Long-lived scalar field dark matter clouds around black holes.

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Scalar fields are a common theme in modern cosmology. They play a central role in inflation, and they have been frequently used to describe dark energy in place of the cosmological constant. Moreover, even if the usual description of dark matter is done in terms of weakly interacting massive particles (WIMPs), given the persistent uncertainty about the real nature of dark matter, one can not a priori reject one candidate in favor of another. Classical scalar fields have been proposed as possible candidates for the dark matter component of the universe.

Given the fact that super-massive black holes seem to exist at the center of most galaxies, in order to be a viable candidate for the dark matter halo a scalar field configuration should be stable in the presence of a central black hole, or at least be able to survive for cosmological time-scales.

We will restrict to the case of a scalar field as a test field in the background spacetime of a Schwarzschild black hole. It is for this reason that I will talk about "scalar field clouds" rather than "galactic halos", in order to maintain a clear perspective of the real problem we are studying.

The dark matter content in the central region of a galactic halo is very diluted, so it is to be expected that this approximation could give some light on the nature of the dark matter problem.

We will assume that the energy associated with the scalar field configuration is very small compared to the mass of the black hole, so that the gravitational back-reaction associated to the scalar field distribution can be disregarded (to be consistent with the test field approximation).

The equation of motion for that field is given by the Klein-Gordon equation

$$(\Box - \mu^2)\phi = 0 \; ,$$

Working in Schwarzschild coordinates, we can write the spacetime metric as

$$ds^2 = -N(r)dt^2 + \frac{dr^2}{N(r)} + r^2 d\Omega^2 \;, \qquad N(r) := 1 - 2M/r \;,$$

with M the mass of the black hole and $d\Omega^2 := d\theta^2 + \sin^2\theta d\varphi^2$ the standard solid angle element.

To look for solutions of the Klein-Gordon equation above, we will start by considering a decomposition into spherical harmonics:

$$\phi(t, r, \theta, \varphi) = \frac{1}{r} \sum_{\ell,m} \psi_{\ell m}(t, r) Y^{\ell m}(\theta, \varphi) \;,$$

where the $1/r$ factor has been introduced for convenience, and the parameters ℓ and m take the usual values: $\ell \in \{0, 1, 2, \ldots\}$, $-\ell \leq m \leq \ell$.

As a result we obtain the following family of reduced equations

$$\left[\frac{1}{N(r)}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial r}N(r)\frac{\partial}{\partial r} + \mathcal{U}_\ell(\mu, M; r)\right]\psi_{\ell m} = 0 \ ,$$

where we have defined

$$\mathcal{U}_\ell(\mu, M; r) := \frac{\ell(\ell+1)}{r^2} + \frac{2M}{r^3} + \mu^2 \ .$$

In order to look for the stationary solutions we make a further decomposition of the functions $\psi_{\ell m}(t, r)$ into oscillating modes of the form:

$$\psi_{\ell m}(t, r) = e^{i\omega t}u(r) \ ,$$

with ω a real frequency, and $u(r)$ a complex function of r in the interval $(2M, \infty)$.

This equation can be rewritten as the following time-independent Schrödinger-like equation:

$$\left[-\frac{\partial^2}{\partial r^{*2}} + V_{\rm eff}(r^*)\right] u(r^*) = \omega^2 u(r^*) \ , \qquad -\infty < r^* < \infty,$$

with the effective potential $V_{\rm eff}(r^*)$ defined as

$$V_{\rm eff}(r^*) := N(r)\,\mathcal{U}_\ell(\mu, M; r) \ , \qquad r = r(r^*).$$

and where we have introduced the Regge-Wheeler tortoise coordinate $r^* := r + 2M\ln(r/2M - 1)$

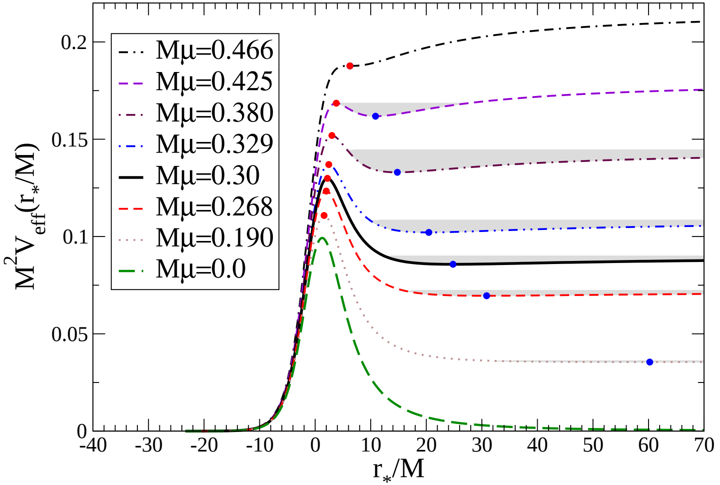


Fig: The effective potential $M^2 V_{\rm eff}$ for $\ell = 1$, and different values of the parameter $M\mu$.

The mode solutions with $0 < \omega^2 < \mu^2$, are localized around the black hole since they decay exponentially at large r^*. This is not the case for the mode solutions with $\omega^2 > \mu^2$, which do not decay at spatial infinity. In the test field limit all configurations have the conserved energy $E = \sum_{\ell,m} E_{\ell m}$, with

Energy associated to the time killing vector field

$$E_{\ell m} = \int_{2M}^{\infty} \rho_E(r) dr \ ,$$

and where

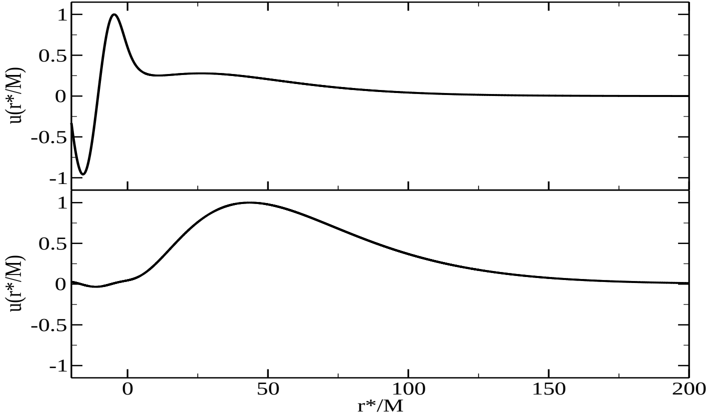
$$\rho_E(r) = \frac{1}{2}\left(\frac{1}{N(r)} \left| \frac{\partial \psi_{\ell m}}{\partial t} \right|^2 + N(r) \left| \frac{\partial \psi_{\ell m}}{\partial r} \right|^2 + \mathcal{U}_\ell(\mu, M; r) |\psi_{\ell m}|^2 \right)$$

A necessary condition for $0 < \omega^2 < \mu^2$ is the existence of the potential well. In order to find under which conditions such a potential well exists we first need to determine the critical points of the effective potential $V_{\rm eff}$. This is reflected in a bound for the values of $(M\mu)^2$

$$
\begin{aligned}
(M\mu)^2 &< -\frac{1}{32}(\ell^2+\ell-1)(\ell^2+\ell+1)^2 \\
&+ \frac{1}{288}\sqrt{3(3\ell^4+6\ell^3+5\ell^2+2\ell+3)^3}\,,
\end{aligned}
$$

Having found under which conditions there is a potential well, we now turn our attention to the existence of resonant modes.
The condition $0 < \omega^2 < \mu^2$ guarantee that the solution for the scalar field decays exponentially at spatial infinity and is “localized” close to the black hole, but the scalar field can still escape towards the black hole horizon.

Figure shows two solutions for particular values of $M\omega$. The first plot corresponds to the solution for $M\omega = 0.295$.
second plot, corresponds to the solution for $M\omega = 0.29619$. The behavior points out that the value $M\omega = 0.29619$ is close to a resonant frequency;



There is one point that we should mention here regarding the relation of the resonant states described above and the quasi-normal modes of massive scalar fields. Quasi-normal modes are solutions of the Klein-Gordon equation in a Schwarzschild or Kerr background for purely outgoing waves at spatial infinity and ingoing at the horizon. It is precisely this choice of boundary conditions what produces a discrete quasi-normal mode spectrum. What we are considering is somewhat different: we are looking for solutions of the Klein-Gordon equation, but that the scalar field has an exponential decay.

The stationary solutions, even the resonant ones, have a divergent energy due to their oscillatory behavior close to the horizon.

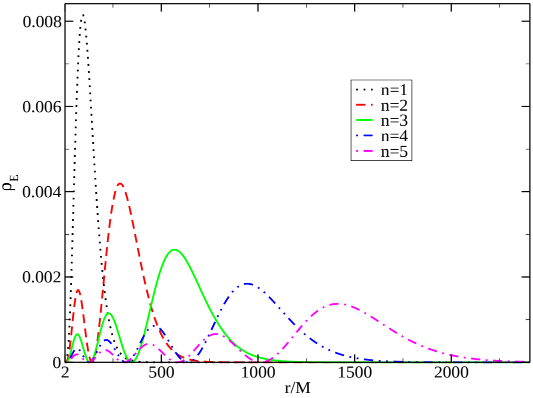
However, one can always construct physical configurations that are arbitrarily close to the stationary solutions and that can survive the black hole for arbitrarily long times:

Choose any stationary solution that decays at spatial infinity and set it to zero by hand in the interval $r \in (2M, 2M + \epsilon)$, for some small parameter $\epsilon > 0$ with dimensions of length.

The resulting configuration can be seen as a combination of the stationary solution plus a perturbation around the horizon, and it will be stationary for the domain of dependence of $[2M + \epsilon, \infty)$.

These configurations will then be used as initial data to be evolved numerically so that we can study the energy decay in a compact region outside the black hole. As boundary condition we impose at $r = R_{\rm out}$ the relation $u' = -|k|u$ with $k = \sqrt{\mu^2 - \omega^2}$ to ensure that $u(r)$ falls exponentially at large r. For the left boundary condition we just impose $u(R_{\rm in}) = 1$, since the equation is homogeneous so $u(r)$ can be rescaled afterwards. Finally, $u(r)$ is simply set to 0 for $r < R_{\rm in}$.

We concentrate on resonant configurations which, are the ones that show long lasting scalar field distributions outside the black hole. We construct pseudo-resonant initial data configurations for different values of ℓ and $M\mu$, and for each pair of these parameters we study the states up to the fifth mode.



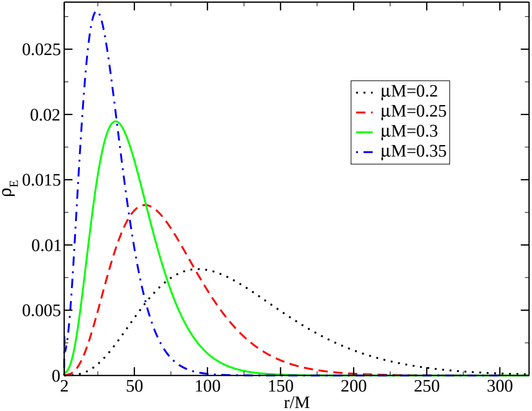


Fig: Radial energy density ρ_E for $\ell = 1$ and the first pseudo-resonant mode $n = 1$, for configurations with different values of $M\mu$. The solution has been normalized so that $E_{lm} = 1$.

The eigenvalues ω_n^2 of the resonant modes accumulate at $\omega^2 = \mu^2$ from below. It seems likely that there is in fact an infinite number of resonant modes n, although we have not investigated in any detail if this is indeed the case.

We study numerically the evolution of the scalar field $\psi_{lm}(t, r)$ for different initial data configurations. Although our attention is mainly focused on the evolution of the long lasting pseudo-resonant states, we also consider other configurations for comparison. We start by evaluating total energy loss and studying some spectral characteristics of the different configurations by means of a time Fourier analysis, and finish with more explicit considerations about how long such configurations can last.

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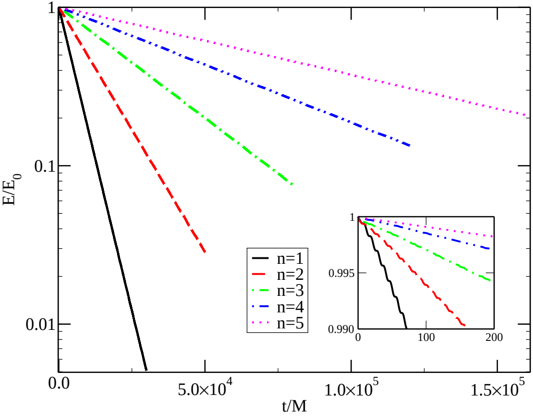
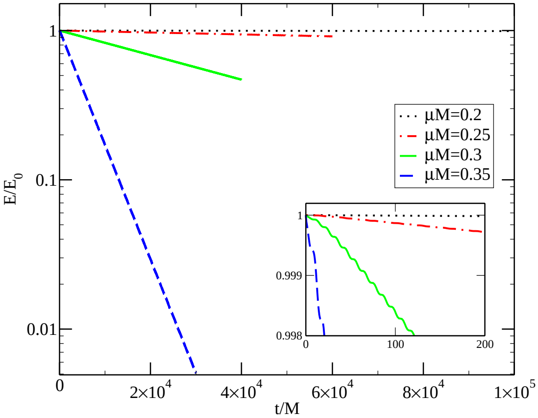
In order to evaluate how long a scalar field configuration remains confined in a compact region we evaluate the energy E_{lm}. The energy shows a slow exponential decay of the form

$$E(t) = E_0 \exp(-s\, t/M) \ ,$$

with s constant, except for some very small oscillations that remain during the whole evolution. Given the exponential decay that dominates the overall behavior, we can perform a linear fit of $\ln(E/E_0)$ as a function of t/M to calculate the parameter s.

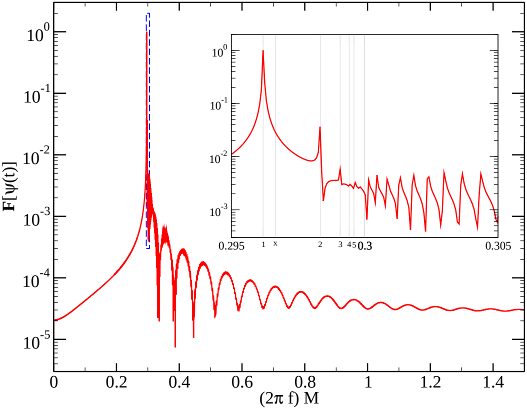
Energy of the scalar field vs. time for the evolution of pseudo-resonant initial data with $\ell = 1$, $M\mu = 0.35$ and modes $n = (1, 2, 3, 4, 5)$.





Energy of the scalar field vs. time for the evolution of the initial data corresponding to the first $(n = 1)$ pseudo-resonant mode with $\ell = 1$, and $M\mu = 0.2$, 0.25, 0.3 and 0.35.

Discrete Fourier transform in time for the evolution of non-resonant data with $M\mu = 0.3$, $\ell = 1$, and frequency $\omega_x = 0.29664794$.



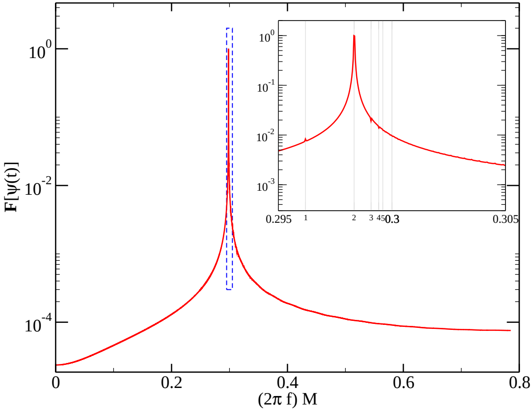


Fig: The Fourier transform in time vs. frequency for the evolution of the resonant data with $M\mu = 0.3$, $\ell = 1$, and frequency ω_2 corresponding to the second pseudo-resonant mode.

Changing back from geometric units, and considering a black hole mass of $10^8 M_\odot$, we get a characteristic time of order

$$t_{1/2} \approx \frac{680}{s} \text{ seconds} .$$

For the value $s = 7 \times 10^{-9}$ that corresponds to $\ell = 1$, $M\mu = 0.2$, $n = 4$, we obtain $t_{1/2} \approx 3,000$ years.

This is still a very small time when compared to cosmological time-scales, however, values of the parameter μ motivated by dark matter scalar field models correspond to $\hbar\mu \sim 10^{-24}\text{eV}$, or $M\mu \sim 10^{-6}$ in geometric units. One might expect that configurations with values of $M\mu$ motivated by dark matter scalar field models will have half-life times of cosmological scales.

However we were unable to study cases with such small μ, so the preceding argument is purely hypothetical.

However... If one imposes the condition of no waves coming from the region close to the horizon, while keeping the requirement of exponential decay at spatial infinity, it turns out that the Schrödinger-like is only satisfied for a discrete set of complex frequencies. These solutions have been called *quasi-resonances* in the literature (Ohashi:2004).

Both the stationary resonances and quasi-resonances are in fact non-physical solutions.

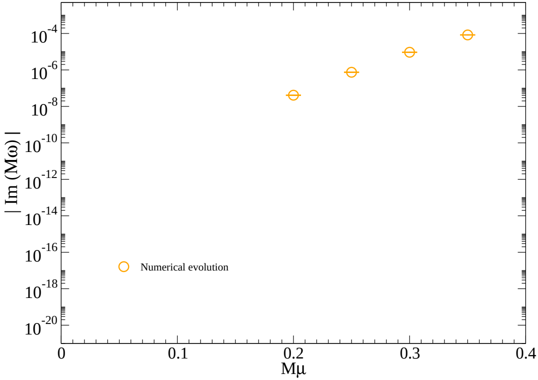
All purely stationary solutions, *i.e.* those with real ω, require waves to move outward from the horizon region to compensate for the waves that tunnel out through the barrier and move toward the horizon (otherwise the situation would not be stationary). Imposing the condition of no waves coming out from the horizon clearly improves the situation at the cost of introducing complex valued frequencies ω. This makes sense physically since now the solutions must decay in time as the waves tunnel out of the potential well and fall toward the BH, with this decay represented by the imaginary part of the frequency. Nevertheless, such solutions are still non-physical, as one can show that the energy density diverges at the horizon for both types of solutions.

There is yet a third class of resonant solutions as a result of the numerical evolutions of regular initial data that has not such divergence of the energy density at the horizon,

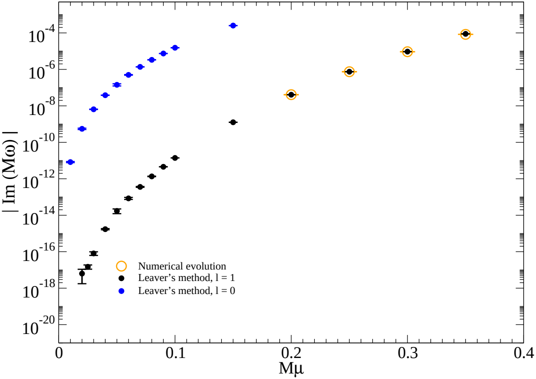
We call these solutions dynamical resonances. We found that the real part of the frequency of the quasi-resonant modes coincides with the frequency of oscillation of the stationary and dynamical resonances, and that the imaginary part coincides with the decay rate of the dynamical resonances. This is a nontrivial result since, after all, the dynamical solutions are in some sense "infinitely different" to both the stationary resonances and the quasi-resonances because they are regular solutions with finite energy.

For the KG Potential, the quasi-resonant modes can be obtained semi-analytically in several ways, the most common are the continued fraction method introduced by (Leaver:1985), and the WKB approach (Iyer:1986)

In the numerical evolutions we were not able to evolve the configurations for small values of $M\mu$, because of the propagation of the errors in the numerical approximation, and because the time required for such evolutions becomes prohibitive.

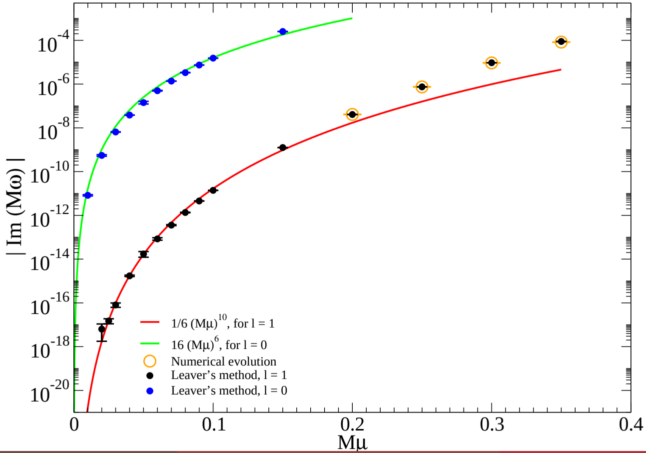


Leaver's method allows us to obtain results for parameter values that could not be reached with the numerical evolutions. The oscillations of the dynamical resonances have frequencies equal (up to numerical error) to the real part of the frequencies of the quasi-resonant modes, while the decay rates are equal to the imaginary part of such frequencies.



However numerical roundoff errors make it still prohibitive to obtain accurately very small values for the imaginary part of the quasi-resonant frequencies.

An analytic expression valid for small values of the combination $M\mu$ was obtained by Detweiler in (Detweiler:1980). He found analytic expressions for the spectrum of quasi-resonant modes in the limit $M\mu \ll 1$. For $\ell = 1$, the imaginary part of the frequency for the first quasi-resonant mode in this limit is given by $\mathrm{Im}(M\omega) = (M\mu)^{10}/6$.



We have calculated the quasi-resonant mode frequencies using Leaver's method up to the smallest values of $M\mu$ allowed by the roundoff errors, and we have found that the imaginary part of such frequencies matches the Detweiler approximations (within errors) at small values of $M\mu$. Hence, we can use Detweiler expressions to determine the decay rate of the dynamical resonances with very small values of $M\mu$, which cannot be reached with the other two methods.

But, is this a fine tunning solution?
In order to test the evolution of a variety of scalar field distributions, we construct a two-parameter family of initial data, of the form

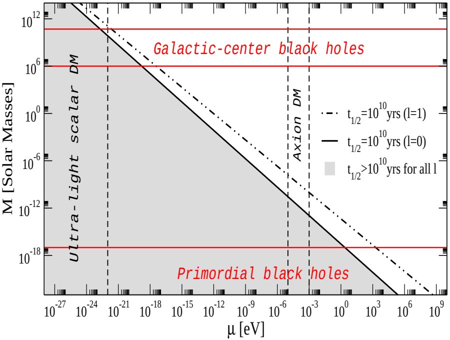
$$u_0(r) = \begin{cases} N(r-R_1)^4(r-R_2)^4 & \text{for } R_1 \le r \le R_2 \\ 0 & \text{otherwise} \end{cases} ,$$

During the initial stages of the evolution some SF accretes into the BH, while some is radiated away, both with rates that depend greatly on the initial data chosen, and can be very large in some cases. However, at late times, all evolutions show a similar steady behaviour with slow accretion into the BH. Similar results are obtained when studying the long-term evolution of a variety of configurations, some of them very different in size and spatial distribution.

Since the decay rate of the dynamical resonances is related to the imaginary part of the quasi-resonant frequencies, then its half-life time is inversely proportional to $\mathrm{Im}(M\omega)$. In particular we can find Black hole and scalar field masses such that $t_{1/2} \sim 10^{10}$ yrs.

There are two distinct regions of the parameter space of physical interest for which the configurations live longer than the age of the Universe:

- A scalar field mass smaller than 1 eV and black hole mass smaller than $10^{-17} M_\odot$, consistent with primordial black holes with an axion distribution (Sikivie:2009);
- An ultra-ligh scalar field with mass smaller than 10^{-22} eV (Hu:2000, Matos:2000) and a supermassive black holes with mass smaller than $5 \times 10^{10} M_\odot$, as could be the case for a dark matter halo surrounding a black hole at a galactic center (Arbey:2001).



The interaction of a Klein-Gordon field with a Schwarzschild black hole has been considered in the test field approximatiom. We have found the existence of long-lived scalar field configurations that, for values of $M\mu \lesssim 10^{-3}$, can survive in the vicinity of the BH for cosmological time scales.

Although we were unable to reach such small values for the combination $M\mu$ by means of numerical simulations we match our results with other methods to show that such configurations are possible.

Also our results seem to indicate that, at late times, even quite generic distributions evolve as a combination of the dynamical resonant modes, which can last for cosmological time-scales.