Dynamical AdS Chern Simons Black branes : QNMs

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dCS gravity

Modification to Einstein gravity

$$S_{dCS} = \kappa \int \sqrt{-g}\,(R - 2\Lambda)\, d^4x + \frac{\alpha}{4} \int \sqrt{-g}\phi({}^*RR) d^4x - \frac{\beta}{2}\sqrt{-g}\,(\nabla_a\phi\nabla^a\phi\,)\, d^4x,$$

where ${}^*RR = R_{abcd}\epsilon^{baef} R^{cd}{}_{ef}$ is the Pontryagin density.

$\phi =$ Cst. $\Rightarrow$ AdS Einstein-Hilbert + Topological term.

Parity breaking analog to (dilaton-) Gauss Bonnet term.

dCS equation of motion

Equation of motion :

$$\begin{aligned} R_{ab} &= -\alpha C_{ab} + \frac{\beta}{2}\nabla_a\phi\nabla_b\phi, \\ \Box\phi &= -\frac{\alpha}{4\beta}({}^*RR), \\ C_{ab} &= \nabla_c\phi\epsilon^{cde(a}\nabla_e R^{b)}{}_d + \nabla_c\nabla_d\phi\ {}^*R^{d(ab)c}, \end{aligned}$$

- C : 4 dimensional analog to Cotton tensor.
- plays nontrivial role for non spherically symmetric configurations.

Linear dCS dynamic

Background :

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Sigma_k^2, \ \phi = 0,$$

Σ_k : angular sector, planar ($k = 0$), spherically symmetric ($k = 1$).

Is a solution to AdS dCS gravity for

$$f(r) = \frac{r^2}{\ell^2} + k - \frac{r_0}{r}, \ \ell = -\frac{3}{\Lambda}.$$

Linear dCS dynamic

Perturbation : Regge-Wheeler sector

$$\begin{aligned} ds^2 &= -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Sigma_k^2 \qquad (1) \\ &\quad +2h_0(r)e^{-i\omega t}\mathbb{T}dt + 2h_1(r)e^{-i\omega t}\mathbb{T}dr, \qquad (2) \\ \phi &= \frac{\sigma}{r} Y^{lm} e^{-i\omega t}, \end{aligned}$$

- Y^{lm} : scalar harmonics on Σ_k,
- $\mathbb{T} = -\frac{1}{F_k(\theta)}\partial_\varphi Y^\kappa_{lm} d\theta + F_k(\theta)\partial_\theta Y^\kappa_{lm} d\varphi$, : vector harmonics.
- $F_1 = \sin\theta, \; F_0 = 1.$

Linear dCS dynamic

Equations : Coupled linear ODE system for h_0, h_1, σ + constraints
Reduce to 2 coupled equations :

$$\frac{d^2}{dr_*^2}Q + (\omega^2 + V_{RW})Q + S_{RW}\sigma = 0,$$
$$\frac{d^2}{dr_*^2}\sigma + (\omega^2 + V_S)\sigma + S_S Q = 0,$$

- r_* : tortoise coordinate $dr_*^2 = dr^2/f(r)$,
- V_{RW}, V_S : Regge-Wheeler and scalar potentials,
- S_{RW}, S_S : source terms,
- $Q = \omega f h_1/r$: Regge-Wheeler master function.

Boundary conditions

- QNMs : typical of dissipative systems.
- Black holes / planes : absorbs energy at the horizon.
- QNMs : ingoing waves @ horizon, outgoing waves @ infinity.

Near Horizon ($r_* \to -\infty$) :

$$Q \approx Q_H^{out} e^{i\omega r_*} + Q_H^{in} e^{-i\omega r_*}, \ \sigma \approx \sigma_H^{in} e^{i\omega r_*} + \sigma_H^{in} e^{-i\omega r_*}$$

Infinity $r_* \to 0$:

$$Q \approx \frac{Q_\infty^{reg}}{r} + Q_\infty^{irr}, \ \sigma \approx \frac{\sigma_\infty^{reg}}{r^2} + \sigma_\infty^{irr} r.$$

Boundary conditions :

$$Q_H^{out} = \sigma_H^{out} = Q_\infty^{irr} = \sigma_\infty^{irr} = 0.$$

Frobenius expansion

How to find QNMs in AdS ?
$\Rightarrow$ Frobenius expansion.

$$Q(r) = e^{-i\omega r_*}\tilde{Q}(r),$$

- Write $\tilde{Q}$ in a series close to the horizon,
- Solve recursively the series,
- Can be proved that the series converges everywhere,
- Impose the boundary condition at infinity from the series.

Frobenius expansion

Extension of Frobenius expansion to coupled system :

$$f(r) = \frac{(r - r_h)\left(\kappa\ell^2 + r^2 + r r_h + r_h^2\right)}{\ell^2 r},$$

$r_h + r_h^3/\ell^2 = r_0$. Factorize the horizon behavior :

$$Q(r) = e^{-i\omega r_*}\tilde{Q}(r), \qquad \sigma(r) = e^{-i\omega r_*}\tilde{\sigma}(r),$$

define

$$x = \frac{1}{r}, x_h = \frac{1}{r_h}, s_1(x) = s_2(x) = \frac{x^4 f}{x - x_h},$$

$$f_1 = f_2 = 2x^3 f - x^2 f' - 2i\omega x^2,$$

$$v_{RW} = (x - x_h)\frac{V_{RW}}{f}, t_{RW} = (x - x_h)\frac{T_{RW}}{f},$$

$$v_S = (x - x_h)\frac{V_S}{f}, t_S = (x - x_h)\frac{T_S}{f}.$$

Frobenius expansion

Finally, expand unknown functions :

$$\tilde{Q}(x) = \sum_{k=0}^{\infty} q_k \left(x - \frac{1}{r_h} \right)^k , \quad \tilde{\sigma}(x) = \sum_{k=0}^{\infty} \sigma_k \left(x - \frac{1}{r_h} \right)^k .$$

Equations → recursion relations for $q_k, \ \sigma_k$:

$$\begin{aligned} q_n &= -\frac{1}{P_n^1} \sum_{k=0}^{n-1} \left[\left(k(k-1) s_{1,n-k} + k f_{1,n-k} + v_{RW,n-k} \right) q_k + t_{RW,n-k} \sigma_k \right] , \\ \sigma_n &= -\frac{1}{P_n^2} \sum_{k=0}^{n-1} \left[\left(k(k-1) s_{2,n-k} + k f_{2,n-k} + v_{S,n-k} \right) \sigma_k + t_{S,n-k} q_k \right] , \end{aligned}$$

$P_n^i = n(n-1)s_{i,0} + n f_{i,0}$, and coefficients expanded as

$$X(x) = \sum_{k=0}^{\infty} X_k (x - x_h)^k .$$

CFT dispersion relation

$d-$dimensional CFT, dispersion relations shear, sound mode in the hydrodynamic (small momentum $q \to 0$) limit [Natsuume, 2008] :

$$\begin{aligned}
\omega_{\text{shear}} &= -iD_\eta q^2 - iD_\eta^2(\tau_\pi + \tau_3)q^4 + \mathcal{O}(q^6)\,, \\
\omega_{\text{sound}} &= v_s q - i\frac{d-2}{d-1}D_\eta q^2 + \\
&+ \frac{d-2}{2(d-1)v_s}D_\eta\left(2v_s^2\tau_\pi - \frac{d-2}{d-1}D_\eta\right)q^3 + \mathcal{O}(q^4)\,,
\end{aligned}$$

$D_\eta = \eta/(\epsilon + p)$ and η, ϵ, p, v_s : shear viscosity, energy density, pressure and speed of sound, $\mathcal{O}^1$ hydro.

Higher order hydro : relaxation time $\tau_\pi \to \mathcal{O}^2$, $\tau_3 \to \mathcal{O}^4$.

QNMs dispersion relation

AdS/CFT : CFT Dispersion ⇔ AdS black branes QNMs hydro limit :

$$\begin{aligned}\tilde{Q}(r) &\approx \tilde{Q}_0(r) + \tilde{Q}_2(r)q^2 + \tilde{Q}_4(r)q^4 + \tilde{Q}_\alpha(r)q^4\alpha^2\,,\\ \tilde{\sigma}(r) &\approx \tilde{\sigma}_\alpha(r)q^4\alpha\,.\end{aligned}$$

Then

$$\begin{aligned}\omega_{\text{shear}} &\sim -i\frac{q^2}{3r_h} - i\left(\gamma\frac{\ell^2}{r_h^3} - \chi\frac{\alpha^2}{\ell^2 r_h^3}\right)q^4 + \mathcal{O}(q^6)\\ \omega_{\text{sound}} &\sim \pm\frac{q}{\sqrt{2}\ell} - i\frac{1}{6r_h}q^2 + +\frac{\sqrt{2}\ell}{9r_h^2}\left[\frac{7}{9} - \frac{\log 3}{2} + \frac{\sqrt{3}\pi}{2}\right]q^3 + \mathcal{O}(q^4),\end{aligned}$$

$\gamma = \frac{9-9\log 3+\sqrt{3}\pi}{162} \sim 0.028$, $\chi = \frac{3}{640}\left(201 - 20\sqrt{3}\pi - 60\log 3\right) \sim 0.123.$

Comparing CFT and QNMs dispersion relation, extract hydrodynamical quantities :

$$\tau_3 = -\tau_\pi + \frac{27\gamma}{4\pi T}\left[1 - \frac{\chi}{\gamma L^4}\alpha^2\right] ,$$

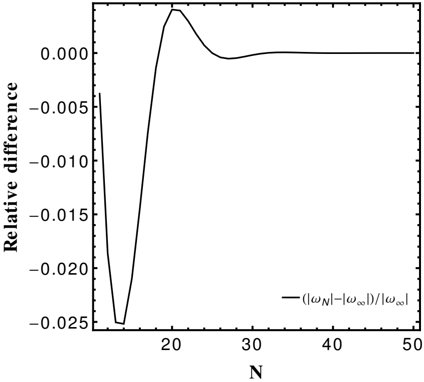
$T = 3r_h/4\pi$, $\epsilon + p = sT$ and

$$\tau_\pi = \frac{18 - 9\log 3 + \sqrt{3}\pi}{24\pi T} ,$$

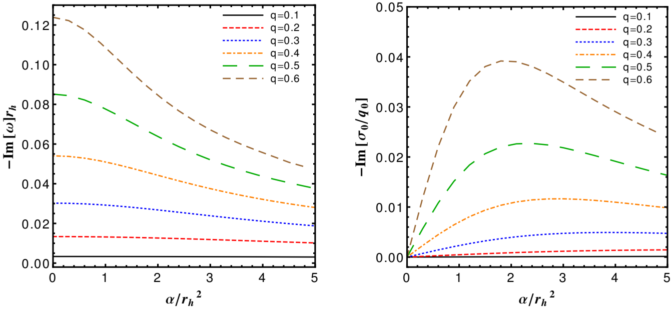
is unaffected by dCS correction.

Numerical results

Truncation (N terms) of the Frobenius series converges :

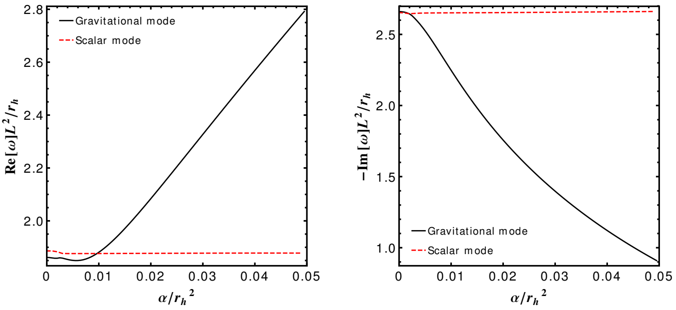


Numerical results : Hydro modes



Hydrodynamic mode for Schwarzschild black branes as a function of the DCS coupling α, for different values of the momentum q.

Numerical results : Overtone



First overtones for scalar and gravitational modes as functions of α for $q = 3$. The scalar mode is roughly independent from α. Different values of q give the same qualitative result.

Conclusion / Summary / Extensions

- Extension of Frobenius series to coupled system, straightforwardly appliable to arbitrary number of equations.
- Potential applications for holographic superconductors, modes of AdS rotating objects, ...
- Modification to hydrodynamical mode appear at higher order hydrodynamic.
- Universality of shear to entropy is safe in dCS.
- Extension to 5D, dual theory better known.

Conclusion / Summary / Extensions

Obrigada pela vossa atenção

e Feliz Natal ! ! !