Introduction to Cosmology

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LECTURE 1 - The observed Universe

Cosmology is a fascinating subject that has witnessed an explosion of activity during the past decades with more and more precision data becoming available with both ground- and space-based astrophysical observations. Its great appeal for both theoretical and experimental physicists is associated with the broad range of physical processes required for a description of the observable universe and its dynamical evolution, bringing together a large number of different concepts in gravitational and nuclear/particle physics.

The purpose of this course is to give an introductory overview to modern topics in cosmology, in particular the extremely successful Hot Big Bang model that defines the standard paradigm for the description of the universe from its early stages to the present day. The discussion will be mostly self-contained but requires some basic knowledge of Einstein's theory of general relativity, quantum field theory and thermodynamics, although we will review the necessary concepts as we proceed.

In this first lecture I will start by giving an overview of what we know about our Universe and the observational grounds on which the Hot Big Bang model stands. The remainder of this course will have a more theoretical approach, but the various observations that we will discuss in this lecture will become relevant as we explore different aspects of the cosmological evolution. This is not intended as a comprehensive review of observational cosmology, but rather as a description of the basic principles derived from observation that allow us to derive the standard cosmological paradigm.

Cosmological expansion

The most fundamental fact in modern cosmology is that the Universe is expanding, a critical discovery made by Edwin Hubble, working at the Mount Wilson Observatory, California, in 1929.

In the beginning of the 20th century, the prevailing picture of the Universe was very different from the one we have today, and astronomers were still struggling to determine the size of our own galaxy, originally thought to be no more than a few tens of thousands of light years across, with the solar system occupying a fairly central position. Shapley dramatically changed this picture by using Cepheid variables to determine the distance to globular clusters within our galaxy. Cepheid stars, named after the constellation where they were originally found, are pulsating giant stars that can be used as *standard candles*, since their absolute luminosity can be determined from the period of variation of their brightness[1]. As the apparent magnitude of a star should decrease with the square of its distance to the observer, one may infer the luminosity distance

$$d_L = \left(\frac{\mathcal{L}}{4\pi\mathcal{F}}\right)^{1/2} \tag{1}$$

from the absolute luminosity $\mathcal{L}$ and the observed flux $\mathcal{F}$. Using this technique, Shapley showed that our galaxy was in fact tens or even hundreds of times larger than previously estimated and could in fact contain the entire Universe, with the Sun significantly displaced from its centre.

[1] This relation can be calibrated using nearby Cepheids whose distance is measured by paralax, i.e. measuring the shift in position of a star in the sky at different times of the year.

At the time, however, astronomers already knew the existence of faint nebulous objects known as *nebulae* and that could be resolved into groups of stars. Hubble found a Cepheid variable in the Andromeda nebulae M31 and used Shapley's technique to show that it was about 300 000 parsecs away (1 pc $\simeq$ 3.6 ly $\simeq 3.1 \times 10^{16}$ m) and so well beyond the established limits of our own galaxy. Hubble identified several other Cepheid variables and convincingly showed that nebulae were in fact *island universes* or independent galaxies as we know them today.

The even more revolutionary proposal that the universe is expanding came later on when Hubble and his student Milton Humason combined distance and velocity measurements for spiral nebulae. Velocities can be computed from the Doppler shift of the spectral lines of these nebulae, a technique pioneered by Percival Lowell and his assistant Vesto Slipher. Hubble determined that most galaxies, except for a few nearby ones which we now know are gravitationally bound to the Milky Way, are moving away from us since their spectral lines are *redshifted*. Moreover, the more distant galaxies were receding at a faster rate than the nearby ones. The following figure shows Hubble's original diagram, from which he inferred a linear relation between distance and velocity.

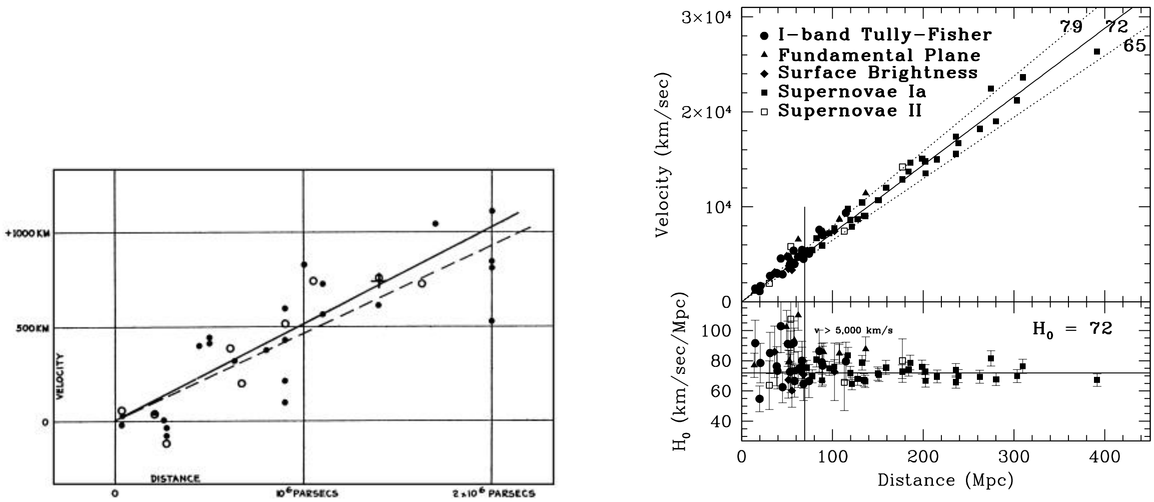

Figure 1: Hubble's original diagram with the relation between velocity and distance [1] (left) and the modern Hubble diagram obtained using the Hubble Space Telescope [2] (right).

Although it is somewhat surprising that Hubble derived a linear relation from his original data, observations collected over the years came to confirm his conclusion, as shown on the right panel of the figure above, and drastically modified our view of the Universe. This relation implies that galaxies are not just moving away from us, as if the Milky Way occupied a central position in the Universe, but that space itself must be expanding.

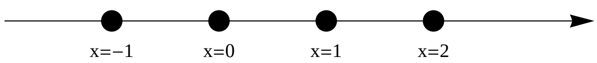

Figure 2: One-dimensional toy universe with fixed galaxies labelled by a coordinate x with neighbouring galaxies separated by a distance $a(t)$.

This can be seen in a simple one dimensional toy-model, where galaxies are placed at equal distance a from each other and labelled by a fixed coordinate x. If space itself is expanding the scale factor will depend on time $a = a(t)$ and the relative velocity between two galaxies at distance $d = a\Delta x$ apart is simply $v = \dot{a}\Delta x$, such that:

$$v = \frac{\dot{a}}{a} d = H d \; . \tag{2}$$

The proportionality factor H is known as the Hubble constant or, more correctly, the Hubble parameter. It is a constant in the sense that it does not depend on the particular galaxies one is considering, although it may (and as we will see it does) vary in time. We can easily convince ourselves that this argument can be easily extended into three dimensions if one assumes homogeneity and isotropy, as we discuss below. Of course galaxies are not exactly at fixed coordinate distances in the real Universe and exhibit *peculiar velocities* that can be accurately measured, but this simple model illustrates how space itself must be expanding in order to explain the Hubble law.

We now know that the present value of the Hubble parameter is significantly different from Hubble's original determination of about 500 $\mathrm{kms^{-1}Mpc^{-1}}$, which was plagued with several systematic errors, and is measured to be [3]:

$$H_0 = 100h \ \mathrm{kms^{-1}Mpc^{-1}} \ , \qquad h = 0.704 \pm 0.025 \ , \tag{3}$$

where h has conventionally been used to express the uncertainty in the measurement.

The idea of an expanding Universe had been around even before Hubble announced his results, with the advent of Einstein's theory of general relativity describing space-time as a dynamical continuum evolving according to the local content of energy and momentum. Solutions to Einstein's equations corresponding to expanding universes had in fact been found by Willem de Sitter, Alexander Friedmann and Georges Lemaître, but the general opinion, including Einstein himself, was that the real Universe should be static. Einstein is in fact famous for introducing a "cosmological constant" term in his gravitational field equations to stabilize the ubiquitous non-static solutions, and after Hubble's discovery he came to acknowledge this as his "biggest blunder". As we will see later on, Einstein's "mistake" turned out to yield one of the most interesting puzzles in modern cosmology.

The fact that the Universe is presently expanding implies that it was much smaller in the past and the logical extrapolation is that it must have emerged from a very hot and dense state, or even in fact an initial singularity of infinite density and temperature that was dubbed the 'Big Bang'. The laws of physics have only been tested up to energies of about 1 TeV, a boundary that is now being pushed at the Large Hadron Collider, in CERN, so that any description of the Universe at temperatures above this threshold is presently no more than theoretical speculation. The term 'Big Bang' has nevertheless been used since the 1950's to denote the standard cosmological paradigm.

Large scale homogeneity and isotropy

The picture of the Universe proposed by Hubble and subsequently developed by many other physicists and astronomers depicts a world where all galaxies are drifting away from each other at the same rate at a given time, where no observer occupies a special place in the Universe - very far from the geocentric view that prevailed for so many centuries. Homogeneity and isotropy are also two key features of the standard cosmological paradigm and a consequence of the so-called *cosmological principle* that the universe looks the same everywhere to all observers.

The assumption of homogeneity and isotropy goes back to Einstein's original work, based on theoretical simplicity rather than any firm observational grounds. However, there is ample evidence for an isotropic and homogeneous universe within the presently observable universe, whose size is determined by the present Hubble radius $cH_0^{-1} \simeq 3000h^{-1}$ Mpc as we will see in the next lecture. Of course the sky does not exactly look the same everywhere and we can easily identify our own Milky Way. Homogeneity and isotropy are properties of the Universe on large scales, greater than a few tens of Mpc.

The best evidence we have for large-scale homogeneity and isotropy comes from measuring the Cosmic Microwave Background (CMB) radiation, a relic of the Big Bang that we will discuss below that yields the most perfect black body spectrum ever found. The temperature of the CMB is basically uniform throughout the sky, exhibiting tiny fluctuations of the order of 1 part in 100 000 that are crucial to our understanding of the energy density profile in the early Universe.

Additional evidence for a homogeneous and isotropic Universe comes from the X-ray background (to about 5%), the distribution of faint radio sources and that of galaxies themselves. Large galaxy surveys have been performed in the past decade, such as the Sloan Digital Sky Survey (SDSS) and the 2dF Galaxy Redshift Survey, that measured the spectra of hundreds of thousands of objects and obtained precise three-dimensional maps of the deep sky. The statistical distribution of galaxies in these surveys indeed exhibits a large degree of isotropy and homogeneity on scales of a few to almost 100 Mpc.

Measurements of the peculiar velocity field in the Universe, i.e. of galactic velocities subtracted of the Hubble flow, have also given some (rough) evidence for homogeneity on scales as large as $60h^{-1}$ Mpc, yielding local matter inhomogeneities of the order of $\delta\rho/\rho \sim 0.1$.

Age of the Universe

The Hubble constant sets a time scale $H_0^{-1} \simeq 9.8h^{-1}$ Gyr. As we will see in the next lectures, the knowledge of the present expansion rate and the matter and energy content of the Universe allows one to determine its age, i.e. to extrapolate back in time to determine the moment of the primordial 'bang' For example, in a matter-dominated model one finds $t_0 = (2/3)H_0^{-1}$, with more complicated expressions if the Universe is filled with different forms of matter, radiation and other exotic forms of energy. The age of the Universe thus provides an important test of cosmological models, and the presently accepted value is of 13.77 ± 0.13 Gyr [3].

Hubble's original determination of the expansion rate, which we know today differs significantly from the presently measured value, place the age of the Universe at around 2 Gyr, which created a huge problem for cosmological models, since this was less than the estimated age of the solar system of about 4.5 Gyr. This "age crisis" motivated the birth of alternative theories such as the *steady state* cosmology, proposed by Hoyle, Bondi and Gold, where an 'ageless' and unchanging universe resulted from the continuous creation of matter and energy. Interestingly, Fred Hoyle was the one to coin the term 'Big Bang' in 1950, as a kind of mockery of what he thought was an "irrational" way of describing the Universe. We now know that the current measurements of the expansion rate are consistent with other methods of determining the age of the Universe and that the Big Bang model has passed inumerous observational tests with flying colours, whereas the unchanging steady-state cosmology fails for example to explain the origin of the CMB.

The age of the Universe can be determined, or at least constrained, by several other different methods, such as:

- the age of oldest globular clusters, which can be determined by looking at the transition from main sequence to red giant phase stars in the corresponding Hertzprung-Russell diagram;
- astrophysical abundances of radioactive isotopes, in particular elements produced by fast neutron capture (r-process) in an early generation of stars such as Uranium isotopes;
- cooling time for white dwarf stars.

All these techniques have inherent difficulties and associated errors but are consistent with a Universe between 10 and 20 Gyr old.

Light-element abundances

In 1946, George Gamow and Ralph Alpher used the recent developments in nuclear physics to make detailed calculations of nuclear reaction rates in the early universe. They assumed the Universe evolved from an initial state of an infinite density and temperature by expanding and cooling, such that stable nuclei could be formed from a primordial soup of protons, neutrons and electrons that they called 'Ylem', from a medieval word for matter. Under these assumptions they were able to predict correctly the observed abundances of Hydrogen and Helium, the latter accounting for a quarter of the luminous matter in the Universe and the former for almost all remaining mass. Primordial nucleosynthesis or Big Bang nucleosynthesis (BBN) is the earliest and one of the most stringent tests of Big Bang cosmology, with the relevant nuclear reactions taking place from $t \simeq 0.01$ to 100 sec, corresponding to temperatures of $T \simeq 10$ MeV to 0.1 MeV.

Alpher and Gamow's results for the abundances of light elements were extremely successful and the primordial soup is able to produce substantial amounts of Deuterium (D/H $\sim 10^{-5}$), ^{3}He (^{3}He/H $\sim 10^{-5}$), ^{4}He (mass fraction $Y \simeq 0.25$) and ^{7}Li (^{7}Li/H $\sim 10^{-10}$). Helium and deuterium are of particular importance, since there are no contemporary astrophysical processes that are capable of producing the observed amounts - the contribution of stellar processing to the ^{4}He abundance can be at most 5% in particular environments and even such a small abundance of Deuterium would be easily destroyed in stellar cores at temperatures of several million Kelvin, since it is very weakly bound. However, BBN cannot produce considerable amounts of elements heavier than Helium, which was originally seen as an argument against the Hot Big Bang model. Today we know that heavier elements can be produced in stellar interiors and supernovae explosions, accounting for the observed abundances, so that BBN is truly a remarkable success of Big Bang cosmology.

The predicted abundances of the light elements depend on the baryon-to-photon ratio η, i.e. the ratio between the number of ordinary nucleons such as protons and neutrons and the number of light particles. The observed abundances then imply $\eta \simeq 5 - 7 \times 10^{-10}$. This corresponds to the number of baryons that have not annihilated with their corresponding anti-particles within the primordial soup and one of the main challenges in modern cosmology, which we will address later on, is to explain how such a small but non-vanishing number arises. This typically requires non-trivial extensions of the Standard Model of particle physics and is an area of active research.

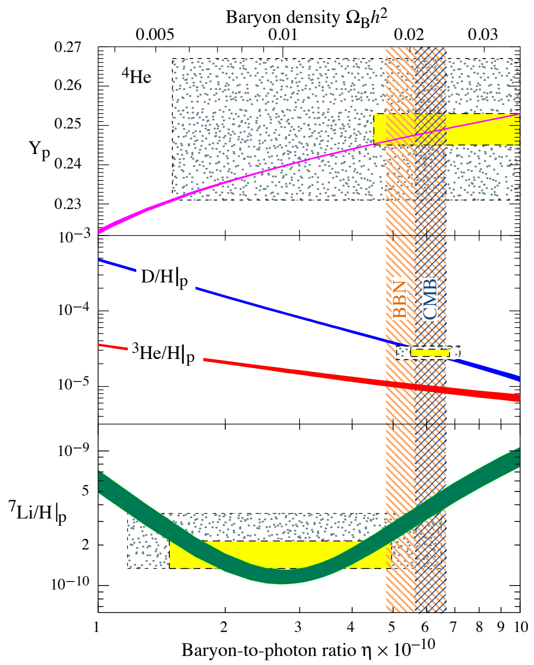

Figure 3: Light-element abundances predicted by Big Bang nucleosynthesis. Boxes indicate the observed light element abundances (smaller boxes: 2σ statistical errors; larger boxes: $\pm 2\sigma$ statistical and systematic errors). The narrow vertical band indicates the CMB measure of the cosmic baryon density [4].

The success of BBN also poses severe constraints on any departures from standard cosmology in the temperature range mentioned above, in particular constraining the existence of additional light particle species ($\lesssim$ MeV) common in many extensions of the Standard Model. Although we will not explore this possibility in detail during this course, it is worth mentioning that the BBN prediction for the abundance of Lithium is about 3-4 times larger than what is observed in old stars in the Milky Way halo, which constitutes the modern 'Lithium problem' and may require some (minor) modification of the standard cosmological evolution [5].

Cosmic Microwave Background

The CMB is perhaps the best evidence we have for the Hot Big Bang paradigm and at the same time the best tool for precision measurements in cosmology. As opposed to e.g. the steady state model discussed previously, in the Hot Big Bang model the Universe cools down as it expands and the density of matter and energy decreases. As first predicted by Ralph Alpher and Robert Herman in 1948, when the Universe was about 379 000 years old, the temperature of the Universe was about 3000 K, which is sufficiently low for protons and electrons to combine into neutral atoms. At this point, photons - the *quanta* of light - stopped interacting with matter and became allowed to travel freely through space. This is called the epoch of *recombination* and *decoupling* and also referred to as the *last scattering surface*, since light from this epoch has basically remained unaltered to the present day. The Universe remained basically neutral until much later when the first stars formed and their light broke some of the neutral atoms into Hydrogen ions in the interstellar medium - this is called the epoch of *reionization*.

The CMB was discovered in 1964 by two American radio astronomers, Arno Penzias and Robert Wilson, who were later awarded the 1978 Nobel Prize in Physics. Penzias and Wilson accidentally found a faint background of radiation in the microwave range while working in the detection of radio waves bounced off echo balloon satellites. This mysterious noise seem to be evenly distributed throughout the sky, did not originate from any known source and persisted even after a family of pigeons nesting in their antenna were carefully evicted and their droppings removed - what Penzias amusingly described as "white dielectric material". At the same time, a group of Princeton astrophysicists including Jim Peebles, Robert Dicke and David Wilkinson were preparing to search for microwave radiation in this range, realizing that the relic radiation from the Big Bang should be redshifted by expansion and presently lie in this part of the electromagnetic spectrum. When they learned about Penzias and Wilson's observation it became clear that it must correspond to the CMB - the definitive evidence in favour of a Hot Big Bang model as opposed to the competing steady state hypothesis.

Many experiments have been devoted to measuring the properties of the CMB, so that we will contempt ourselves with mentioning a few of the most important ones. NASA's Cosmic Background Explorer (COBE) satellite, launched in 1989, was the first space-based experiment dedicated to measuring the CMB spectrum and spatial distribution, showing that the relic radiation from the Big Bang has the most perfect black-body spectrum ever found in Nature - a consequence of the Universe having enough time to settle in the most probable thermal state by the time of last scattering - , with a temperature of 2.7255 K, corresponding to a present photon density of 422 cm^{-3}. COBE also showed that, apart from a dipole anisotropy resulting from our motion relative to the cosmic rest frame, the CMB is uniform up to tiny fluctuations of 1 part in 10^5 as mentioned earlier. The spectrum of these fluctuations reflects the matter density distribution at the time of last scattering and points towards a primordial scale-invariant spectrum that could result from a period of inflation in the early universe, as we will discuss in this course. COBE inaugurated an era of precision cosmology and its principal investigators, George Smoot and John Mather, were awarded the 2006 Nobel Prize.

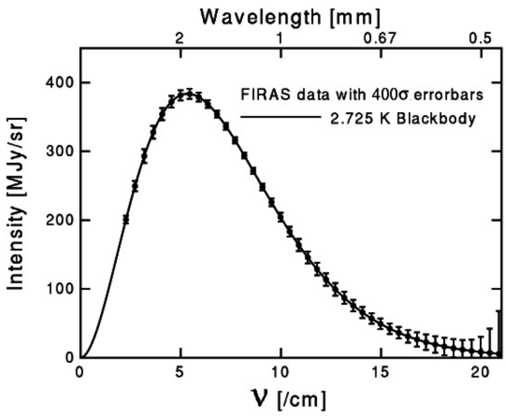

Figure 4: The black-body spectrum of the CMB radiation measured with the FIRAS instrument of the COBE satellite (data from [6]).

The state-of-the-art in precision observations of the CMB is currently given by the Wilkinson Microwave Anisotropy Probe (WMAP), another space mission that has been collecting data since 2001 and that, besides temperature anisotropies, has also began to accurately measure the polarization of the relic radiation. The polarization of the CMB was first detected with the Degree Angular Scale Interferometer (DASI) telescope in 2002 and reflects the distribution of charge at last scattering, being one of the most promising observables in modern cosmology. WMAP has recently released its final 9-year observation results with tight constraints on cosmological parameters, but will soon be overthrown by the European Planck mission, launched in 2009 and expected to bring precision cosmology to a new level in early 2013.

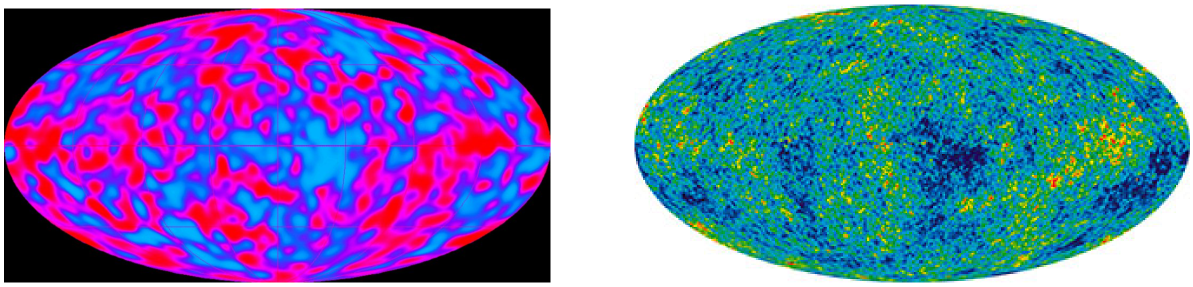

Figure 5: CMB temperature sky maps obtained with the COBE satellite after 2 years of data collection (left) and with the WMAP satellite after 9 years of data collection. The colour code corresponds to temperature fluctuations of a few micro-Kelvin about the uniform 2.7255 K background, after the Milky Way foreground has been removed [7].

Visible and dark matter

An important observable in cosmology is the amount of matter present in the Universe, since as we will see this determines the expansion rate and consequently the cosmological evolution. In theory, the average density of the Universe could be measured by determining the number of galaxies in the Hubble volume and the average galaxy mass:

$$\langle \rho \rangle = n_G \langle M_G \rangle \ . \tag{4}$$

The mass of a galaxy can be determined from its gravitational effect, in particular measuring distances and velocities of stars within the galaxy and using Kepler's third Law:

$$GM(r) = v^2 r \ , \tag{5}$$

where r is the distance to the galactic centre, v the orbital velocity and $M(r)$ the total mass within a sphere of radius r, assuming spherical symmetry. Using this technique for spiral galaxies and taking the largest distance within which most of the galaxy light is emitted, one finds that luminous matter provides less than 1% of the critical density required for a flat Universe:

$$\rho_c = \frac{3H_0^2}{8\pi G} = 1.88 \times 10^{-29} h^2 \ \mathrm{gcm}^{-3} \ . \tag{6}$$

The meaning of this critical density will become clearer later on this course, but for now it suffices to say that observations suggest that the Universe has in fact a very flat geometry, in particular the position of the first acoustic peak in the CMB power spectrum. This implies that luminous matter in galaxies cannot account for the matter density of our Universe!

Moreover, when the rotational curves of galaxies are extended beyond the above mentioned luminous boundary, for example by measuring velocities of rare stars or 21 cm emission from neutral Hydrogen gas clouds, one finds that these curves flatten, i.e. $v \simeq const.$ at large distances, which from Eq. (5) implies $M(r) \propto r$. Thus, there seems to be a lot of “dark” mass extending beyond the visible limits of virtually all spiral galaxies, as first shown by Jan Oort in 1932 for the Milky Way, and there is (weak) evidence for it being spherically distributed, with $\rho_{DM} \propto r^{-2}$. This dark halo contributes at least 3 to 10 times the mass of visible matter and some fraction of it can be accounted for by non-luminous baryonic matter, since BBN and CMB measurements indicate baryonic matter should yield about 4-5% of the critical density. This corresponds for example to “dark” objects such as Jupiter-like planets, white dwarfs, neutron stars, black holes, etc. However, this means that most of the matter and energy in the Universe is unaccounted for!

The average galactic mass in a cluster of galaxies can also be determined by other means. Clusters are gravitationally-bound systems and, assuming they had sufficient time to relax, one can use the virial theorem to show that:

$$GM = 2\frac{\langle v^2 \rangle}{\langle r^{-1} \rangle} . \tag{7}$$

Thus, measuring average velocity of galaxies and the average inverse distance between galaxies in a cluster can be used to infer its total gravitational mass, as first shown by Fritz Zwicky in 1933. This technique yields fractions of the critical density in clusters of about 10-30%, consistently with other techniques such as local distortions of the Hubble flow. Measurements of the CMB anisotropy spectrum are perhaps the most accurate way of determining the amount of matter in the Universe, and the best fit to the data yields 24% of the critical density in pressureless cold (non-relativistic) dark matter, implying that the remaining 71% must correspond to an unknown component which is 'unclustered', i.e. smoothly distributed, and which we briefly discuss below.

Non-baryonic dark matter is one of the most interesting puzzles in modern cosmology, pointing towards new elementary particles and extensions of the Standard Model of particle physics. A lot of effort has recently been devoted to building theoretical models to describe this non-luminous component with the observed abundance and in devising experimental ways to directly and/or indirectly detect any dark matter particles flowing within our local environment. Although there are some interesting 'hints' for dark matter, the results have so far been inconclusive and we will devote one of our final lectures to this topic. It is important to mention at this stage, however, that alternative hypothesis such as modified theories of gravity are currently less widely accepted, in particular given recent weak-lensing maps of galaxy clusters, such as the Bullet cluster, where the separation between gravitational and luminous mass is evident.

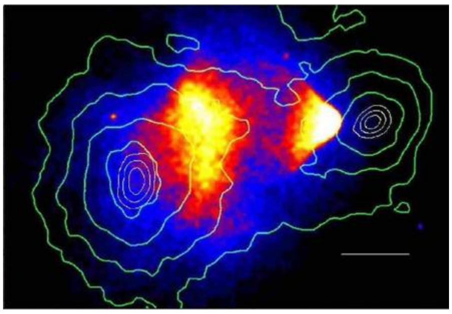

Figure 6: Image of the Bullet cluster obtained by the Chandra satellite. Shown in green are the gravitational mass contours reconstructed from weak-lensing observations [8].

Large-scale structure

As we have been discussing so far, on large scales the Universe is well-described by a homogeneous and isotropic model, but on smaller scales it exhibits several features and a particular structure from which we can learn a great amount. In particular, as mentioned above, from the large galaxy surveys that have been performed in the past decade we have been able to learn a great deal about the distribution of matter in the Universe. Galaxies are, to a first approximation, distributed uniformly throughout the sky, but have a tendency to cluster. This can be quantified by the galaxy-galaxy correlation function, ξ_{GG}, defined as the probability in excess of a random (Poisson) distribution of finding two galaxies at a distance r apart. For distances $0.1h^{-1}$ Mpc $\lesssim r \lesssim 16h^{-1}$ Mpc, the following power-law has been derived from the SDSS data [9]:

$$\xi_{GG} \simeq \left(\frac{r}{6.1h^{-1}\ \mathrm{Mpc}} \right)^{-1.75} . \tag{8}$$

Also, about 10% of the observed galaxies are found in gravitationally-bound clusters, such as the nearby Virgo and Coma clusters. The Abell catalogue combines over 4000 galaxy clusters and classifies them according to their richness, which roughly corresponds to the number of galaxies in the cluster. A cluster-cluster correlation function of the same form as Eq. (8) can be derived from this catalogue, valid for distances $10h^{-1}$ Mpc $\lesssim r \lesssim 50h^{-1}$ Mpc:

$$\xi_{CC} \simeq \left(\frac{r}{25h^{-1}\ \mathrm{Mpc}}\right)^{-1.8} . \tag{9}$$

More recent cluster data indicates a slightly smaller correlation length $16-19h^{-1}$ Mpc and a slightly steeper power-law, depending on the richness class, but the difference between the correlation lengths in the galaxy and cluster correlation functions seems to suggest that light may only be a 'biased' tracer of matter, which may give us important clues on the distribution of dark matter in the Universe [10].

There are also seem to exist larger structures, such as superclusters which are more loosely-bound and non-virialized, with densities about twice the average density of the Universe. These include our own Local Supercluster, centered in Virgo, Hydra-Centaurus and Pisces-Cetus. On the opposite end, large voids with virtually no matter (luminous or dark) have been found in the surveys, ranging from $10-100$ Mpc in diameter, the largest confirmed one being the Boötes void, which is about $75-100$ Mpc across. Even larger voids are conjectured to exist, in particular the 'Great Void' or 'Eridanus Supervoid', which could be associated with 'cold spot' observed in the CMB temperature anisotropy distribution.

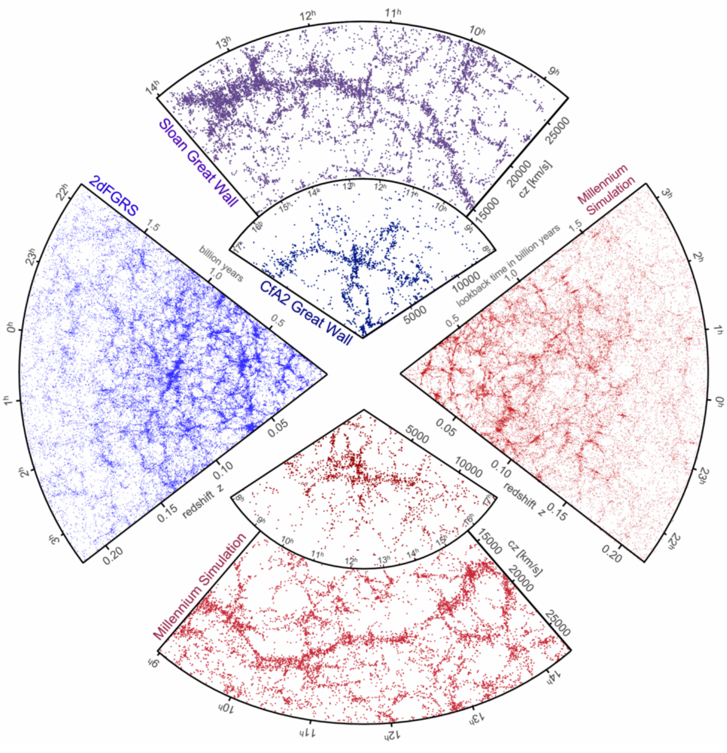

Figure 7: The galaxy distribution obtained from spectroscopic redshift surveys and from mock catalogues constructed from cosmological simulations [12].

A lot of progress has also been made in modeling the growth of structure in the Universe using N-body numerical simulations, performed for example by the Millenium or the more recent DEUS collaborations (see e.g.[11] for a recent review). These simulations follow the (non-linear) gravitational evolution of a primordial density profile of collisionless cold dark matter particles and have shown the formation of different types of structures such as filaments, walls and voids, in good agreement with observational data as illustrated in figure 7.

Besides galaxy surveys, other techniques are currently being used to study the large-scale structure of the Universe, namely weak gravitational lensing and distant quasar absorption line (Lyman-α, etc) surveys, which will hopefully shed a new light on the difference between the dark and luminous matter distributions.

Evidence for acceleration

As we discussed previously, evidence for an expanding Universe requires determining distances and velocities of standard candles - objects for which the absolute luminosity is known or can be calibrated. The Cepheid variables are the prime example of such objects and have been detected at distances as far as 10 Mpc. However, to determine the expansion history of the Universe to earlier times, we need to find standard candles at distances at least 100 times larger. Already in 1938, Baade and Zwicky proposed that supernovae could provide standard candles to very large distances, since they are extremely bright and, for a short period, can outshine a whole galaxy.

Type Ia supernovae are particularly suitable for this task, occurring in binary systems when a low-mass white dwarf star accreting matter from a companion exceeds the Chandrasekhar limit of 1.4 solar masses and becomes unstable, leading to a thermonuclear explosion producing an enormous amount of energy and leaving a neutron star remnant. Their brightness evolves over periods of a few weeks and, within our galaxy, supernovae can even be observed with the naked eye, as recorded for example by Chinese astronomers in 1054! The spectra and light curves of type Ia supernovae are extremely uniform, which indicates a common origin and absolute luminosity, which can be determined from the relation between their peak brightness and decay time.

Although extremely rare events for a single galaxy, statistically significant samples of supernovae data have been collected since the 1990's, in particular by the Supernova Cosmology Group (SCP), led by Saul Perlmutter of the Lawrence Berkeley National Laboratory (USA), and the High-z Supernova Search Team (HZT), led by Brian Schmidt of the Mount Stromlo Observatory in Australia. In the beginning of 1998, both groups published results for redshift and distance measurements of 42 and 16 type Ia supernovae, respectively, the latter having analyzed mainly by Adam Riess, a postdoctoral researcher at UC Berkeley (USA). Their results were crucial in determining the extension of the Hubble law into large distances.

As we will derive later on, the relation between the luminosity distance and redshift in an expanding Universe is given by:

$$d_L = H_0^{-1}\left(z + \frac{1}{2}(1-q_0)z^2 + \ldots\right) , \tag{10}$$

where for a spectral line of wavelength λ_1 emitted at time t_1 and observed at time t_0 with wavelength λ_0 the redshift is given by:

$$z = \frac{\lambda_0 - \lambda_1}{\lambda_1} = \frac{a(t_0)}{a(t_1)} - 1 , \tag{11}$$

with $a(t)$ denoting the scale factor of the Universe, since waves simply stretch with expansion. As we can see, at low-redshifts the linear Hubble law is recovered, while at larger redshifts some deviations may be expected, depending on whether expansion is accelerating or decelerating. In principle, ordinary matter can only make expansion slow down due to its gravitational pull, so that $q_0 = -\ddot{a}(t_0)a(t_0)/\dot{a}^2(t_0)$ is historically known as the *deceleration parameter*. However, the data from the SCP and HZT groups, including supernovae up to $z \lesssim 1$, showed quite the opposite - high redshift supernovae are dimmer than expected with the linear Hubble law by a factor of about $10-15$ %, which means the Universe is unexpectely accelerating! For this surprising discovery, Perlmutter, Schmidt and Riess were awarded the 2011 Nobel Prize in Physics.

Accelerated expansion cannot be produced by ordinary matter and radiation, in fact requiring an exotic fluid generically known as *dark energy*, and whose negative pressure counterbalances the gravitational attraction. The simplest example of dark energy is in fact Einstein's cosmological constant, typically denoted by Λ, and which corresponds to vacuum energy. This creates a huge theoretical problem, as crude estimates of the quantum vacuum energy in the Standard Model yield a cosmological constant which is about 120 orders of magnitude larger than the value required to explain the supernovae data!

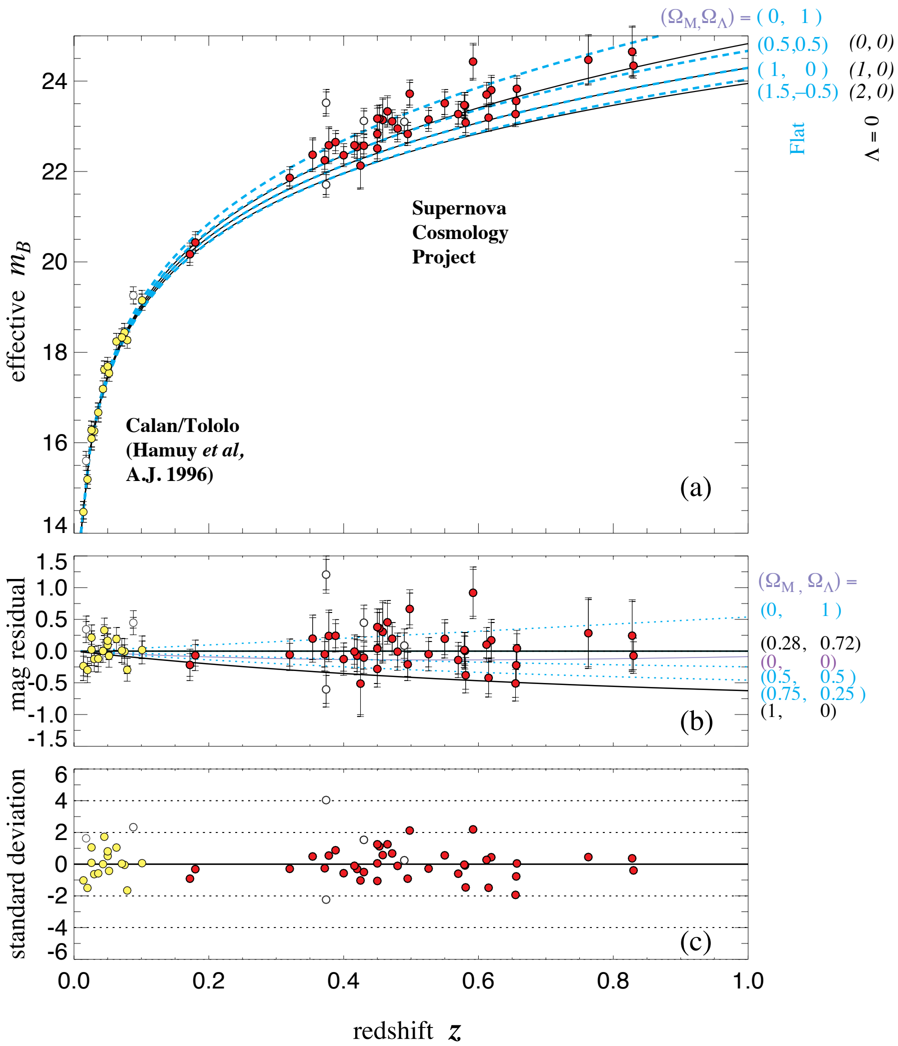

Figure 8: Hubble diagram obtained with 42 high-redshift type Ia supernovae from SCP and 18 low-redshift supernovae from the Calan-Tololo Supernova Survey. The solid and dashed curves corresponds to different cosmological models with different matter and cosmological constant abundances relative to the critical density, denoted by Ω_M and Ω_Λ, respectively [13].

The supernovae data is actually consistent with CMB and large-scale structure observations, and dark energy provides the missing smooth component leading to a critical density in the Universe. The simplest model consistent with the observational data so far is thus the so-called ΛCDM or concordance model, with the present energy density of the Universe divided into 71% of dark energy (cosmological constant), 24% of cold dark matter and 5% of ordinary baryonic matter. The agreement between this model and observations is remarkable, but it remains an enormous theoretical challenge to explain the observed value of the cosmological constant and the so-called coincidence problem - dark energy only became the dominant component in the Universe very recently (on cosmological time scales!), as studies of supernovae at $z > 1$ have confirmed that the Universe was matter-dominated at earlier times.

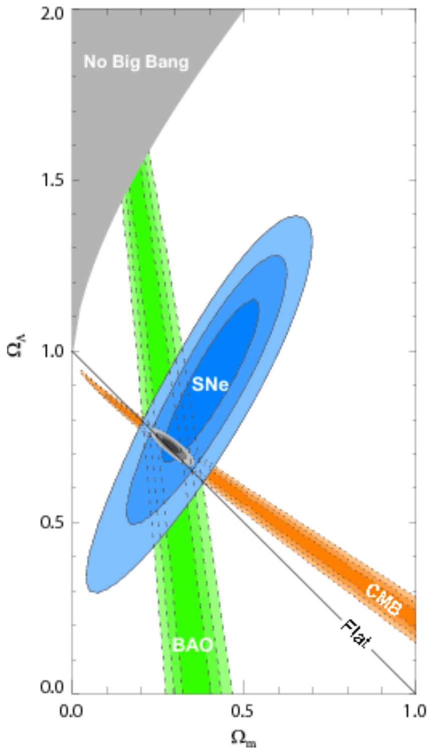

Figure 9: The concordance model or ΛCDM model putting together observations from type Ia supernovae, CMB temperature anisotropies and large-scale structure, in particular the so-called baryon acoustic oscillations (BAO). The solid line corresponds to a flat Universe with matter and a cosmological constant $\Omega_\Lambda + \Omega_m = 1$ [14].

We will discuss possible models of dark energy in more detail in one of our last lectures, but it is important to mention at this stage that there are alternative models, such as inhomogeneous cosmologies. These models make use of the fact that the expansion rate is larger in overdense regions, so that if we happen to live inside a particularly large void in the Universe we will measure a lower value of H_0 in our vicinity than the average value in the Universe (see e.g. [15]). Although other particle physics scenarios and modified gravity theories may also account for the supernovae dimming, the dark energy hypothesis remains the most widely accepted possibility.

Summary

In summary, observations can tell us a lot about the properties, structure and evolution of Universe, covering its history as far back as 0.01 seconds. Cosmological observables thus include:

- the present Hubble parameter H_0 and deceleration parameter q_0
- the age of the Universe t_0
- the present energy density ρ_0 and composition (baryonic and dark matter, radiation, cosmological constant, etc)
- the CMB temperature and polarization spectra, as well as other cosmological background radiations (IR, UV, X-ray, γ-ray, etc)
- the cosmological abundance of light-elements (H, D, ^{3}He, ^{4}He, ^{7}Li)
- the baryon-to-photon ration, η
- the statistical distribution of galaxies, clusters and larger structures

In the next lectures we will describe the standard cosmological paradigm based on these observables and towards the end of the course we will look at its shortcomings and open problems, and what they may tell us about particle and gravitational physics at high temperature/energy scales.

Problem 1

Assuming the virial theorem for a gravitationally-bound cluster of $n \gg 1$ galaxies of equal mass m_G, $\langle K\rangle = -\langle V\rangle/2$, derive Eq. (7) that allows one to determine the total cluster mass $M = nm_G$.

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