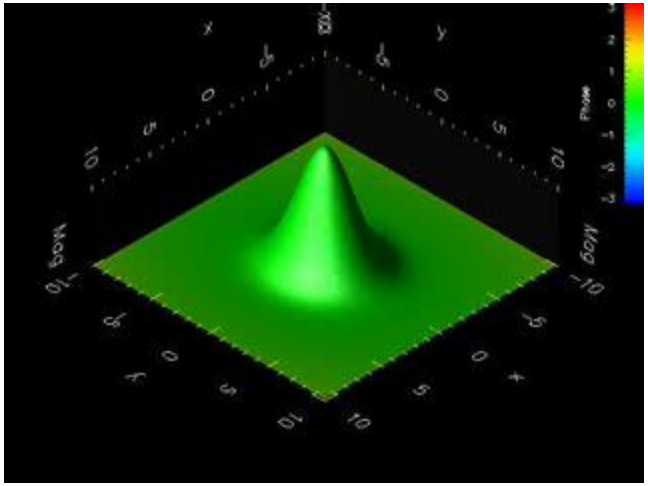
Deformation Method and Applications

Laercio Losano

Departamento de Física, Universidade Federal da Paraíba
Caixa Postal 5008, 58051-970 João Pessoa, Paraíba, Brasil
E-mail: losano@fisica.ufpb.br



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Outline

1. Introduction
2. Deformation Method
3. Extentions and Applications

- Deformed defects for scalar fields with polinomial interactions
- New family of sine-Gordon models
- Construction of topological defect networks with complex scalar fields
- Traveling wave soluitons of nonlinear partial differential equations

1. Introduction

- In standard Field Theory the model is defined by the choice of the field potential.

$$\mathcal{L} = \frac{1}{2}\partial_{\mu}\phi\partial^{\mu}\phi - V(\phi)$$

- The Deformation Method is a procedure that makes it possible to obtain new potentias and the static solutions, straightforwardly from one known potential and its static solutions.

$$V(\phi) \rightarrow \tilde{V}(\chi)$$

$$\phi(x) \rightarrow \chi(x)$$

- This procedure can be extended to obtain traveling wave solutions of systems described by like-KdV equations, or by high-order partial differential equations.

2. Deformation Method

[D. Bazeia, L. Losano, and J.M.C. Malbouisson, PRD66,101701(R) (2002)]

1) Let us consider a first system described by the real scalar field $\phi(x,t)$

lagrangean density

$$\mathcal{L} = \frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right)^2 - \frac{1}{2}\left(\frac{\partial\phi}{\partial x}\right)^2 - V(\phi)$$

potential $V(\phi)$, metrics (+,-) , adimensional field and coordenates.

We suppose that $V'(\bar{\phi}) = 0 \quad and \quad V(\bar{\phi}) = 0$

The equations of motion $\frac{\partial^2\phi}{\partial t^2} - \frac{\partial^2\phi}{\partial x^2} + \frac{dV}{d\phi} = 0$ for $\phi=\phi(x)$ $\frac{d^2\phi}{dx^2} = \frac{dV}{d\phi}$

with the boundary condictions

$$\phi(x \to -\infty) \to \bar{\phi}$$

$$\frac{d\phi}{dx}(x \to -\infty) \to 0$$

is solved by $\frac{d\phi}{dx} = \pm\sqrt{2V}$

2) Let us consider a second system described by the real scalar field, $\chi(x,t)$, whose lagrangian density is

$$\mathcal{L} = \frac{1}{2}\left(\frac{\partial\chi}{\partial t}\right)^2 - \frac{1}{2}\left(\frac{\partial\chi}{\partial x}\right)^2 - \tilde{V}(\chi)$$

static obey $\frac{d^2\chi}{dx^2} = \frac{d\tilde{V}}{d\chi}$ solved by (same b.c) $\frac{d\chi}{dx} = \pm\sqrt{2\tilde{V}}$

3) Now, doing $\phi = f(\chi)$ (deformation function invertible and differentiable)

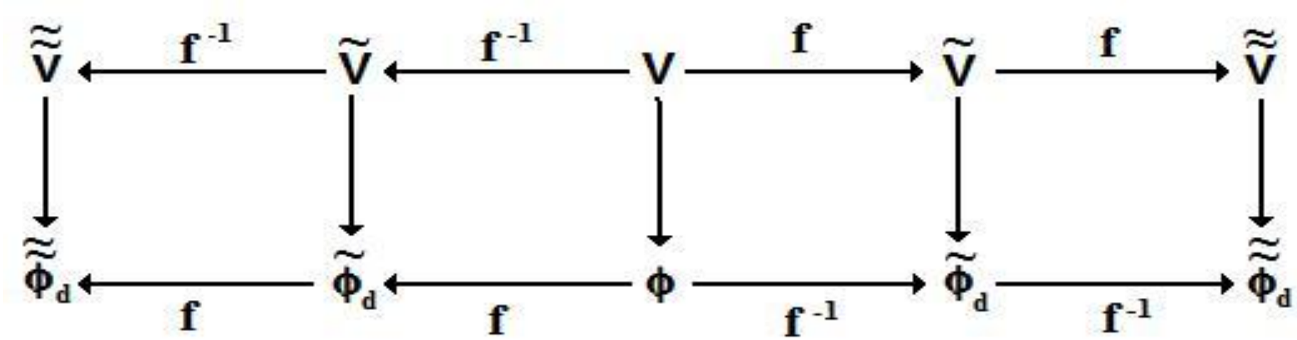
If we take in the 2th model

$$\tilde{V}(\chi) = \frac{V(\phi \to f(\chi))}{\left(\frac{df}{d\chi}\right)^2}$$

Its statics solutions are directly obtained by

$$\chi(x) = f^{-1}(\phi(x))$$

4) And doing again and again



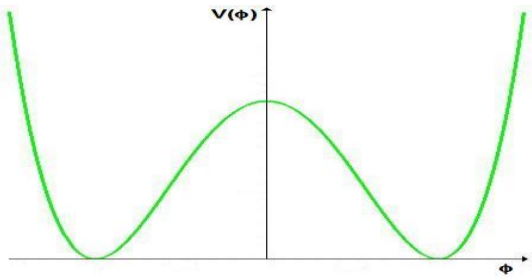
Example

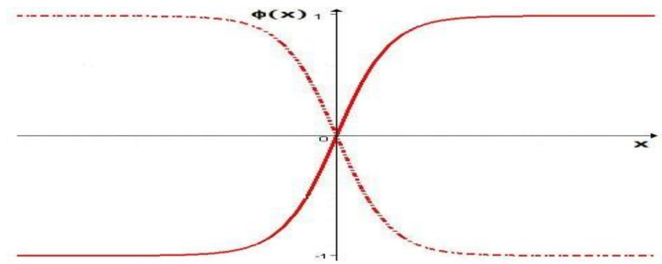
Starting from
(ϕ^4 model)

$$V(\phi)=\frac{1}{2}(1-\phi^2)^2$$

and

$$\phi(x)=\pm\tanh(x)$$





deforming with

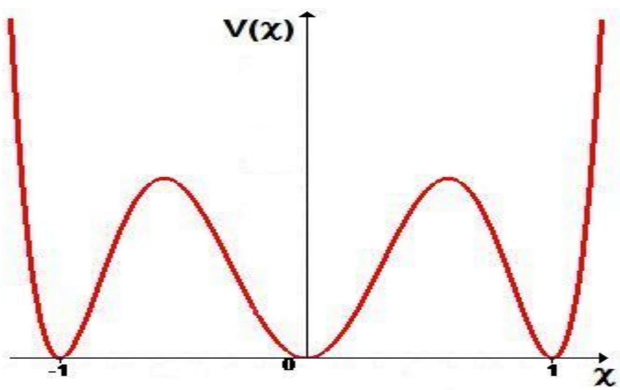
$$\phi=f(\chi)=2\chi^2-1$$

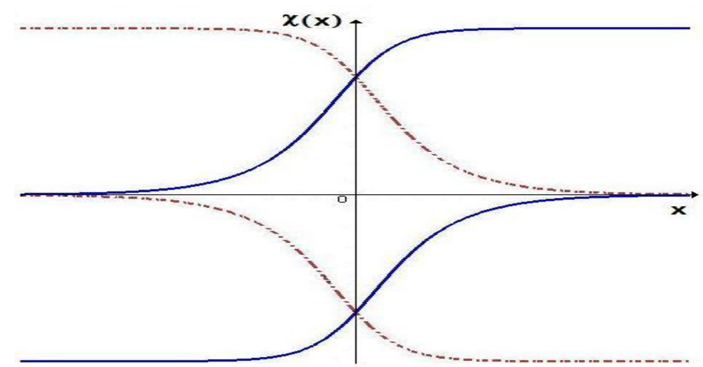
we obtain
(χ^6 model)

$$\tilde{V}(\chi)=\frac{V(\phi=f(\chi))}{(df/d\chi)^2}=\frac{1}{2}\chi^2(1-\chi^2)^2$$

and

$$\chi(x)=\pm\sqrt{\frac{(1\pm\tanh(x))}{2}}$$



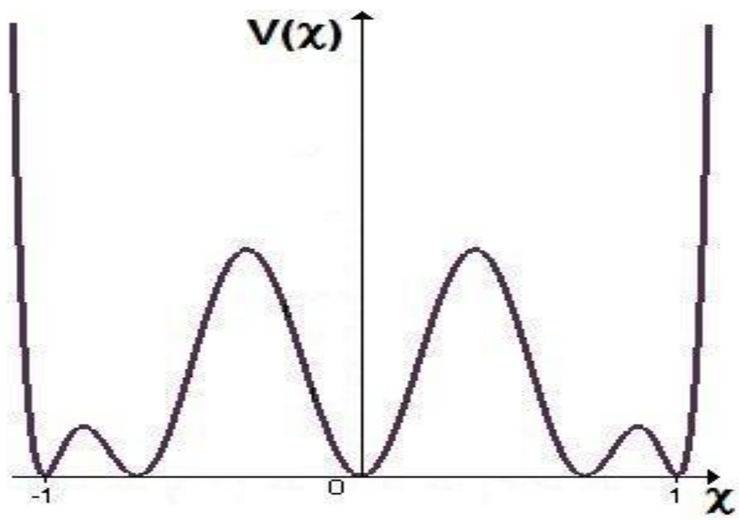


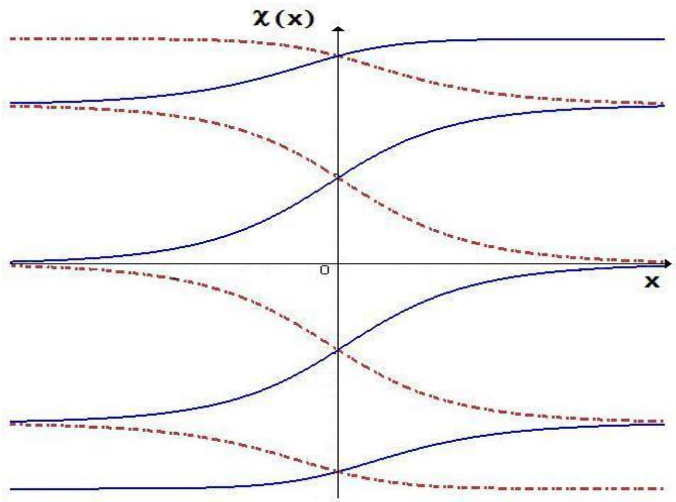
Example

starting from (ϕ^6 model) $V(\phi) = \frac{1}{2}\phi^2(1-\phi^2)^2$ and $\phi(x) = \pm\sqrt{\frac{[1 \pm \tanh(x)]}{2}}$

deforming again with $\phi = f(\chi) = 2\chi^2 - 1$

we obtain (χ^{10} model) $\tilde{V}(\chi) = \frac{1}{2}\chi^2(1-\chi^2)^2(1-2\chi^2)^2$ and $\chi(x) = \pm\sqrt{\frac{(1+\phi(x))}{2}}$





3. Extentions and Applications

1) "Deformed Defects for sacalar fields with Polinomial Interactions"

D. Bazeia, M. A. González León, L. Losano, and J. Mateos Guilarte, PRD73, 105008 (2006)

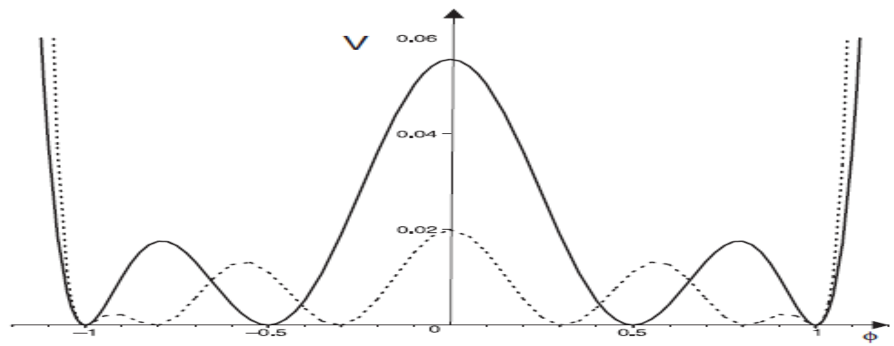
-The model χ^4 : $U(\chi) = \frac{1}{2}(1-\chi^2)^2$ is deformed by $f(\phi) = \cos(a \arccos(\phi) - m\pi)$

[two families of polinomial potential, degree is determinated by the parameter a]

sine family – a odd

$$V^a_{\sin}(\phi) = \frac{1}{2a^2} \prod_{j=1}^{(a+1)/2} \left(1 - \frac{\phi^2}{Z_j^{a\,2}}\right)^2$$

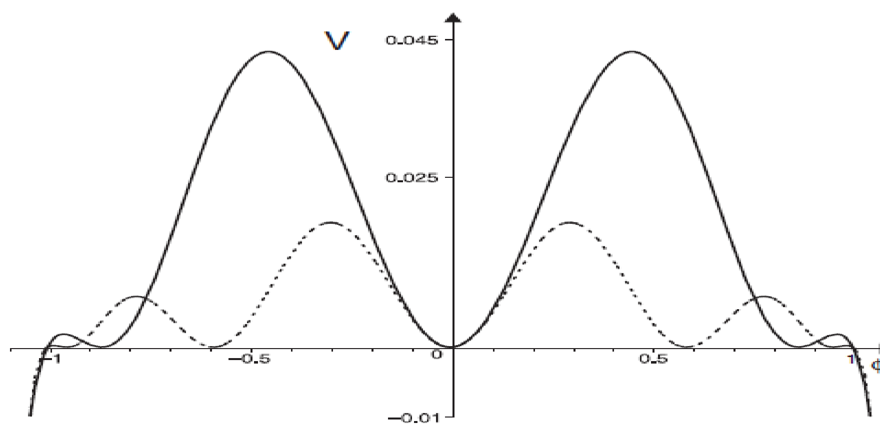
$$V^3_{\sin}(\phi) = \frac{8}{9}\left(\frac{1}{4} - \phi^2\right)^2 (1 - \phi^2)^2$$



cosine family – a odd

$$V^a_{\cos}(\phi) = \frac{1}{2}\,\phi^2(1-\phi^2) \prod_{j=1}^{(a-1)/2} \left(1 - \frac{\phi^2}{Z_j^{a\,2}}\right)^2$$

$$V^3_{\cos}(\phi) = \frac{8}{9}\,\phi^2\left(\frac{3}{4} - \phi^2\right)^2 (1 - \phi^2)$$



2) "New family of sine-Gordon models",

D. Bazeia, L. Losano, R. Menezes, and M.A.M. Souza,
EPL 87, 2101 (2009)

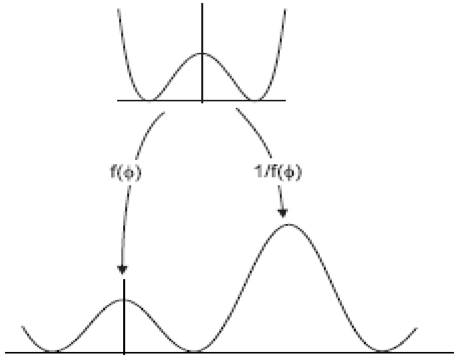
The model χ^4 : $U(\chi)=\frac{1}{2}(1-\chi^2)^2$ deformed by $\chi=f(\phi)$ or $\chi=1/f(\phi)$ give the same $V(\phi)=\frac{1}{2}(1-f^2)^2/(f')^2$

(i) deforming by

$$f_1(\phi)=f=r\tan(\phi)$$
$$f_2(\phi)=1/f=1/r\cot(\phi)$$

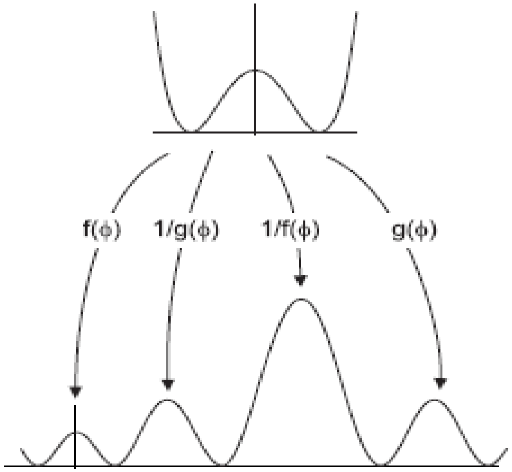
give double sine-Gordon

$$V(\phi)=\frac{1}{2r^2}\left((1+r^2)\cos^2(\phi)-r^2\right)^2$$



(ii) deforming by

$$f_1(\phi)=f=\tan(s\arctan(r\tan(\phi)))$$
$$f_2(\phi)=1/f=\cot(s\arctan(r\tan(\phi)))$$
$$f_3(\phi)=g=\tan(s\arctan(1/r\cot(\phi)))$$
$$f_4(\phi)=1/g=\cot(s\arctan(1/r\cot(\phi)))$$



-for s= 1,2,3,...., give double-sG, triple-sG, quadruple-SG,....

3) "Construction of topological defect networks with complex scalar fields "

V.I. Afonso, D. Bazeia, M.A. Gonzalez Leon, L. Losano, and J. Mateos Guilarte, NPB 810, 427(2009), PLB 662, 75(2008).

Initial model

$$\chi(x,t) = \chi_1(x,t) + i\chi_2(x,t)$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu \chi \partial^\mu \bar{\chi} - V(\chi,\bar{\chi})$$

where

$$V(\chi,\bar{\chi}) = \frac{1}{2} W'(\chi)\overline{W'(\chi)}.$$

Deformed model

$$\mathcal{L}_D = \frac{1}{2}\partial_\mu \phi \partial^\mu \bar{\phi} - \frac{V(f(\phi),\overline{f(\phi)})}{f'(\phi)\overline{f'(\phi)}}$$

where

$$\phi(x,t) = \phi_1(x,t) + i\phi_2(x,t)$$

related to Inicial model by $\chi = f(\phi) = f_1(\phi_1,\phi_2) + i f_2(\phi_1,\phi_2)$

then for $\mathcal{V}(\phi,\bar{\phi}) = \frac{V(f(\phi),\overline{f(\phi)})}{|f'(\phi)|^2} = \frac{1}{2}\frac{W'(f(\phi))}{\overline{f'(\phi)}}\frac{\overline{W'(f(\phi))}}{f'(\phi)} = \frac{1}{2}\mathcal{W}'(\phi)\overline{\mathcal{W}'(\phi)}$

the kink BPS soluiton is $\phi^K(x) = f^{-1}(\chi^K(x))$

Deformation of symmetric Wess–Zumino models

$$W(\chi) = \chi(x,t) - \frac{\chi^{N+1}(x,t)}{N+1}$$

taking $f(\phi) = \mathcal{W}(\phi)$ constrains $f(\phi)$ to obey

$$f'(\phi)\overline{f'(\phi)} = \sqrt{2V(f(\phi), \overline{f(\phi)})}$$

In the $N=3$ case, we have

$$W(\chi) = \chi - \frac{1}{4}\chi^4, \qquad V(\chi,\bar{\chi}) = \frac{1}{2}(1-\chi^3)(1-\bar{\chi}^3)$$

separated eqs. of constrain

$$f'(\phi)^2 = f(\phi)^3 - 1$$

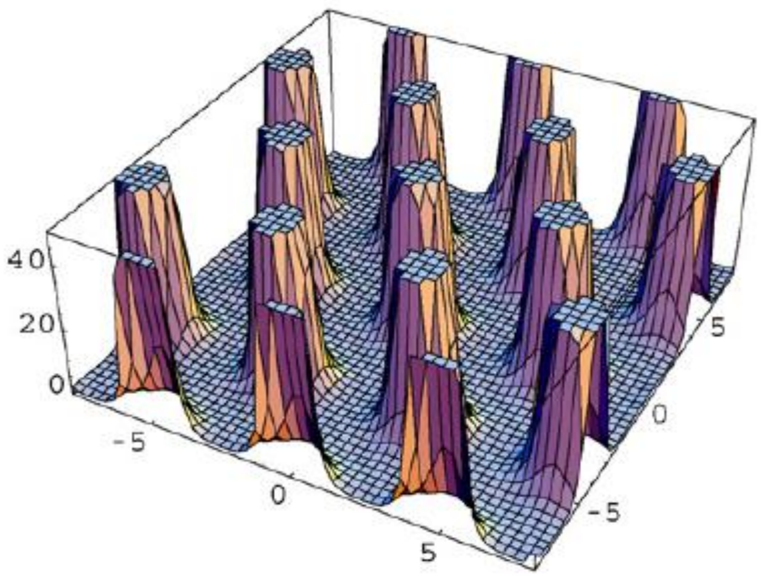
$$\overline{f'(\phi)^2} = \overline{f(\phi)^3} - 1$$

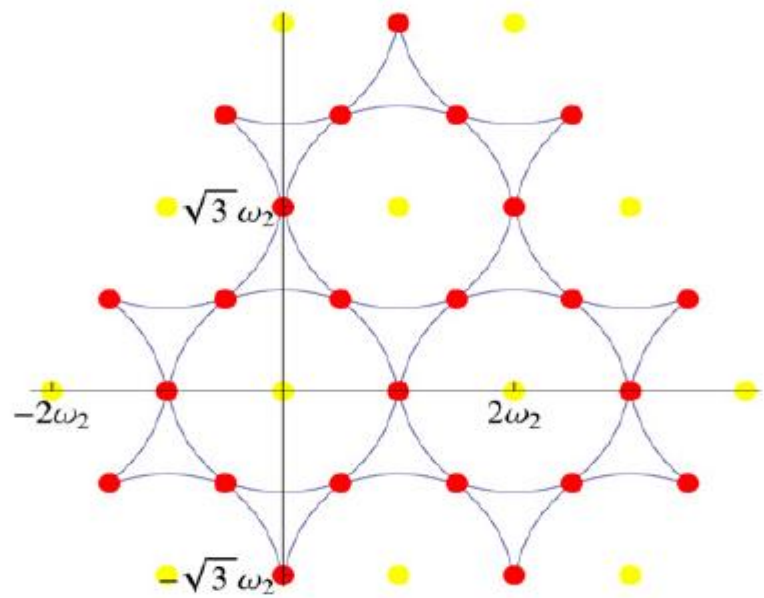
give holomorphic solution

$$\mathcal{W}(\phi) = f(\phi) = 4^{\frac{1}{3}}\mathcal{P}(4^{-\frac{1}{3}}\phi; 0, 1)$$

(Weierstrass P function)

deformed potential : $\mathcal{V}(\phi,\bar{\phi}) = \frac{1}{2}\mathcal{P}'_{01}(4^{-\frac{1}{3}}\phi)\overline{\mathcal{P}'_{01}}(4^{-\frac{1}{3}}\phi)$





4) "Traveling wave soluitons of nonlinear partial differential equations",
D. Bazeia, Ashok Das, L. Losano, and M.J. Santos,
App. Math. Lett. 23, 681 (2010)

We propose a simple algebraic method for generating classes of traveling wave solutions for a variety of partial differential equations

g-Rosenau equation

$$v_t + (P(v))_x + (Q(v)v_{xx})_x = 0$$

trial wave

$$u(x,t) = \cos(kx - \omega t)$$

map

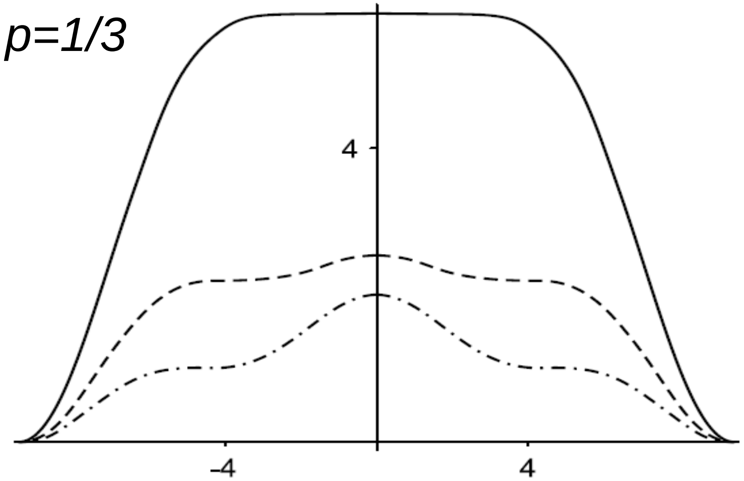
$$u = a + v^p,$$

solutions

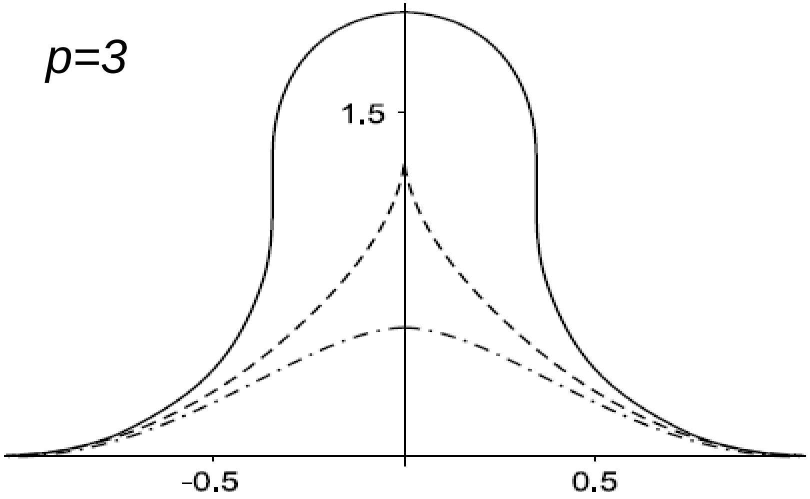
$$v = \begin{cases} (\cos p(x-\alpha_1 t) - a)^{\frac{1}{p}}, & \text{for } |p(x-\alpha_1 t)| \le \pi \\ -(1+a)^{\frac{1}{p}}, & \text{otherwise,} \end{cases}$$

- compactons(no tail)
- doble compactons
- peakon compactons

p=1/3



p=3



Thank you!