 1 sec after reheating, neutrinos decouple from the primordial plasma and are not affected by subsequent entropy transfers into the photon fluid, making their present temperature lower than that of the CMB. The neutron-to-proton ratio becomes essentially fixed apart from a few neutron decay events.

(d) *Big Bang Nucleosynthesis* (0.1 MeV $\lesssim T \lesssim$ 1 MeV): during the first few minutes, nuclear reactions between protons and neutrons lead to the thermal production of light nuclear elements. As these reactions freeze-out, the light element abundances become constant, leaving in particular 25% of the baryonic mass fraction in Helium-4 and the remainder in Hydrogen nuclei (protons), with small traces of other light elements.

During the radiation era, the small adiabatic perturbations in the radiation and matter fluids grow on super-horizon scales, with growth of CDM perturbations stagnating inside the Hubble horizon, where photon and baryon perturbations undergo (damped) acoustic oscillations.

3. **Matter era**

Non-relativistic matter becomes the dominant component when the temperature of the universe drops to about $T \sim 1$ eV, when the universe is around 10^4 years old. During this period, when the universe cools below $T \lesssim 0.3$ eV ($\sim 300\,000 - 400\,000$ years), electromagnetic interactions keeping protons and electrons in thermal equilibrium freeze-out, and they can effectively combine into neutral Hydrogen atoms. The neutral universe is therefore transparent to photons, which may thus propagate freely afterwards, leading to the emission of the Cosmic Microwave Background, whose perfect blackbody spectrum is the best evidence that we have today for local thermal equilibrium in the early universe. Due to Hubble expansion, its present temperature is $T_0 = 2.73$ K, with small fluctuations of about 1 part in 10^5 that result from the growth of inflationary perturbations. The spectrum of anisotropies of the CMB exhibits a series of peaks and throughs that reflect the acoustic oscillations of the photon fluid at the time of last scattering.

During the matter era, perturbations in the dominant cold dark matter component grow on both sub- and superhorizon scales, eventually becoming non-linear and undergoing gravitational collapse that leads to the formation of the large scale structure in the universe. The light of the first stars reionizes the intergalactic plasma, ending the 'dark ages" since the emission of the CMB after about 10^9 years of cosmic history.

4. **Late time acceleration**

The present expansion rate is accelerated, as evidenced by observations of type Ia supernovae. Although this may be the consequence of a slightly more anisotropic universe in our cosmological vicinity than we expected, it is widely believed that this is the result of a new form of energy, with the required negative pressure ($w < -1/3$), which is generically known as dark energy. The simplest form of dark energy is a cosmological constant ($w = -1$), arising from the zero-point energy of vacuum, and which only recently in the cosmic history ($z \sim 1$) has become the dominant component in the universe, making it grow exponentially fast. However, a simple estimate in quantum field theory gives a vacuum energy density which could be as large as M_P^4, which is about 120 orders of magnitude larger than the observed value! This cosmological constant problem is one of the most fundamental questions in modern cosmology, being also associated with the puzzling coincidence that it only became dominant at recent times. A cosmological constant, and in general accelerated expansion, halts the growth of cosmic structures, so a larger value than what we measure today may even preclude our own existence. Many alternative models of dark energy have been proposed in the literature, based for example in the dynamics of scalar fields as in the description of the early inflationary period, such as the simple *quintessence* or *k-essence* models, or even exotic *phantom fluids* with $w < -1$ (see e.g. [7]).

The cosmic history and the properties of our universe are not yet fully understood, despite the great success of the Hot Big Bang model in describing many of the main physical processes. The fundamental questions are intrinsically related to the description of particle physics and gravity at high energies and associated with the nature and origin of dark matter, dark energy, baryon asymmetry, as well as the mechanism driving inflation and several other issues. Astrophysical observations, both with ground- and space-based observatories, as well as particle accelerators such as the LHC, have evolved significantly in the last few years, with several planned and undergoing experiments that may help us address these problems. So keep tuned for more developments!

References

[1] P. A. R. Ade *et al.* [Planck Collaboration], arXiv:1303.5075 [astro-ph.CO].

[2] D. J. Eisenstein and W. Hu, Astrophys. J. **496**, 605 (1998) [astro-ph/9709112].

[3] S. Weinberg, Phys. Rev. Lett. **59**, 2607 (1987).

[4] D. Baumann, arXiv:0907.5424 [hep-th].

[5] M. Tegmark *et al.* [SDSS Collaboration], Phys. Rev. D **69**, 103501 (2004) [astro-ph/0310723].

[6] P. A. R. Ade *et al.* [Planck Collaboration], arXiv:1303.5076 [astro-ph.CO].

[7] E. J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D **15**, 1753 (2006) [hep-th/0603057].