Quantum entropy of supersymmetric black holes, overview and recent developments (arXiv:1111.2025)

João Gomes

LPTHE, Université Pierre et Marie Curie

20 December 2011

IV Black Hole Workshop, UA

Motivation

It is well known that, in Einstein's general relativity, the entropy of a black hole is proportional to the area of the horizon

$$S_{BH} = \frac{A}{4G}, \; (\hbar = c = 1)$$

To give a statistical interpretation to the Beckenstein-Hawking entropy (S_{BH}) is an old problem in quantum gravity.

Can we describe the black hole as an ensemble of quantum states, such that the Boltzmann relation holds?

$$S = \ln \Omega$$

Ω: number of accessible microstates.

Motivation

By now there is a good statistical understanding of the Beckenstein-Hawking entropy in string theory.

- Construct a supersymmetric black hole specifying certain quantum numbers (charges, angular momentum) and then compute its BH entropy.
- Find a microscopic system (branes, strings, other solitonic objects) with the same quantum numbers. Working in the weak string coupling regime we can switch off gravity and count BPS states. *It is possible to compute Ω_{micro} exactly (many examples in $\mathcal{N} = 4, 8$ theories).*
- Supersymmetry ensures that $S_{BH} = \ln \Omega_{micro}$, in the thermodynamic limit.

Microscopic counting

Motivation

For large horizon radius or large charge both computations simplify.

- On the Black Hole side we can neglect higher derivative corrections. The entropy is then given by the area formula.
- On the microscopic side we can use an asymptotic expansion/Cardy limit of the degeneracy.

Both computations match perfectly in the regime of large charges [Strominger, Vafa]

$$S_{BH} = \ln \Omega_{micro}(Q \gg 1)$$

Motivation

Our main interest is to compute finite charge corrections to the Bekenstein-Hawking entropy.

$$S = \frac{A}{4G} + a\ln(A) + \mathcal{O}(A^{-1})$$

Since in string theory *corrections to Einstein's gravity theory are different in different phases of the theory*, the black hole entropy can give useful information about the microscopic details of the phase.

Macroscopic counting.

Motivation

If we have the right tools, we may compute the black hole entropy *exactly by including all perturbative and non-perturbative corrections.*

Equality of both microscopic and macroscopic computations is equivalent as establishing an *exact holography in the context of AdS_2/CFT_1 correspondence*,

$$S_{quantum} = \ln \Omega_{micro}$$

valid for any value of the charges.

Index VS degeneracy

- We use an index to count the BPS states of the microscopic theory (when $g_s \to 0$)

$$\Omega_{micro} = \mathrm{Tr}(-1)^F e^{-\beta H + i\theta J}|_{\text{fixed charges } Q,P}$$

- Since it is an index it does not depend on β, θ or even on g_s!

$$\Omega_{micro} = \mathrm{Tr}(-1)^F = n_+ - n_-$$

- On the black hole side the Wald entropy counts the logarithm of the degeneracy

$$\Omega_{macro} = \mathrm{Tr}(1)$$

- For black holes that preserve at least four supercharges the near horizon geometry has spherical symmetry

$$AdS_2 \times S^2$$

and therefore

$$\mathrm{Tr}(-1)^{2J} = \mathrm{Tr}(1)$$

Index=Degeneracy

Wald entropy

- Once we start including higher derivative corrections the area formula is not valid anymore. The Wald formalism allows to compute the entropy by incorporating the contribution from *local terms* in the effective action. [Wald,Iyer]
- However there can be *non-local/non-analytic contributions* to the effective action from integrating over massless modes. These contributions are essential for duality covariance.
- The inclusion of *non-perturbative corrections*, is in this context conceptually more difficult.

We need some formalism that can account for these issues!

Quantum entropy

We need to use a quantum version of the Wald entropy based on AdS_2/CFT_1 correspondence [Sen],

$$W(q,p) = \left\langle \exp[-iq_I \oint A^I] \right\rangle_{AdS_2}^{\text{finite}}$$

- In AdS_2 the gauge field behaves like $A \sim er + c$. The electric field is a non-normalizable mode and has to be fixed (microcanonical ensemble).
- AdS_2 has infinite volume which induces IR divergences. We introduce a cuttof at $r = r_0$ and keep only the cuttoff independent piece.
- The observable $W(q,p)$ equals the degeneracy $d(q,p)$. On the CFT we compute $\lim_{r_0 \to \infty} \mathrm{Tr} \exp[-2\pi r_0 H]$. It counts the number of ground states in a fixed charge sector.
- In the classical large charge limit $W(q,p) \simeq \exp S_{Wald}$

Quantum entropy

Our goal is to evaluate $W(q,p)$ by performing the path integral over the string fields in AdS_2.

This is highly non-trivial and may seem foolish since we don't even understand well string field theory.

Nevertheless we may find instructive to use the supergravity action.

Amazingly, localization techniques can simplify the path integral enormously.

Supergravity

We then need to compute the supergravity action together with the Wilson lines.

Here is the action:

$$
\begin{aligned}
& (-i(X^I \bar{F}_I - F_I \bar{X}^I))(-\frac{1}{2}R) + \big[i\nabla_\mu F_I \nabla^\mu \bar{X}^I \\
+ \quad & \frac{1}{4} i F_{IJ} (F^{-I}_{ab} - \frac{1}{4}\bar{X}^I T^{ij}_{ab}\, \varepsilon_{ij})(F^{-abJ} - \frac{1}{4}\bar{X}^J T^{ij}_{ab}\, \varepsilon_{ij}) \\
- \quad & \frac{1}{8} i F_I (F^{+I}_{ab} - \frac{1}{4} X^I T_{abij}\, \varepsilon^{ij}) T^{ij}_{ab}\, \varepsilon_{ij} - \frac{1}{8} i F_{IJ} Y^I_{ij} Y^{Jij} - \frac{i}{32} F\, (T_{abij}\, \varepsilon^{ij})^2 \\
+ \quad & \frac{1}{2} i F_{\hat{A}} \hat{C} - \frac{1}{8} i F_{\hat{A}\hat{A}} (\varepsilon^{ik} \varepsilon^{jl} \hat{B}_{ij} \hat{B}_{kl} - 2 \hat{F}^-_{ab} \hat{F}^-_{ab}) \\
+ \quad & \frac{1}{2} i \hat{F}^{-ab} F_{\hat{A}I} (F^{-I}_{ab} - \frac{1}{4}\bar{X}^I T^{ij}_{ab}\, \varepsilon_{ij}) - \frac{1}{4} i \hat{B}_{ij} F_{\hat{A}I} Y^{Iij} + \mathrm{h.c.} \big] \\
- \quad & i(X^I \bar{F}_I - F_I \bar{X}^I)(\nabla^a V_a - \frac{1}{2} V^a V_a - \frac{1}{4} |M_{ij}|^2 + D^a \Phi^i{}_\alpha D_a \Phi^\alpha{}_i) \;.
\end{aligned}
$$

Semiclassical approach is very hard!

Localization

Localization means deforming the action by a Q exact term [Witten, Duistermaat,Heckamn, Schwarz, Zaboronsky]

$$S \to S + tQV$$

- Q is a fermionic generator which squares to a bosonic symmetry of the theory $Q^2 = \mathcal{L}_\phi$.
- Since V is by construction $\mathcal{L}_\phi$ invariant, $Q^2 V = 0$.
- Since $QS = 0$ and $Q^2 V = 0$ we can show that
$$\frac{d}{dt}\int e^{-S-tQV} = 0$$

Then when we send $t \to \infty$ the theory localizes into the critical points of QV and the semiclassical aproximation becomes exact.

$$\int e^{-S} = \sum_{\sigma \in \delta(QV)} e^{-S(\sigma)} \mathrm{sdet}(\delta^2 QV)$$

Localization

To apply localization in our problem we need to identify a fermionic symmetry Q.

The near horizon geometry of the black hole is invariant under the supergroup $SU(1,1|2)$.

Apart from the bosonic generators of $SL(2,\mathbb{R}) \times SU(2)$ symmetries of $AdS_2 \times S^2$ it contains in addition 8 supercharges Q which close into the $1+1$ dimensional $\mathcal{N}=4$ superconformal algebra.

We pick an element which squares to $Q^2 = L - J$. [Banerjee, Banerjee,Gupta,Mandal, Sen]

Localization in supergravity

We consider the computation of $W(q,p)$ in supergravity coupled to N_v+1 vector-multiplets.

To apply localization we need an off-shell representation of the fermionic symmetry Q.

In general off-shell supergravity is notorously complicated but for $\mathcal{N}=2$ vector multiplets there is an elegant construction. [de Wit, Van Proeyen, Van Holten]

The various couplings in the supergravity action are enconded in a sole function: the prepotential $F(X)$. This function is different in different phases of the theory and encodes also higher derivative corrections.

Localization in supergravity

In the localization procedure we choose V to be of the form

$$V = (Q\Psi, \Psi)$$

where Ψ denotes all fermion fields in the theory. [Pestun, Banerjee et. al.]

The bosonic part of the QV action is then

$$QV|_{bosonic} = (Q\Psi, Q\Psi) = (Q\Psi)^{\dagger} Q\Psi$$

which is a sum of perfect squares.

The critical points correspond to the BPS equations in field space

$$Q\Psi = 0$$

Since the SUSY variations don't depend on the prepotential, the localization locus will be independent of the phase we are considering!

Localization in supergravity

We find that the scalars in the vector multiplets are allowed to go “off-shell” at the cost of exciting the auxiliary fields [Dabholkar, Gomes, Murthy]

$$ds^2_{AdS_2} = d\eta^2 + \sinh^2\eta\, d\theta^2$$

$$X^I = X^I_* + \frac{C^I}{\cosh\eta}, \ \bar{X}^I = \bar{X}^I_* + \frac{C^I}{\cosh\eta}, \ K^I = \frac{2C^I}{\cosh^2\eta}$$

We see that the solutions are smooth everywhere inside AdS_2 and respect the boundary conditions.

The scalars are excited above their attractor values at the cost of exciting the auxiliary fields.

There is a continuous family of localizing solutions labelled by $N_v + 1$ parameters C^I.

Localization in supergravity

After removing the divergent piece we get

$$W(q,p) = \int D\phi e^{S_{ren}(\phi,q,p)}$$

The "zero mode" ϕ is basically the value of the scalar at the origin of AdS_2.

The renormalized action is

$$S_{ren} = -\pi q_I \phi^I - 2\pi i \left[F\left(\frac{\phi^I + ip^I}{2}\right) - \bar{F}\left(\frac{\phi^I - ip^I}{2}\right) \right]$$

the OSV [Ooguri,Strominger,Vafa] conjectured integrand!

$\mathcal{N} = 8$ Black holes

- 1/8 BPS black holes in Type IIB on T^6;
- The microscopic degeneracy is given by

$$\sum c(4n - j^2) q^n y^j = \frac{\vartheta_1(\tau, z)^2}{\eta(\tau)^6}, \; q = e^{2\pi i \tau}, \; y = e^{2\pi i z}$$

$$\vartheta_1(\tau, z) = q^{\frac{1}{8}}(y^{\frac{1}{2}} - y^{-\frac{1}{2}}) \prod_{n=1}^{\infty}(1 - q^n)(1 - yq^n)(1 - y^{-1}q^n)$$

$$\eta(\tau) = q^{\frac{1}{24}} \prod_{n=1}^{\infty}(1 - q^n)$$

- For large charges the Wald entropy gives

$$S_{BH} = \pi\sqrt{\Delta} = \pi\sqrt{4n - j^2}$$

- The R^2 higher derivative corrections vanish identically in this background (for T^6). The prepotential has a simple expression

$$F = \frac{X^1}{X^0} C_{ab} X^a X^b$$

$\mathcal{N} = 8$ Black holes

The degeneracy can be written as an exact Rademacher expansion

$$d(\Delta = 4n - j^2) = (-1)^{\Delta+1} \int \frac{\vartheta_1(\tau, z)^2}{\eta^6(\tau)} e^{-2\pi i n\tau - 2\pi i j z} d\tau dz$$

$$d(\Delta) = \sum_c c^{-9/2} Kl(\Delta, c)\, I_{\frac{7}{2}}(\frac{\pi\Delta}{c})$$

$$I_n(t) = \frac{1}{2\pi i} \int_{\epsilon - i\infty}^{\epsilon + i\infty} \frac{dt}{t^{n+1}} e^{t + \frac{z^2}{4t}}$$

The Kloosterman sums $Kl(\Delta, c)$ are defined as

$$Kl(\Delta, c) = \sum_{d \in \mathbb{Z}^*/\mathbb{Z}_c} e^{2\pi i \Delta \frac{d}{c} - \frac{d^{-1}}{c}}$$

$\mathcal{N} = 8$ Black holes

We define a measure on the space of X's

$$\int dX^I \sqrt{G} e^{-\int G_{ab}(X)\dot{X}^a \dot{X}^b}$$

The induce metric M_{ab}on the localization locus reveals crucial for duality invariance of the degeneracy [Dabholkar, Gomes, Murthy]

$$W(q,p) = \int d\phi \sqrt{M(p,\phi)} e^{S_{ren}(q,p,\phi)}$$

which generalizes previous works [de Wit, Cardoso, Mahapatra]. After performing the remaining gaussian integrals we recover the leading Bessel function (c=1)

$$I_{7/2}(\pi\Delta).$$

This result goes farher beyond the semiclassic approximation! Moreover, the inclusion of additional $\mathbb{Z}_c$ orbifolds of $AdS_2 \times S^2$ reproduces the non-perturbative corrections

$$I_{7/2}(\frac{\pi\Delta}{c}) \sim e^{\frac{\pi\sqrt{\Delta}}{c}}, \ \Delta \gg 1$$

Conclusions

- Localization allows to reduce a very complicated path integral in AdS_2 to a finite integral.
- Integration variables ϕ correspond to some normalizable mode that can be excited at the cost of exciting some auxiliary field! Importance of off-shell gravity!!
- For Black holes in theories without gravity corrections it is possible to reproduce, up to group theoretic numbers, the microscopic answer for any value of the charges!! Can be seen as an example of Exact holography!
- We see that holography is able to probe the microscopic details of black holes, going much farther than the thermodynamic limit!

Thank You!

"Not everything that counts can be counted, and not everything that can be counted counts.", Albert Einstein