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Daniele Malafarina

From Black Holes to Naked Singularities in Gravitational Collapse

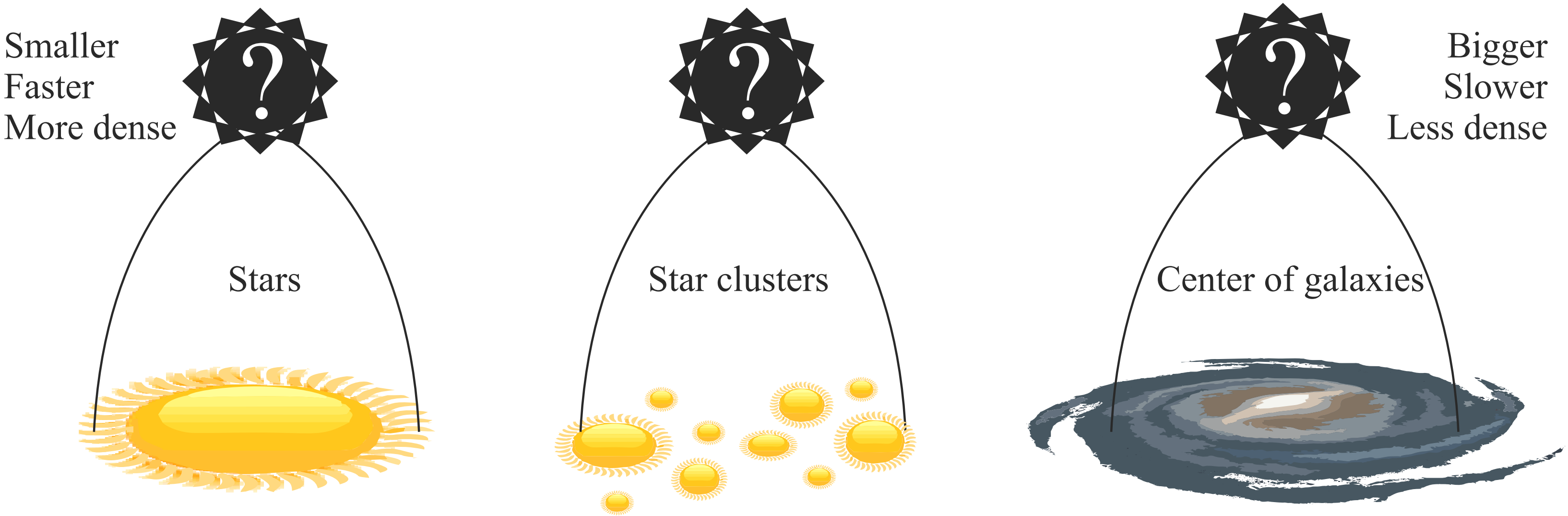
work done in collaboration with
Pankaj S. Joshi (Tata Institute of Fundamental Research, Mumbai, India)

Abstract

We give a brief overview of naked singularities, how these can form from gravitational collapse and their possible relevance in astrophysics. We investigate how the black hole formation process described by the Oppenheimer-Snyder scenario is affected once small pressures are introduced in the collapsing matter cloud. We show that the presence of pressures and inhomogeneities, however small, can change drastically the structure of the apparent horizon and the final fate of collapse.

Gravitational Collapse

- It can describe the final stages of the life of a star and other processes that happen at bigger scales like those leading to the formation of supermassive black holes.
- The true nature of the compact objects that result from complete gravitational collapse remains a mystery.
- These processes involve a broad range of sizes, time scales and densities.



Some Open Issues in Collapse

Is a black hole necessarily the only possible endstate of complete collapse of a massive body? If not, what other possibilities are there?

- Naked singularities?
- Compact objects composed of some exotic matter?
- Compact objects composed of ordinary matter?

If naked singularities exist in the universe then interesting astrophysical questions are:

- How do they form?
- What causes a collapse to end in a black hole rather than a naked singularity?
- What observational features do they have? Can we detect them?
- Is there any way to detect and distinguish them from black holes?

Cosmic Censorship Conjecture

Any singularity that arises as the endstate of physically realistic processes must be hidden within an horizon.

We know that singularities are a general feature of exact solutions of Einstein's field equation.

If the hypothesis is true:
What is the mechanism by which all singular solutions of the field equations become unphysical? *(a proof is missing)*

If the hypothesis is not true:
What are the naked singularities that might arise from physical processes?
(there exist counterexamples to CCC)

What are naked singularities?
What kind of effects can they have on the outside universe?

Open Problems within Collapse Models

- How generic is the occurrence of naked singularities?
- How stable are these models under perturbation of the initial data?
- What role do pressures, rotation and deformations play in collapse?
- What type of matter fields can we consider?
- What is the nature of compact objects being observed in astrophysics?
- Can the collapse of a massive star lead to the formation of a naked singularity?
- Is there any mechanism by which naked singularity formation can be avoided?
- If naked singularities do exist, what kind on observational signature do they have?
- Can effects occurring near the singularity reach outside observers?
- In which cases would a naked singularity be distinguishable from a black hole?
- Could these models be used to explain observed phenomena in the universe?

Black Holes

Theory:

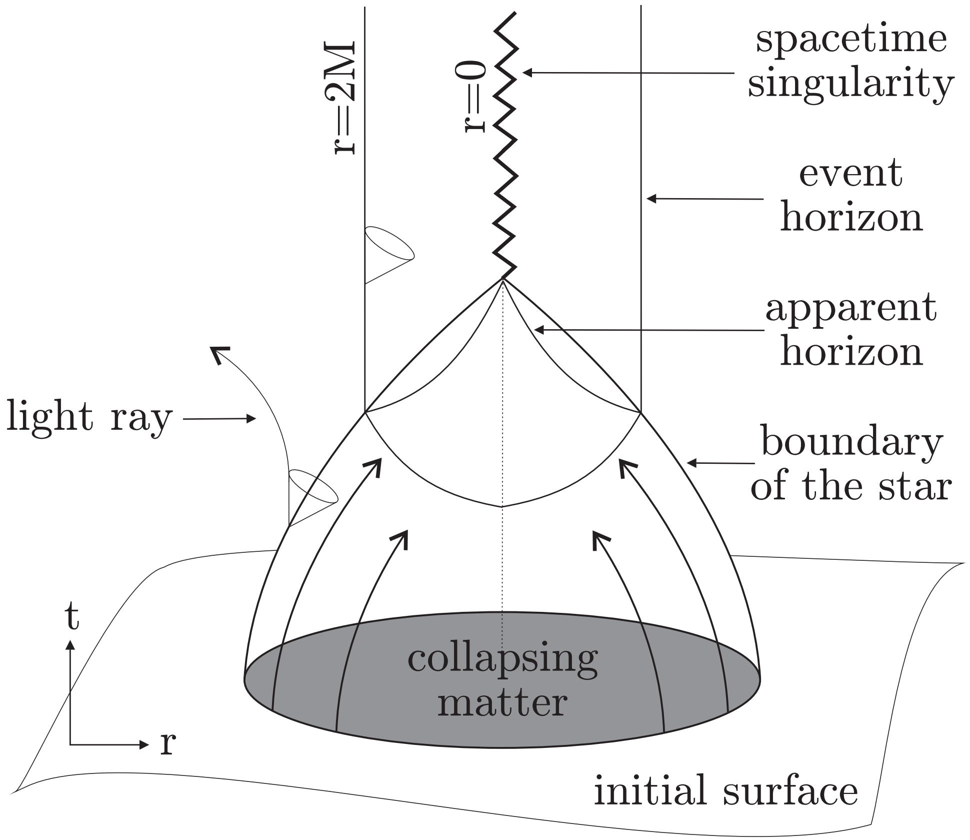
- A black hole in General Relativity is a singularity of the spacetime covered by an event horizon.
- In collapse a black hole results when the infalling matter is trapped by the formation of the horizon before the singularity forms.
- The first theoretical model to be studied was that of a spherical cloud of homogeneous dust.
- More realistic models are needed.

Observation:

- Over the past decades we gained more and more evidence that very compact objects such as black holes exist in the universe.
- We know of black holes as endstate of the life of a star and supermassive black holes that dwell at the center of galaxies.
- How do these ultracompact objects form?

Black Hole Formation

- The matter cloud collapses under its own gravity.
- The trapped surfaces develop before the singularity.
- The singularity curve remains always hidden within the horizon.
- No light ray can escape the singularity to reach far away observers.



Naked Singularities

If collapse doesn't halt then a singularity must be produced but black holes are not the only final product of collapse predicted by General Relativity.

Theory:

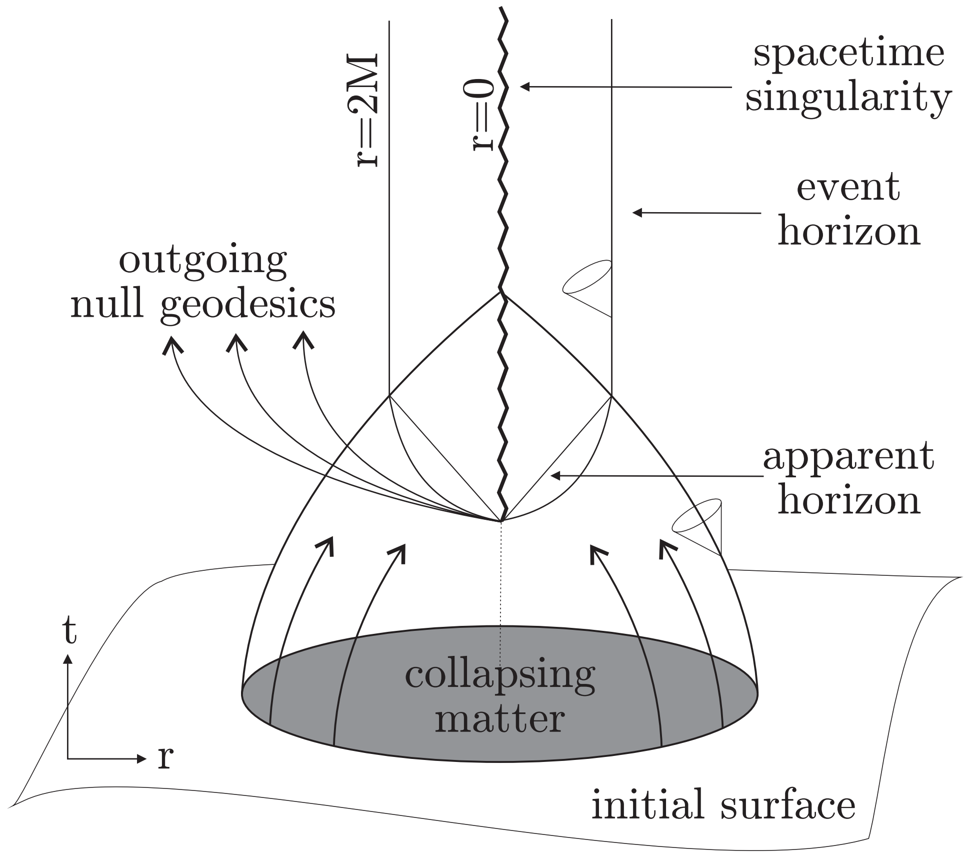
- A naked singularity is a singularity not covered by an horizon.
- In collapse a naked singularity results when the infalling matter becomes singular before the formation of the horizon.
- The first theoretical examples to be found in collapse models were those of a spherical inhomogeneous dust ball.
- Naked singularities are found in many solutions of Einstein equations. The question is if they can form from physically realistic processes.

Observation:

- Very compact sources in the universe could be naked singularities.
- Naked singularities could be at the core of exploding supernovae.
- If they form how do we detect them?

Naked Singularity Formation

- The matter cloud collapses under its own gravity.
- The singularity forms earlier than the trapped surfaces.
- Some light ray can escape the singularity to reach far away observers.
- The horizon develops at a later stage and covers the singularity.



Naked Singularities in Stationary Spacetimes

A naked singularity in astrophysics might be seen as a region of very strong gravitational field and high curvature not covered by an event horizon, where quantum-gravity effects might be visible.

If naked singularities exist they might have observational features very different from black holes. These are currently being investigated in static and stationary geometries.

Gravitational lensing by naked singularities.
(*G.F.R. Ellis, K.S. Virbhadra*).

Properties of accretion disks around a naked singularity.
(*Z. Kovack and T. Harko*).

Shadows cast by Kerr naked singularities.
(*C. Bambi*).

Motion of test particles around Kerr naked singularities.
(*D. Pugliese, H. Quevedo and R. Ruffini*).

Particle acceleration by naked singularities.
(*P.S. Joshi and M. Patil*).

Naked Singularities in Collapse

There are a lot of models describing naked singularities arising from gravitational collapse.

Naked singularities arising in the LTB dust collapse.
(*Christodoulou, ...*).

Naked singularity formation in self-similar gravitational collapse.
(*Ori and Piran, Carr, ...*).

Naked singularities arising in collapse with tangential stresses.
(*Magli, Nakao, ...*).

Naked singularities arising in perfect fluid collapse.
(*Joshi and Goswami, Giambò, Harada, ...*).

Naked singularities in collapse of massless scalar fields.
(*Joshi, ...*).

The choice of different matter sources does not rule out the possibility that naked singularities form from collapse with regular initial data.

History of Naked Singularities

1933 Spherically symmetric dynamical dust solution of Einstein's field equations (*Lemaitre and Tolman*).

1939 Homogeneous dust collapse model always ends in a black hole (*Oppenheimer and Snyder*).

1969 Cosmic Censorship Hypothesis (*Penrose*).

1984 Naked singularities arising in the LTB dust collapse (*Christodoulou, Joshi and Dwivedi*).

1987 Naked singularity formation in self-similar collapse (*Ori and Piran*).

Can they be avoided by the presence of pressures?

'97-'99 Examples of n.s. arising in collapse with tangential stresses (*Magli, Piran*).

'00-... Examples of n.s. arising in more general models (*Joshi and Goswami*).

How stable and generic are these examples?

What is a Naked Singularity?

What we mean by naked singularity?

- A naked singularity for us is a region of ultrahigh densities, where quantum-gravity effect may occur, that is not covered within the horizon.
- Divergence of physical quantities and curvature invariants indicates a breakdown of the model.

Fluid Matter Models

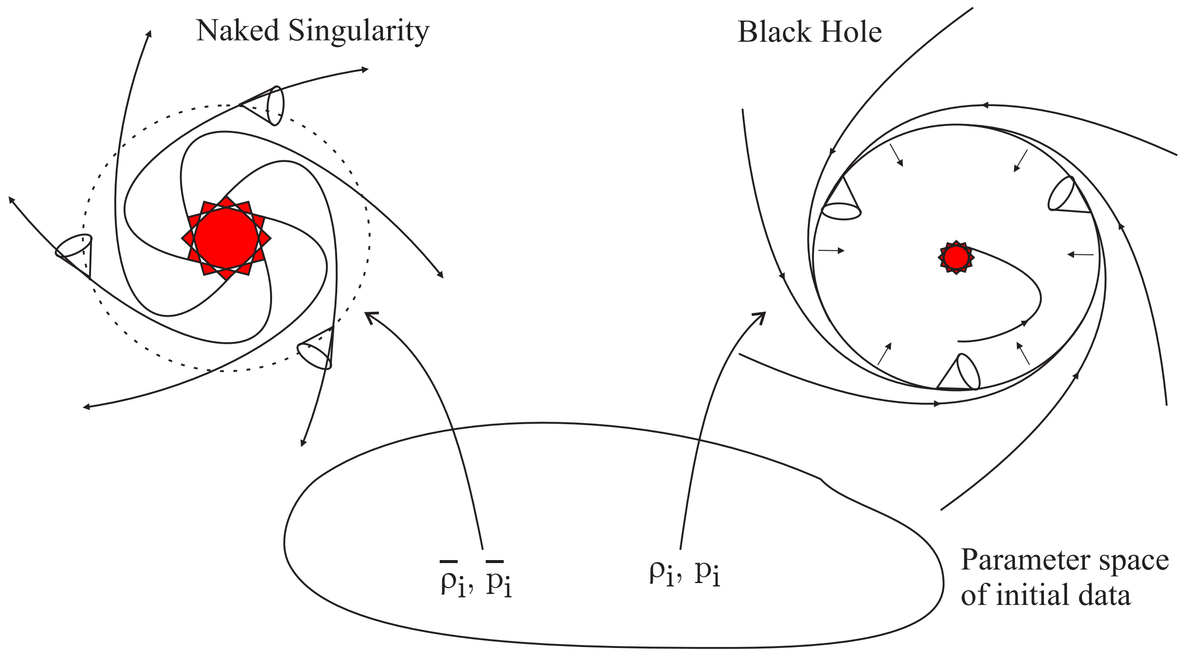
How good are fluid matter models like dust or perfect fluid?

- The fluid approximation for matter has a limit after which microscopic effects must be considered. Does this happen before or after the quantum-gravity regime is achieved?
- It has been suggested that close to the singularity matter must behave like dust.
- Can other type of matter models (like scalar fields or Eistein-Vlasov systems) resolve the singularity?

The Problem of Genericity

Given the existence of solutions with naked singularities:

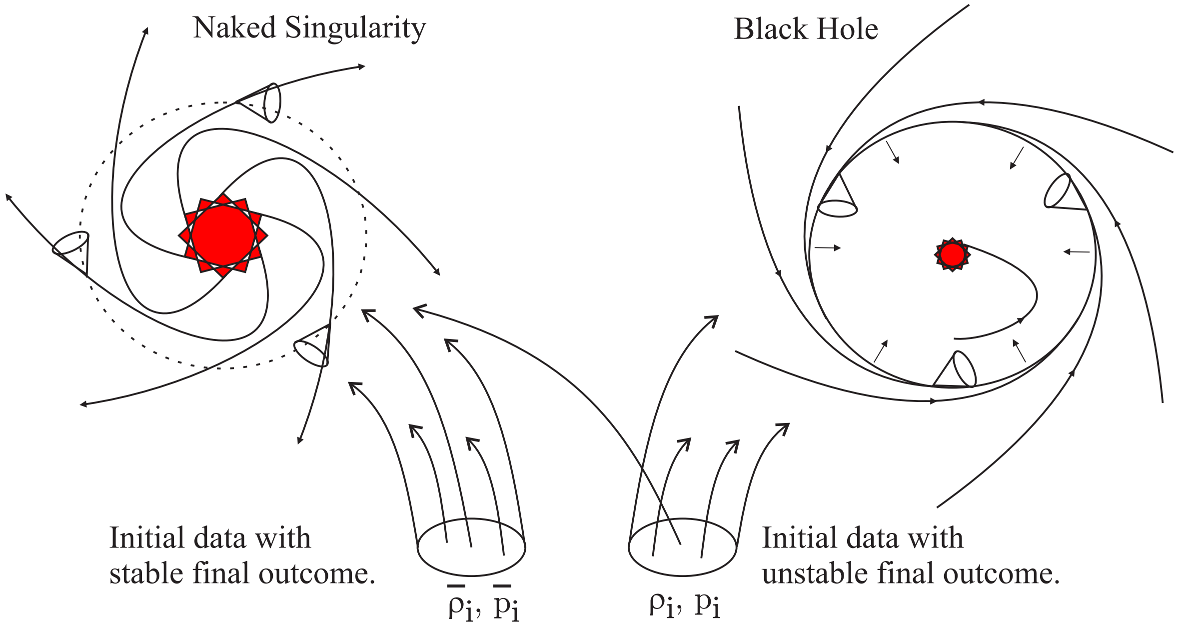
- How 'big' is the set of initial data leading to naked singularities? (as compared to the corresponding set for black holes)
- In which space of parameters is this set measured?



Stability of the Final Outcome

Given a certain matter configuration leading to a BH or a NS:

- What are the parameters that determine the final outcome?
- How stable is it with respect to small changes in the initial data?



Space-time

Spherically symmetric interior describing a collapsing matter cloud in comoving coordinates

$$ds^2 = -e^{2\nu(t,r)}dt^2 + \frac{R'}{G(t,r)}dr^2 + R(t,r)^2 d\Omega^2$$

depends on 3 functions ν, G, R and matches smoothly across the boundary to a known exterior (generalized Vaidya).

Matter

The energy-momentum tensor is composed by

- Energy density: $\rho = -T_t^t$
- Radial pressure: $p_r = T_r^r$
- Tangential pressure: $p_\theta = T_\theta^\theta = T_\phi^\phi$

The metric components are related to the energy-momentum tensor via Einstein's field equations.

Einstein Equations

$$p_r = -\frac{\dot{F}}{R^2\dot{R}}$$
$$\rho = \frac{F'}{R^2R'}$$
$$\dot{G} = 2\nu'\frac{\dot{R}}{R'}G$$
$$\nu' = \frac{2(p_\theta - p_r)}{\rho + p_r}\frac{R'}{R} - \frac{p_r'}{\rho + p_r}$$

where F is the Misner-Sharp mass of the system describing the amount of matter enclosed in the shell r at any given time t and it is given by

$$F = R(1 - G + e^{-2\nu(r,v)}\dot{R}^2)$$

The first equation implies that for dust ($p_r = p_\theta = 0$) and tangential pressures ($p_r = 0$) we must have $F = F(r)$.

Scaling Factor

There is the gauge freedom to fix the scaling factor in the area function $R(r, t)$:

$$R = rv(r, t)$$

with

- initial time t_i: $v(r, t_i) = 1$
- singularity time t_s: $v(r, t_s) = 0$
- collapse: $\dot{v} < 0$

Due to its monotonic behaviour we can use v as a time coordinate:

$$(r, t) \mapsto (r, v)$$

The density diverges along the central shell at $R = 0$ at all times.
With the choice of the scaling factor it diverges only at the singularity.

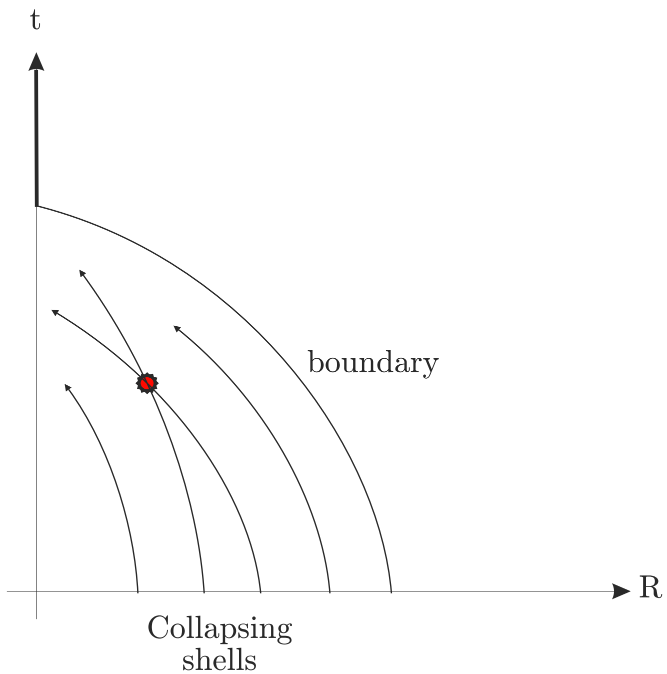
Shell Crossing

The density diverges also at $R' = 0$.

- Shell crossing singularities are weak curvature singularities.
- They are due to a breakdown of the coordinate system.
- Avoidance of shell crossing singularities implies

$$R' = v + rv' > 0$$

- This is always satisfied close to the center.



Velocity Profile

We can integrate the equation for $\dot{G}$ and obtain

$$G = b(r)e^{2A(r,v)}$$

where A is defined by

$$A(r,v)_{,v} := \frac{r\nu'}{R'}$$

and $b(r)$ is a free function called the velocity profile. The function $b(r)$ is related to the velocity or the kinetic energy of the infalling particles.
If we define

$$b(r) = (1 + r^2 b_0(r))$$

We can divide the evolutions in three possible subcases:

- Unbound model $b_0 > 0$
- Marginally bound model $b_0 = 0$
- Bound model $b_0 < 0$

Perfect Fluids

$$p_r = p_\theta$$

The assumption is justified by the fact that near the center matter must behave like perfect fluid. Perfect fluids are widely used in astrophysics.

There is one free function, $F(r, v)$.

The system becomes closed once an equation of state is provided.

$$\nu' = -\frac{p'}{\rho + p}$$

Tangential Pressures

$$p_r = 0$$

Can represent a cloud where particles move on circular orbits. The structure of equations is simpler since $F = F(r)$.

There are two free functions $F(r)$ and $p_\theta(r, v)$.

A linear equation of state is not allowed since it gives a singular spacetime.

$$\nu' = 2\frac{p_\theta}{\rho}\frac{R'}{R}$$

Initial Data

The free functions ($F(r,v)$ for perfect fluid and $F(r)$, $p_\theta(r,v)$ for tangential pressures) must be specified globally.

Through the field equations we can evaluate the evolution of the collapsing cloud once the initial configuration is provided.

$$
\begin{aligned}
\rho_i(r) &= \rho(r, t_i) \\
p_i(r) &= p(r, t_i) \\
R_i(r) &= R(r, t_i) \\
\nu_i(r) &= \nu(r, t_i) \\
G_i(r) &= G(r, t_i)
\end{aligned}
$$

where

- The free functions at t_i may be chosen arbitrarily.
- The choice of v implies $R_i = r$.
- Einstein equations give the relations between the remaining functions.

Energy Conditions

Realistic matter must obey the weak energy conditions:

$$\rho \geq 0 \text{ and } \rho + p \geq 0$$

- The first one implies $F \geq 0$ near the center.
- The second one is satisfied near the center whenever $|p| \ll \rho$.

Negative pressures that satisfy weak energy conditions are possible.

Regularity Conditions

Regularity near the center of the cloud imposes some conditions:

- regularity of the mass implies $F = r^3 M(r, v)$
- regularity of the pressure implies $\nu = r^2 g(r, t)$
- regularity of the density implies $M'(0, v) = 0$
- regularity of the velocity profile implies $b(r) = 1 + r^2 b_0(r)$

Trapped Surfaces

The apparent horizon is the surface that separates light rays directed outwards that are outgoing from those directed outwards that are ingoing. In vacuum it coincides with the event horizon.

$$1 - \frac{F}{R} = 0$$

The apparent horizon curve is given by

$$v_{ah}(r) = r^2 M(r, v_{ah})$$

The timelike region is given by $\frac{F}{R} < 1$.

- If $M = M(r)$ then only the central singularity can be visible.
- If $M(r, v) = M_0$ at $r = 0$ then only the central singularity can be visible.
- If $\frac{M(r,v)}{v} \underset{v \to 0}{\longrightarrow} \mathrm{c} < \frac{1}{\bar{r}^2}$ then the singularity at $\bar{r}$ can be visible.

Solving Einstein Equations

Consider the case of perfect fluid collapse:

$$F = r^3 M(r, v)$$

we get

$$\begin{cases} \rho = \frac{3M + rM'}{v^2 R'} \\ p = -\frac{M_{,v}}{v^2} \end{cases}$$

Equation for ν implies

$$A(r, v) = \int_v^1 r \frac{M_{,vr} v + (M_{,vv} v - 2M_{,v})v'}{(3M + rM_{,r} - M_{,v} v)v} dv$$

From the Misner-Sharp mass equation we get the equation of motion for the collapsing matter shells:

$$\dot{R} = -e^{\nu} \sqrt{\frac{F}{R} + G - 1}$$

Integrating this equation solves the set of Einstein equations.

Collapse Evolution and Final Fate

The equation of motion can be rewritten as

$$\dot{v} = -e^{\nu(r,v)}\sqrt{\frac{M(r,v)}{v} + \frac{b(r)e^{2A(r,v)} - 1}{r^2}}$$

Inverting the equation of motion and solving for $t(r,v)$ we get the time at which the shell r reaches the 'event' v:

$$t(r,v) = \int_v^1 \frac{e^{-\nu}}{\sqrt{\frac{M}{\bar{v}} + \frac{be^{2A} - 1}{r^2}}} d\bar{v}$$

The time curve of the singularity becomes:

$$t_s(r) = t(r,0)$$

Since $t(r,v)$ is in general $\mathcal{C}^2$ we can expand it near the center:

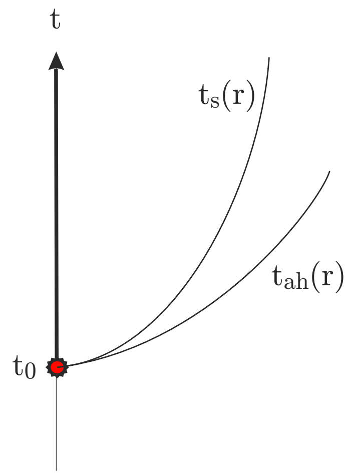
$$t(r,v) = t(0,v) + r\chi_1(v) + r^2\chi_2(v) + o(r^3)$$

Singularity and Apparent Horizon

Performing the expansion of $t_s(r)$ near the center we get

$$t_s(r) = t_0 + \chi_1(0)r + \chi_2(0)r^2 + o(r^3)$$

- Nakedness of the singularity will be decided by the sign of the first non vanishing $\chi_i(0)$
- If the first non vanishing $\chi_i(0)$ is positive then $t_{ah} \geq t_0$.
- The apparent horizon is future directed from the central singularity outwards.



The apparent horizon curve is not necessarily null and future directed.

The time curve of the apparent horizon

$$t_{ah}(r) = t_s(r) - \int_0^{v_{ah}} \frac{e^{-\nu}}{\sqrt{\frac{M}{v} + \frac{be^{2A}-1}{r^2}}} dv$$

Outgoing Geodesics

The singularity forms at the time

$$t_0 = t(0,0)$$

Outer shells are trapped at $t_{ah}(r) > t_0$, leaving the possibility for null geodesics originated at t_0 to escape.

Future directed outgoing radial null geodesics are given by

$$dt = +\frac{R' e^{-\nu}}{\sqrt{G}} dr$$

If outgoing null geodesics terminate at the singularity in the past with a definite tangent and $t_{ng}(r) < t_{ah}(r)$ for $r \to 0$ then the singularity is locally visible.

$$t_{ng}(r) = t_0 + x_0 u$$

with $u = r^{\alpha}$ (take $\alpha = 5/3$). Then

$$x_0 = \lim_{t \to t_s} \lim_{r \to 0} \frac{R}{u} = \left(\frac{3}{2} \sqrt{M_0} \chi_1(0) \right)^{\frac{3}{2}}$$

$$\boldsymbol{\chi_1 > 0} \Rightarrow \textit{the singularity at } \mathbf{r = 0} \textit{ is locally naked.}$$

Same reasoning applies to χ_2 if $\chi_1 = 0$

Black Hole vs. Naked Singularity

In the general case

$$\chi_1(0) = -\frac{1}{2}\int_0^1 \frac{\frac{M'(0,v)}{v} + b_0'(0) + 2a'(0,v)}{\left(\frac{M(0,v)}{v} + b_0(0) + 2a(0,v)\right)^{\frac{3}{2}}} dv$$

where $a(r, v)$ is given by $A(r, v) = r^2 a(r, v)$. The final outcome in terms of black hole or naked singularity is governed by:

- Mass profile $M(r, v)$.
- Velocity profile $b_0(r)$.
- Pressure profile, related to $a(r, v)$.

Black Hole vs. Naked Singularity II

Typically we choose pressure profiles with only quadratic terms:

$$p = p_0 + p_2 r^2 + \dots$$

This implies that $\chi_1(0) = 0$. Therefore visibility of the singularity is decided by the next non vanishing term:

$$\chi_2(0) \ = \ \int_0^1 \Bigg[\frac{3}{8} \frac{(M'(0,v) + 2a'(0,v) + b_0'(0))^2}{\left(\frac{M(0,v)}{v} + b_0(0) + 2a(0,v)\right)^{\frac{5}{2}}} - \frac{1}{2} \frac{\nu''(0,v)}{\sqrt{\frac{M(0,v)}{v} + b_0(0) + 2a(0,v)}} + \\ - \frac{1}{2} \frac{\frac{M''(0,v)}{v} + 2a''(0,v) + 2a(0,v)^2 + 2b_0(0)a(0,v) + b_0''(0,v)}{\left(\frac{M(0,v)}{v} + b_0(0) + 2a(0,v)\right)^{\frac{3}{2}}} \Bigg] dv$$

Genericity

For any mass function M and velocity profile b that must be specified globally we have an initial data sets for collapse near the center leading to black holes ($\chi_j < 0$) and naked singularities ($\chi_j > 0$).

$$I = I_{NS} \cup I_{BH}$$

with

$$I_{NS} := \{(M_j, b_{0j}, p_j) \text{ s.t. } \chi_j > 0\} \text{ and } I_{BH} := \{(M_j, b_{0j}, p_j) \text{ s.t. } \chi_j < 0\}$$

- M_j initial mass profile
- b_{0j} initial velocity profile
- p_j initial pressure profile

The critical surface is given by $\chi_j = 0$.
For points on the critical surface consider the next order χ_{j+1}.
We say that a collapse to a naked singularity (black hole) is generic if I_{NS} (I_{BH}) is an open subset of I with non-zero measure.

Stability

Given $x = (M_j, b_{0j}, p_j)$, $x \in I_{NS}$ we say that the naked singularity is stable if

$$\exists U(x) \;\text{ s.t. } \bar{x} \in I_{NS} \;\forall\; \bar{x} \in U(x)$$

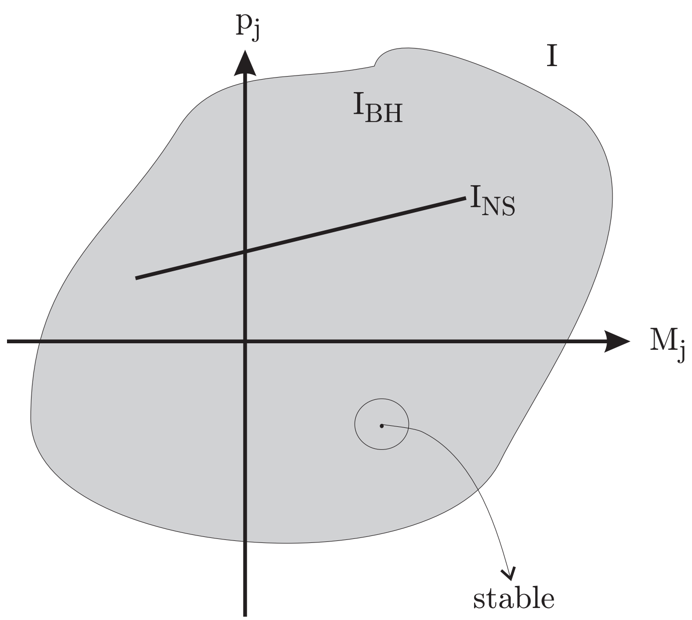
Given $x = (M_j, b_{0j}, p_j)$, $x \in I_{BH}$ we say that the black hole is stable if

$$\exists U(x) \;\text{ s.t. } \bar{x} \in I_{BH} \;\forall\; \bar{x} \in U(x)$$

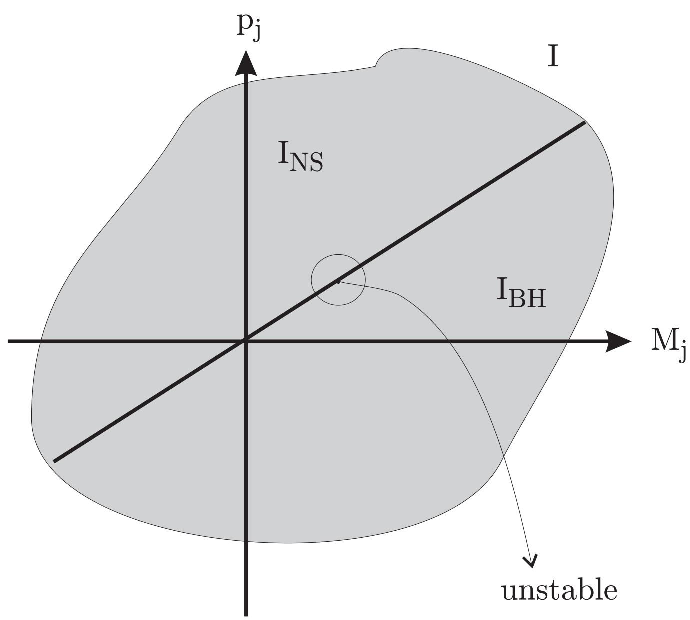
- Points away from the critical surface are stable.
- Simultaneous collapse: $x \in I_{BH}$ such that $\chi_j = 0$ for all j is unstable.
- Oppenheimer-Snyder: $x_{OS} \in I_{BH}$ is unstable.

Stability and Genericity

For every choice of the free function(s) in principle the initial data set can have a final outcome to be stable or non stable, generic or non generic.



I_{NS} non generic



I_{NS} generic

Oppenheimer-Snyder

The Oppenheimer-Snyder model is obtained for homogeneous dust:

- Dust implies $p = 0$.
- Homogeneity implies $\rho = \rho(t)$.

From which follow the conditions:

(i) $M(r) = M_0$

(ii) $v = v(t)$

(iii) $b_0(r) = k$

the line element becomes

$$ds^2 = -dt^2 + v^2 \left[\frac{dr^2}{1 + kr^2} + r^2 d\Omega^2 \right]$$

and

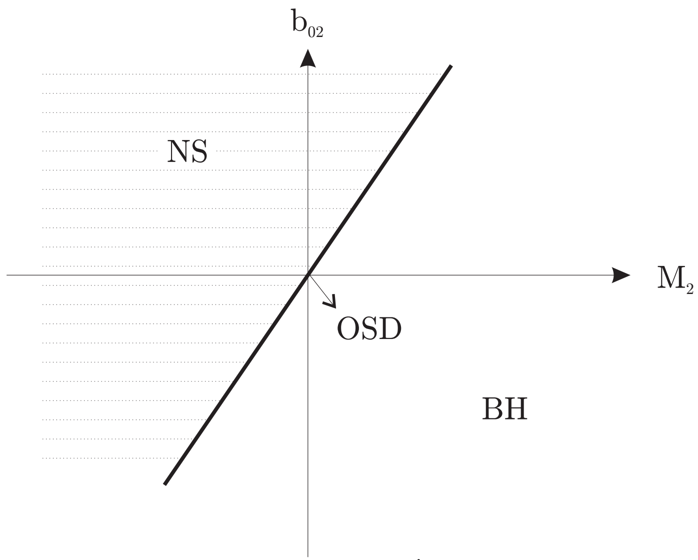
$$\chi_i(0) = 0$$

Collapse is simultaneous and the singularity is covered at all times.

Lemaitre-Tolman-Bondi

The Lemaitre-Tolman-Bondi dust model describes the collapse of a spherical ball of inhomogeneous dust and is obtained for $p = 0$, which implies $\nu = A = 0$.

$$ds^2 = -dt^2 + \frac{R'^2}{1 + r^2 b_0(r)} dr^2 + R^2 d\Omega^2$$



(i) $M = M(r) = M_0 + M_2 r^2 + ...$

(ii) $v = v(r, t)$

(iii) $b_0 = b_0(r) = b_{00} + b_{02} r^2 + ...$

$b_0(r) = k \;\Rightarrow\; \chi_2 = -\frac{1}{3}\frac{M_2}{M_0^{\frac{3}{2}}}$

$$\chi_1(0) = -\frac{1}{2}\int_0^1 \frac{\frac{M'(0)}{v} + b_0'(0)}{\left(\frac{M(0)}{v} + b_0(0)\right)^{\frac{3}{2}}} dv; \quad \chi_2(0) = -\frac{1}{2}\int_0^1 \frac{\frac{M''(0)}{v} + b_0''(0)}{\left(\frac{M(0)}{v} + b_0(0)\right)^{\frac{3}{2}}} dv$$

The singularity is covered or naked depending on $M(r)$ and $b_0(r)$.

Simultaneous Singularity

The condition for simultaneous collapse is

$$t_s(r) = t_0 \Leftrightarrow \chi_i = 0 \text{ for every } i$$

Given a density profile $M(r)$, once we choose b_{00}, we must choose every b_{0i} suitably in order to have $\chi_i = 0$

$$b_{01} = -\frac{\alpha_1}{\beta_1}$$

with $\alpha_1 = \int_0^1 \frac{M_1\sqrt{v}}{(M_0+b_{00}v)^{\frac{3}{2}}}dv$ and $\beta_1 = \int_0^1 \frac{v^{\frac{3}{2}}}{(M_0+b_{00}v)^{\frac{3}{2}}}dv$.

Similarly for b_{02} and the same reasoning applies to every order.

Then the velocity profile $b_0(r)$ for which collapse ends in a black hole is given by

$$b_0(r) = \sum_{i=0}^{+\infty} \frac{b_{0i}}{i!} r^i$$

Small Tangential Pressure Perturbation of OS

Two free functions:

$$M(r) = M_0 + M_1 r + M_2 r^2 + ...$$

and

$$\nu(r, v) = r^2 g(r, v) = g_0(v) r^2 + g_1(v) r^3 + ...$$

that give

$$p_\theta = \frac{r^2}{v R'^2} \left(3 M_0 g_0 + 4 M_1 g_0 r + \frac{9}{2} M_0 g_1 r + ... \right)$$

Note that the cloud must behave like dust at $r = 0$ ($p_\theta \to 0$ as $r \to 0$).

In general we get

$$\chi_1(0) = -\frac{1}{2} \int_0^1 \frac{M_1 + b_{01} v + g_1(v) v}{(M_0 + b_{00} v + g_0(v) v)^{\frac{3}{2}}} \sqrt{v} dv$$

and the final outcome is decided by $M(r)$, $b_0(r)$ and $p_\theta(r, v)$

Small Tangential Pressure Perturbation of OS II

From the conditions that give the OS model we drop (ii) and keep (i) and (iii). For simplicity take $g_0(v) = 0$. Then

$$p_\theta = \frac{9}{2}\frac{M_0 g_1}{v R'^2} r^3$$

with $g_1(v)$ bounded and small.

$$\chi_1(0) = -\frac{1}{2}\int_0^1 \frac{v^{\frac{3}{2}} g_1(v)}{(M_0 + kv)^{\frac{3}{2}}} dv$$

All those pressure profiles for which the function g_1 gives $\chi_1(0) > 0$ will cause the collapse to produce a naked singularity.

The line element near the center becomes

$$ds^2 = -(1 - 2g_1 r^3)dt^2 + \frac{R'^2}{1 + kr^2 - 2g_1 r^3} dr^2 + R^2 d\Omega^2$$

Profiles With Only Quadratic Terms

For more physically reasonable matter models we can require:

- ρ to have no cusps at the center. This implies $M_1 = 0$
- Perfect fluid.
- Pressure and density to have only quadratic terms in r.

Assuming that we can write

$$M(r,v) = M_0(v) + M_2(v)r^2 + .. \text{ we get } A(r,v) = a_2(v)r^2 + a_4(v)r^4 + ...$$

with

$$a_2 = \int_v^1 \frac{2M_{2,v}}{3M_0 - M_{0,v}v} dv$$

Then

$$\chi_2(0) = -\frac{1}{2}\int_0^1 \frac{\frac{M_2(v)}{v} + 2a_4 + 2a_2^2 + 2ka_2 + a_{2,v}v\left(\frac{M_0(v)}{v} + k + 2a_2\right)}{\left(\frac{M_0(v)}{v} + k + 2a_2\right)^{\frac{3}{2}}} dv$$

with $b_0(r) = k$.

Small Perfect Fluid Perturbation of LTB

For simplicity take $b_0 = 0$. Then take

$$M(r, v) = M_0 + (C + \varepsilon(v))\, r^2$$

with $\varepsilon(1) = 0$.

Small perturbation: take the pressure such that at all times

$$M_0 \gg M_2(v)$$

The density and pressure become

$$p = -\frac{\varepsilon_{,v}}{v^2} r^2 \text{ and } \rho = \rho_{LTB} - p + \frac{5\epsilon - \epsilon_{,v} v}{v^2(v + rv')} r^2$$

- The model is dust at the initial time
- $\varepsilon = 0$ implies LTB dust
- $\varepsilon = 0$ and $C = 0$ implies OS

Small Perfect Fluid Perturbation of LTB II

- $\varepsilon \uparrow \Rightarrow \ \varepsilon < 0 \ \Rightarrow p < 0$
- $\varepsilon \downarrow \Rightarrow \ \varepsilon > 0 \ \Rightarrow p > 0$

$$\chi_2(0) = -\int_0^1 \frac{C + \left(\varepsilon + \frac{2}{3}\varepsilon_{,v} v\right) + \frac{8}{9}\frac{\varepsilon v}{M_0^2}\left(\varepsilon + \varepsilon_{,v} v\right)}{\left(M_0 + \frac{4}{3}\frac{\varepsilon v}{M_0}\right)^{\frac{3}{2}}} \sqrt{v} dv$$

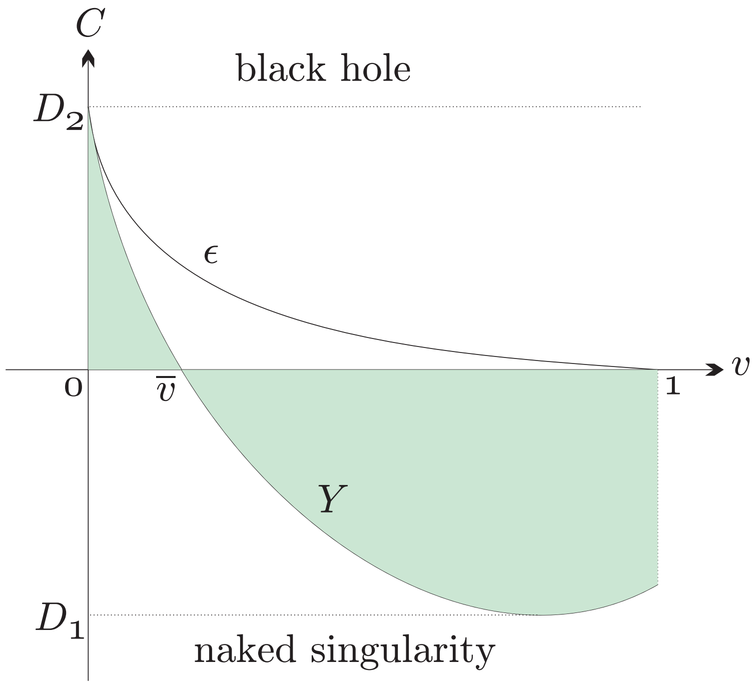
Consider the case where ε is decreasing and p is positive.

It is possible to set some conditions on $\varepsilon(v)$ and M_0 so that the final outcome of collapse is a naked singularity.

Therefore the introduction of an arbitrarily small positive perfect fluid pressure perturbation can turn the final fate of the Oppenheimer-Snyder collapse from black hole to naked singularity.

Small Perfect Fluid Perturbation of LTB III

$$Y(v) = \left(\varepsilon + \frac{2}{3}\varepsilon_{,v} v\right)$$



Energy Emission From the Vicinity of a Naked Singularity

Non central singularities: Models with non vanishing pressures in the form of a perfect fluid show that it is possible for the shells located at $r \neq 0$ to be visible when becoming singular, thus leaving space for new scenarios.

Consider p to be related to ρ via

$$\frac{p}{\rho} = \lambda(r, v) = \lambda_0(v) + \lambda_2(v) r^2 + ...$$

Einstein equations for ρ and p give

$$3\lambda M + \lambda r M_{,r} + [v + (\lambda + 1) r v'] M_{,v} = 0$$

that can be solved to second order for a suitable choice of λ_2.

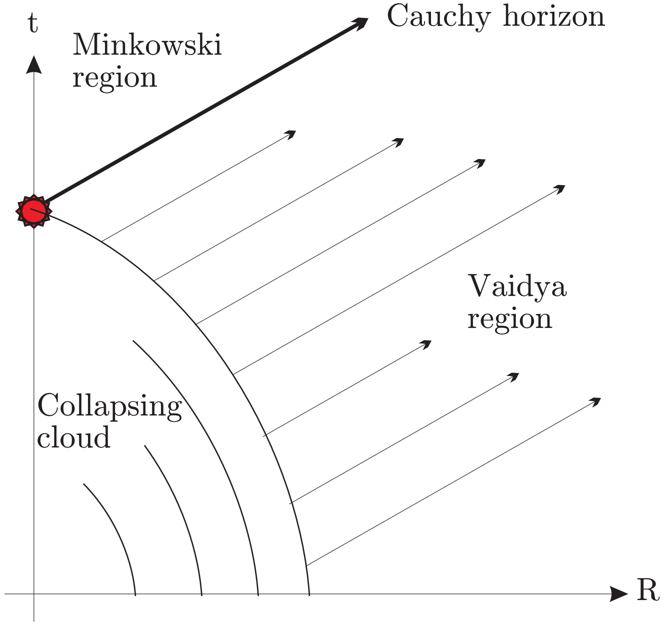
$$M(r, v) = C_1 e^{-3 \int_v^1 \frac{\lambda_0(v)}{v} dv} + r^2 C_2 e^{-5 \int_v^1 \frac{\lambda_0(v)}{v} dv} + ...$$

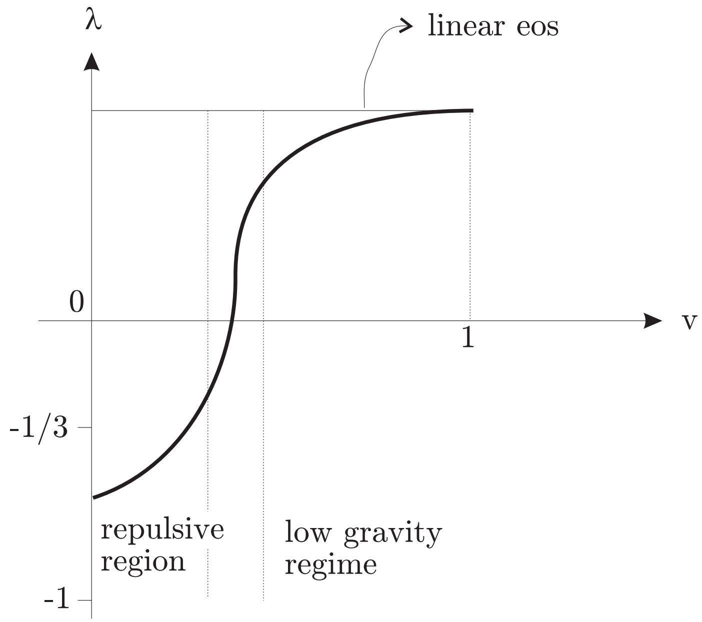
Energy Emission From the Vicinity of a Naked Singularity II

If we take $\lambda_0(v)$ such that

$$\lambda_0 \xrightarrow[v\to 0]{} d_0 \leq -\frac{1}{3} \quad \text{and} \quad \lambda_0 \xrightarrow[v\to 1]{} d_1 > 0$$

then the pressure turns negative during collapse, the mass is radiated away and the singularity curve can be timelike.





$$M(r,v) \xrightarrow[v\to 0]{} 0 \ \text{ such that } \ \frac{r^2 M(r,v)}{v} < 1 \ \text{ and } \ p < 0 \text{ near } v = 0$$

Discussion

There is more to collapse then the black hole paradigm:

- Black holes:
  Effects in the ultradense region near the singularity are hidden.
- Naked singularities:
  Effects in the region surrounding the singularity might be visible.
- Static models and bouncing models:
  For certain choices of the initial data it is possible for collapse to halt before all matter falls into the singularity and for the outer shells to bounce back.

The general formalism to analyze collapse developed here can be applied to many different scenarios.

Naked singularities are a 'generic' feature in collapse with pressures.

The Oppenheimer-Snyder model leading to the formation of a black hole is not stable under small pressure perturbations.

Still a lot needs to be done.

Future Developments

- Do naked singularities of any kind exist in the universe or are they only mathematical examples?
- What are the conditions for global visibility in collapse models?
- What kind of matter fields are physically more ‘realistic’?
- How can rotation and deformations affect this picture?
- If naked singularities exist can they be observed and what would distinguish them from black holes?
- Can these model constitute the basis towards an explanation of some of the observed high energy phenomena in the universe?

If naked singularities exist then some new physics could be hiding in theoretical models and astrophysical observations.