Chiral Phase Transitions around Black Holes

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IV Black Holes Workshop

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Universidade de Aveiro
Portugal

- A. Flachi and T. Tanaka, `Chiral Modulations in Curved Spacetimes I: Formalism', JHEP 02 (2011) 026
- **A. Flachi and T. Tanaka, `Chiral Phase Transitions around Black Holes', Phys Rev D84 061503 (2011) (Rap. Comm.)**
- A. Flachi, `Chiral Modulations in Curved Spacetimes II: Conifolds', to appear JHEP (2011)
- A. Flachi, `(De)confinement Transitions and Hadronization Processes around Black Holes', to appear

Motivations

- Primordial Black Holes

- Mini Black Holes

- QCD

  During the evaporation the Black Hole may sweep through critical points

- ...

Evaporation

Black holes evaporate at a temperature $T_{bh} = 1/(8\pi m_{bh})$

$T_{BH} << m_e$ only photons, gravitons and neutrinos are emitted

$T_{BH} \sim m_e$ electrons start to be emitted; cross section is small; scattering unfrequent

$T_{BH} \sim 100$ MeV ***first muons and pions, then hadrons copiously produced; local thermal equilibrium;***

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``how this happens ?
are hadrons directly emitted by the BH?
elementary particles are emitted instead and produce jets?”

Some Previous work

S. Hawking (1981)
I. Moss, (1985)

electroweak phase transitions

A. Heckler, (1997)

photosphere formation

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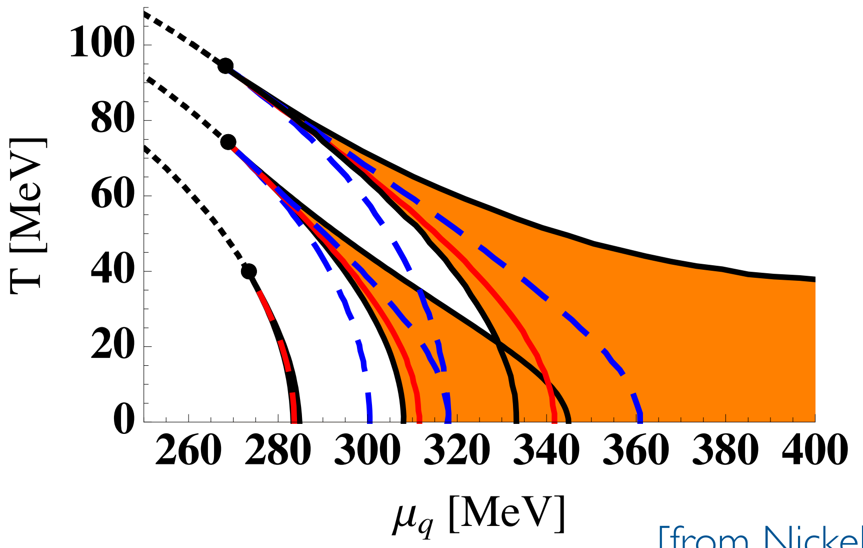
QCD

Similar problem occurs in QCD

Lattice QCD

very difficult (even in flat space);
currently not viable



[from Nickel, PRL '09]

Effective Field Theories

- Low-Energy QCD
- Heavy Ion Collisions
- Superconductivity
- Quark Condensation
- Condensed Matter Precursors

share global symmetries of QCD
chiral symmetry breaking
confinement/deconfinement transitions

amenable of analytical treatment
(mean field; large N)

Inhomgeneous phases

Chiral density Wave Approach
Ginzburg Landau Approximation
Exact Solutions

Strongly Interacting Fermion Effective Field Theories

$$S = \int d^4x \sqrt{g} \left\{ \bar{\psi} i \gamma^\mu \nabla_\mu \psi + \frac{\lambda}{2N} \left(\bar{\psi}\psi \right)^2 \right\}$$

ψ : $(D \times N_f \times N_c)-$dimensional quark spinor

$N \equiv N_f \times N_c$

$g = |\mathrm{Det} g_{\mu\nu}|$

Chiral Symmetry: $\psi \to \gamma^5 \psi$

Mass terms cannot appear without breaking the above symmetry.

When this happens, $< \bar{\psi}\, \psi >$ acquires a non zero VEV and fermion become massive

Dynamics of the condensate $\langle \bar{\psi}\psi \rangle = -N\sigma(x)/\lambda$

Condensate Effective Action

In order to study the condensate dynamics, we need to compute (at finite temperature) and minimize the effective action $\Gamma[\sigma]$

BH geometry

$$ds^2 = f dt^2 + f^{-1} dr^2 + r^2 (d\theta^2 + \sin^2\theta d\varphi^2)$$

Large - N

$$\Gamma = -\int d^4x \sqrt{g} \left(\frac{\sigma^2}{2\lambda} \right) + \mathrm{Tr} \ln \left(i\gamma^\mu \nabla_\mu - \sigma \right)$$

Computation of functional determinants for inhomogeneous backgrounds on BHs

Conformal rescaling

$$d\hat{s}^2 = f^{-1} ds^2$$

$$\Gamma = -\int d^4x\sqrt{g}\left(\frac{\sigma^2}{2\lambda}\right) + \hat{\Gamma} + \delta\Gamma$$

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$$\hat{\Gamma} = \frac{1}{2}\sum_{\epsilon=\pm}\mathrm{Tr}\ln\left[\hat{\Box} + \mathscr{A} + f\sigma_\epsilon^2\right]$$

$$\sigma_\epsilon^2 := \sigma^2 + \epsilon f^{1/2}\sigma'$$

$$\mathscr{A}^{(n)} = f\left((n-2)\Delta\ln f/4 - (n-2)^2(\nabla\ln f)^2/16\right)$$

$$\hat{\Gamma} = \frac{1}{2}\int d^3x\sqrt{\hat{g}}\left[\zeta(0)\ln\ell^2 + \zeta'(0)\right]$$

$$\zeta(s) := \frac{1}{\Gamma(s)}\sum_{n,\epsilon}\int dt\, t^{s-1}\mathrm{Tr}\; e^{-t\left(-\hat{\Delta}+\omega_n^2+\mathscr{A}+f\sigma_\epsilon^2\right)}$$

Co-cycle

$$\delta\Gamma = \lim_{n\to 4}\left(C_n^{(2)}[\hat{g}] - C_n^{(2)}[g]\right)/(n-4)$$

Computation of heat-kernel coefficients

$$C_n^{(2)}[g] = \frac{1}{(4\pi)^{\frac{n}{2}}}\frac{1}{2}\int d^nx\sqrt{g}\left(V^2 - \frac{1}{3}RV + \cdots\right)$$

$$\delta\Gamma = \frac{\beta}{2(4\pi)^2}\sum_{\epsilon=\pm}\int d^3x\sqrt{g}\left[\frac{\sigma_\epsilon^4}{2}\ln f - \frac{2\sigma_\epsilon^2}{f}\lim_{n\to 4}\frac{d\Lambda_n}{dn}\right]$$

$$\Lambda_n = \mathscr{A}^{(n)} - (\hat{R}^{(n)} - fR^{(n)})/6$$

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$$\varpi_\nu(u) := \sum_{n=1}^{\infty}(-1)^n n^{-\nu} K_\nu(n\beta u)$$

$$a_\epsilon := \frac{1}{180}\left(\hat{R}^2_{\mu\nu\tau\rho} - \hat{R}^2_{\mu\nu} - \hat{\Delta}\hat{R}\right) + \frac{1}{6}\hat{\Delta}\left(f\sigma_\epsilon^2\right)$$

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Minimization up to 2nd order

$$\sigma'' + \delta_1\sigma' + \delta_2\sigma'^2 + \mathscr{K} = 0$$

with regular boundary conditions

Higher order terms can be included systematically

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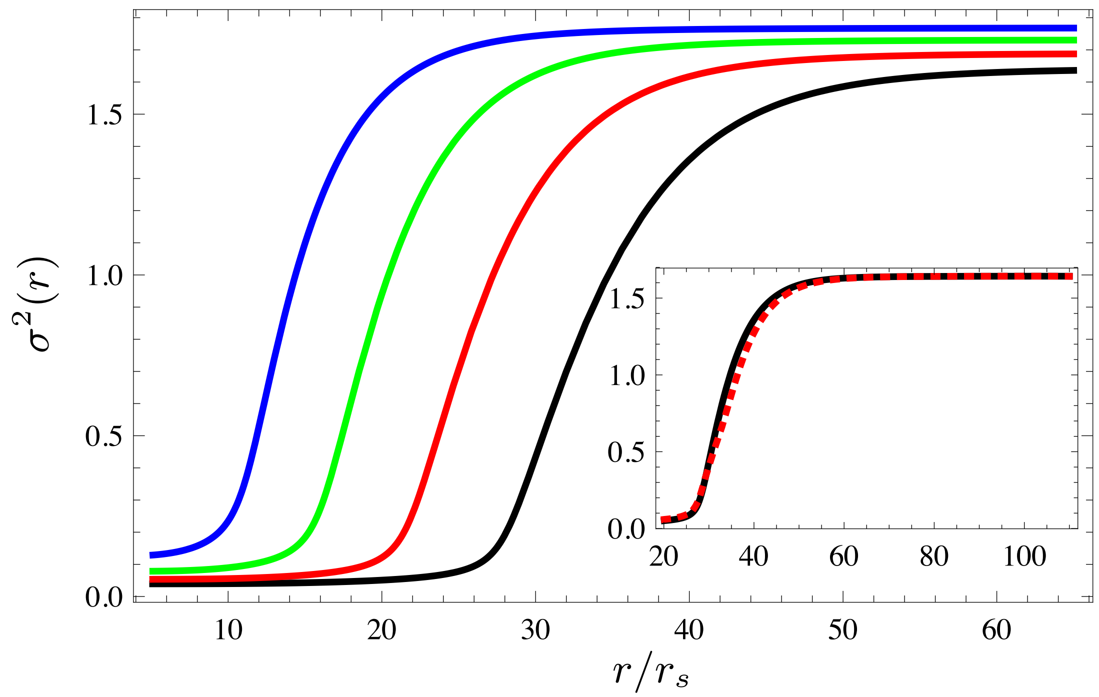
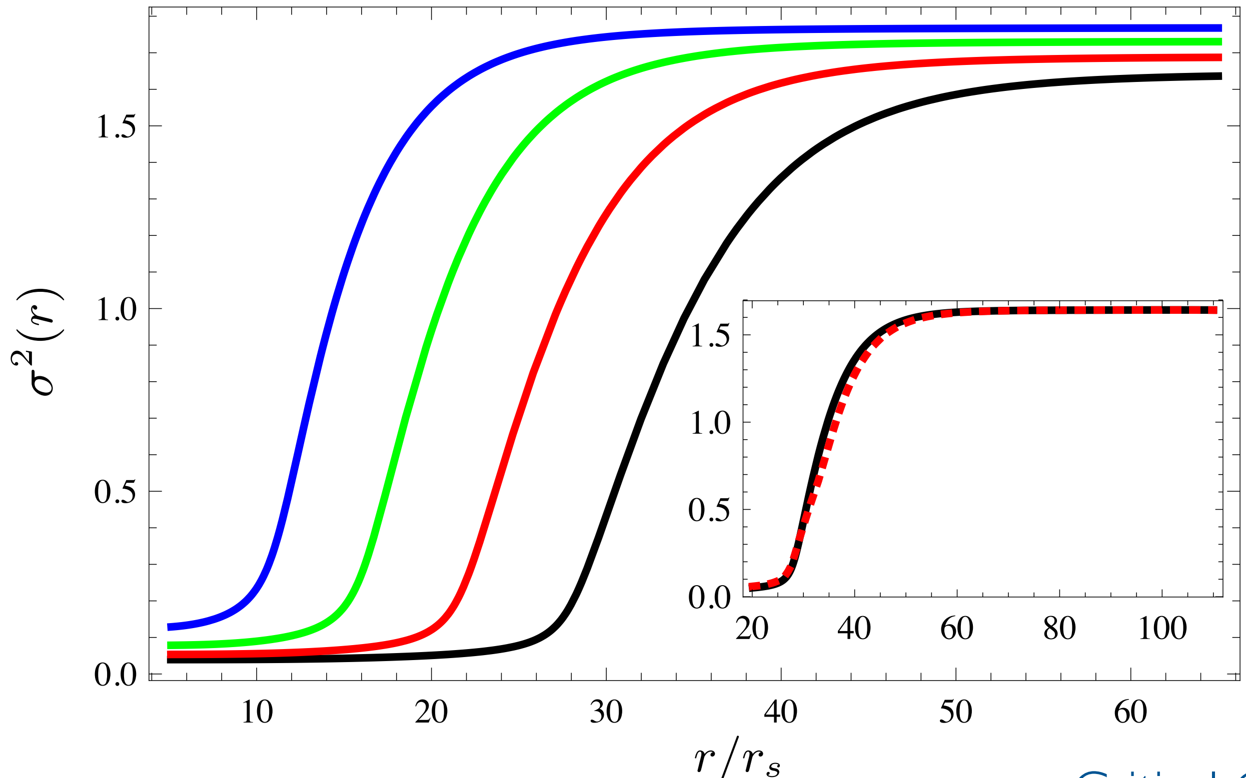
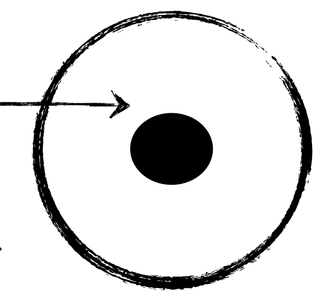


Solutions



The solution has a kink profile that vanishes near the horizon

A region of *chirally restored* symmetry is separated from one of *broken symmetry*

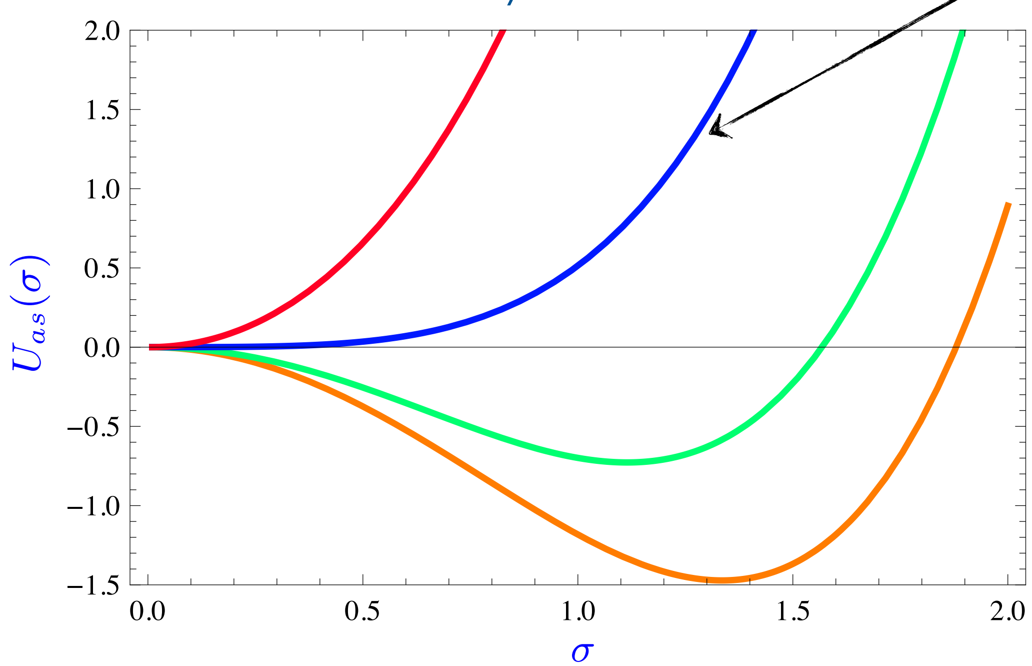


Radius of the bubble

$$r_{bubble} \sim r_s/\left(1 - T_{BH}^2/T_c^2\right)$$

Critical Configuration

Thermodynamic Potential



Thickness of the bubble increases as the temperature approaches T_c

Asymptotic Potential

$$\partial_\sigma U_{as} = -\frac{3\sigma\left(4\lambda\sigma(4\varpi_{-1}(\sigma) + \beta\sigma\ln\left(\sigma/\ell\right) - 2\lambda\beta\sigma^2 + \beta\right)}{2\lambda\beta(-4\beta\sigma\varpi_1(\sigma) - 6\varpi_0(\sigma) + 3\ln\left(\sigma/\ell\right) - 2)}$$

Critical Temperature

$$T_c = \sqrt{3}\lambda^{-1/2}.$$

Chromosphere

Whether the emission of strongly interacting particles may produce a chromosphere

Inside the bubble

In the present situation scattering occurs only in the radial direction and angular quantum numbers do not change

Particles stay almost massless and there are no processes that randomize the particle motion inside the bubble

Only the ***gradient of the effective local temperature*** is important, and particle simply stream away to infinity reducing their velocity due to the gravitational attraction of the black hole

This configuration will be similar to the shock produced by stellar wind

Bubble interface

Scattering occurs non-trivially and particles with energy smaller than the VEV of the condensate outside of the bubble will be reflected back

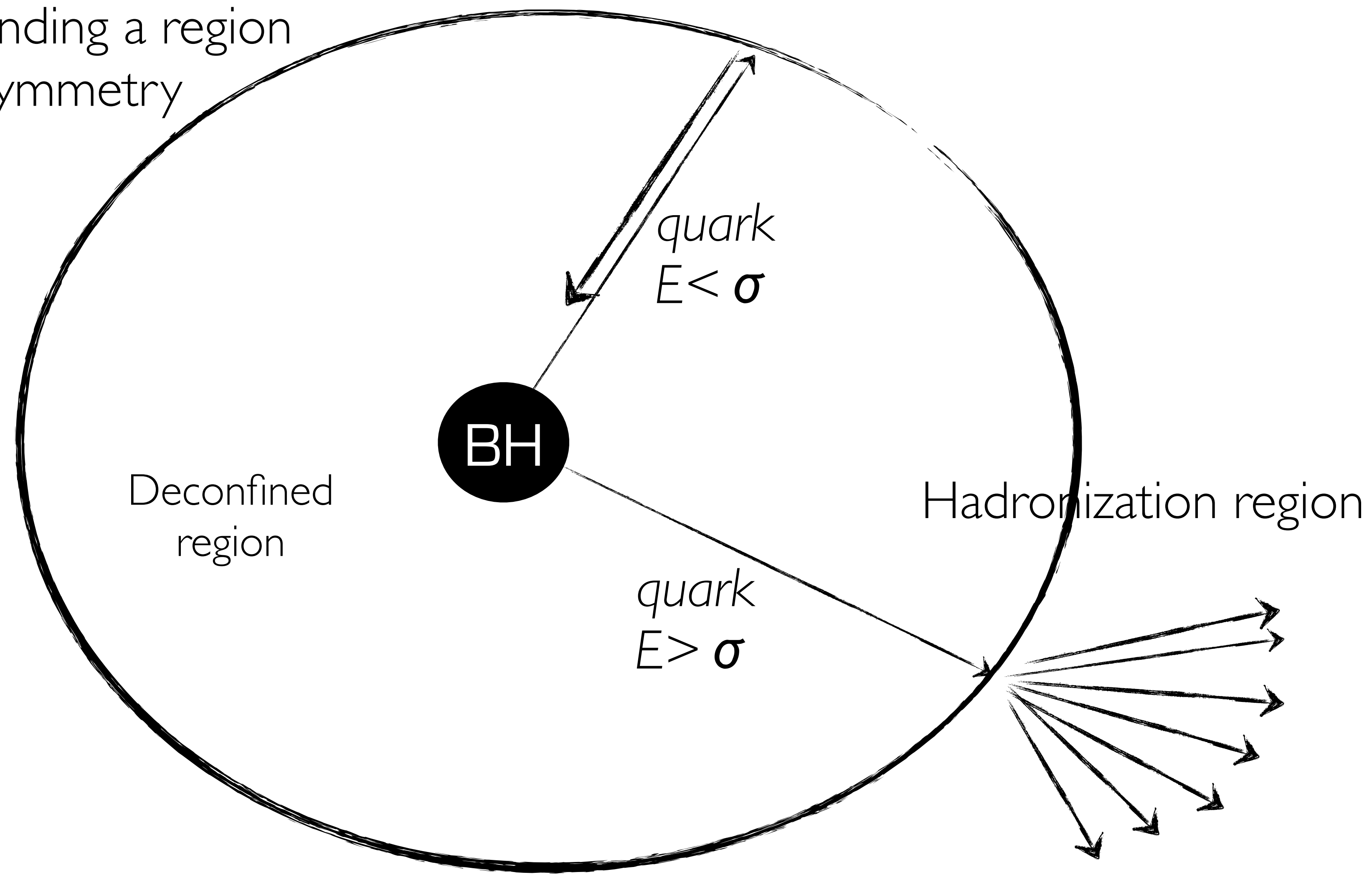
Outside the Bubble

Only hadrons exist

(De)Confinement Transitions and Hadronization

B := Bubble surrounding a region of restored chiral symmetry

σ : VEV of the condensate outside the bubble



Some open questions

Are there other possible inhomogeneous configuration that the condensate may take ?

How does the scattering at the interface occurs and how much of the radiation can actually escape to infinity?

Does the presence of gauge degree of freedom may change the picture ?

We completely ignored back-reaction effects. Does back-reaction change anything ?

What happens to the propagation of particles coupled to quarks when further interactions are added ?

What happens in AdS ?

We know that fermions do not super-radiate. Do interactions change anything ?

Merry
Christmas!