Spin-orbit coupling for tidally evolving super-Earths

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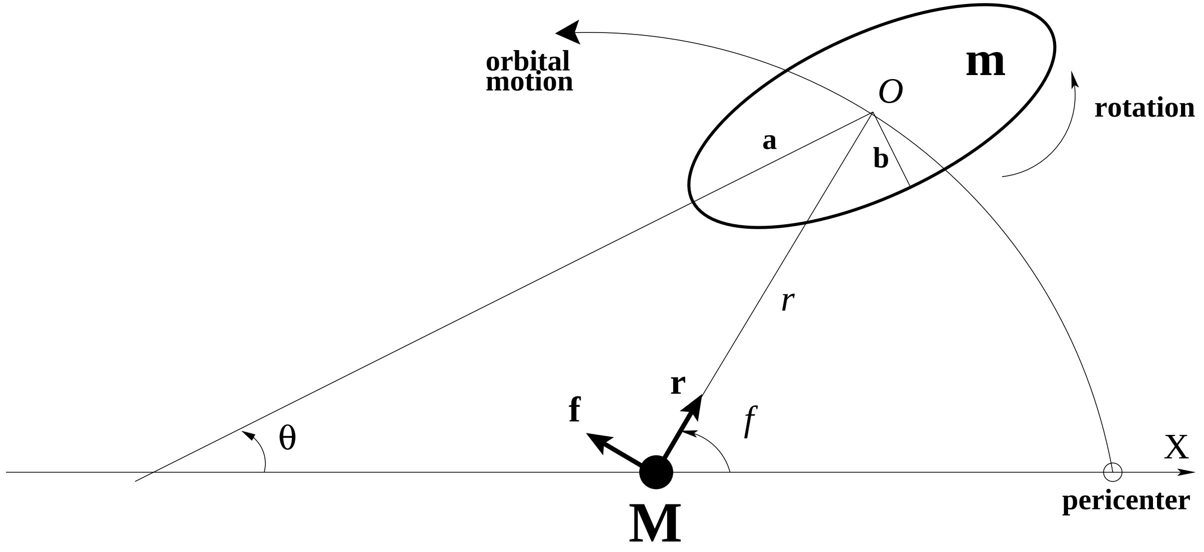
Aveiro, 2012

THE PROBLEM



- A close-in rocky-like planet ($P \leq 4$ days) orbiting its central star.

- Permanent deformation and proximity$\longrightarrow$ two torques acting on the planet rotation: (triaxial and tidal torques).

- Objective: to investigate the spin behavior and the implications on the orbital evolution.

INTRODUCTION

- **What is a Spin-Orbit Resonance (SOR)**?

$\dot{\theta} = \Omega \longrightarrow$ angular velocity of rotation ($2\pi/P_{rot}$)
$n \longrightarrow$ mean orbital motion ($2\pi/P_{orb}$)

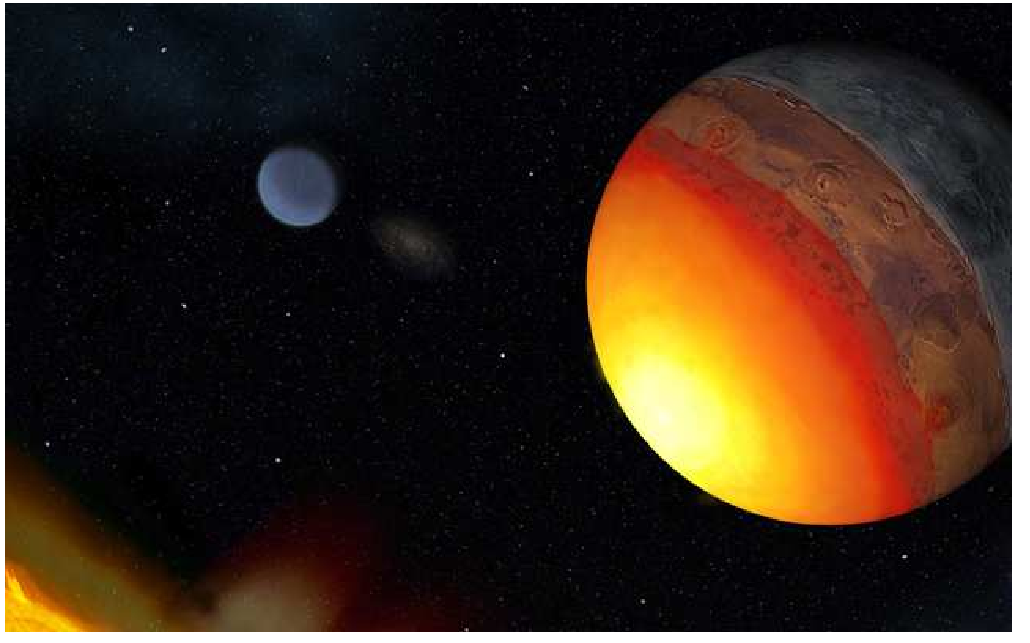
Let p a half-integer such that $\Omega = pn \longrightarrow p$–SOR

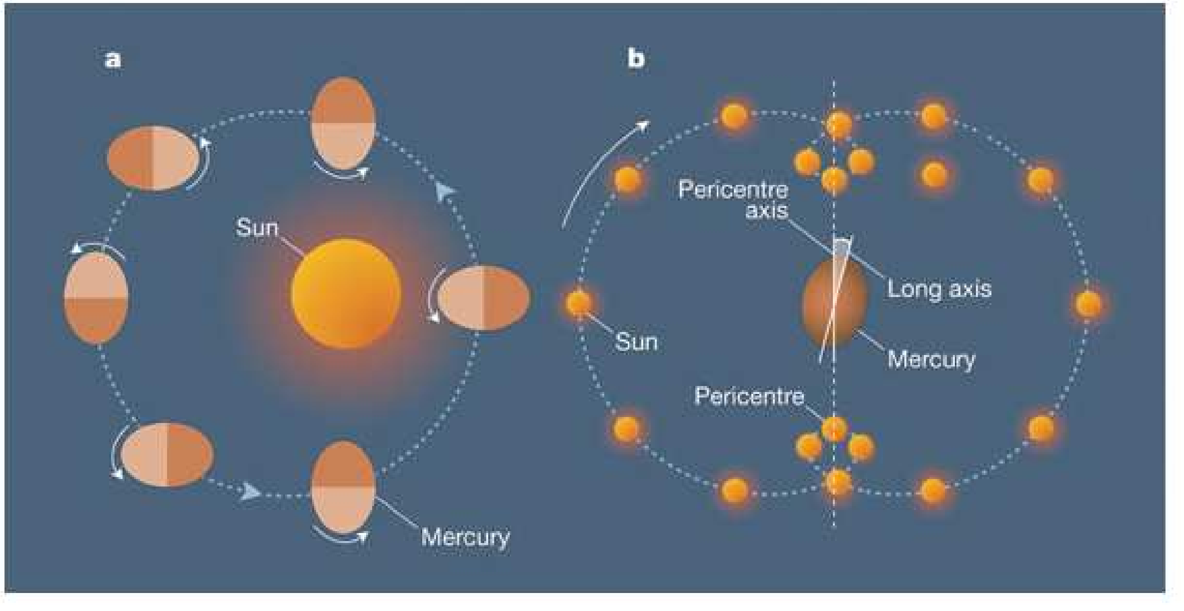
-**Examples**:

The Moon (and other Solar System's moons) $\longrightarrow p = 1$ (*synchronous* rotation, always showing the same face to the planet).

Mercury $\longrightarrow p = 3/2$

INTRODUCTION





INTRODUCTION

How the rotation becomes trapped in a SOR?

- SOR$\longrightarrow$ Stationary solutions of the rotation under combined action of two torques:

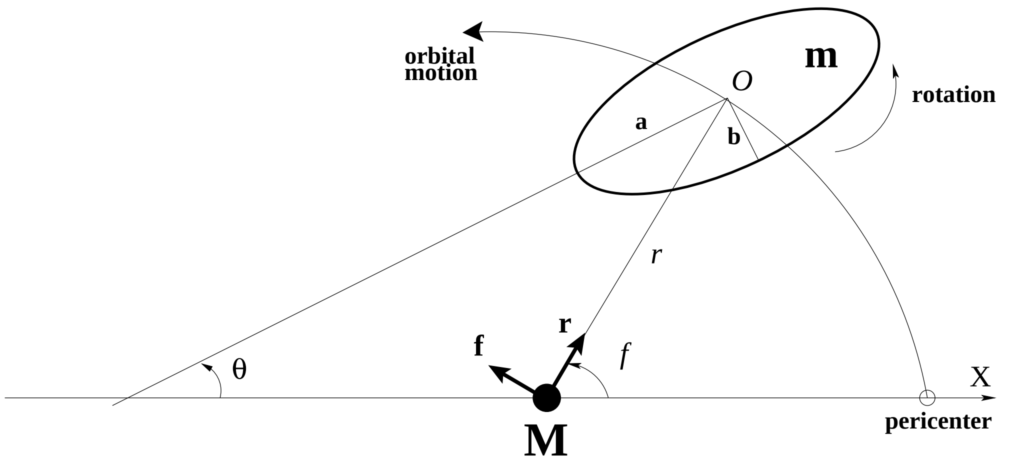
$$C\dot{\Omega} = T_{tide} + T_{triax} = 0$$

- Both torques depends on the orbital eccentricity e, and thus the specific SOR also does.

- For circular orbits ($e = 0$)$\longrightarrow$ only the 1/1 SOR is possible, whereas for $e > 0$ higher order SORs also appears ($e_{Mercury} \simeq 0.20$).

What about exoplanets?$\longrightarrow$ close -in planets, large variety of e values$\longrightarrow$ rotation?

THE MODEL



- Potential of the deformed body$\longrightarrow$ computation of tidal and triaxial forces and torques.

$$\mathbf{T}_{\text{triax}} = 6\,\frac{GmMR^2}{r^3} C_{22} \sin 2(f-\theta)\hat{\mathbf{k}}.$$

$$\mathbf{T}_{\text{tide}} = 3k_2\Delta t \frac{GM^2R^5}{r^8}[-r^2\mathbf{\Omega} + \mathbf{r}\times\mathbf{v}],$$

- C_{22} and $k_2\Delta t \longrightarrow$ parameters related to equatorial deformation (triaxially) and internal viscosity (tides).

NUMERICAL SIMULATIONS

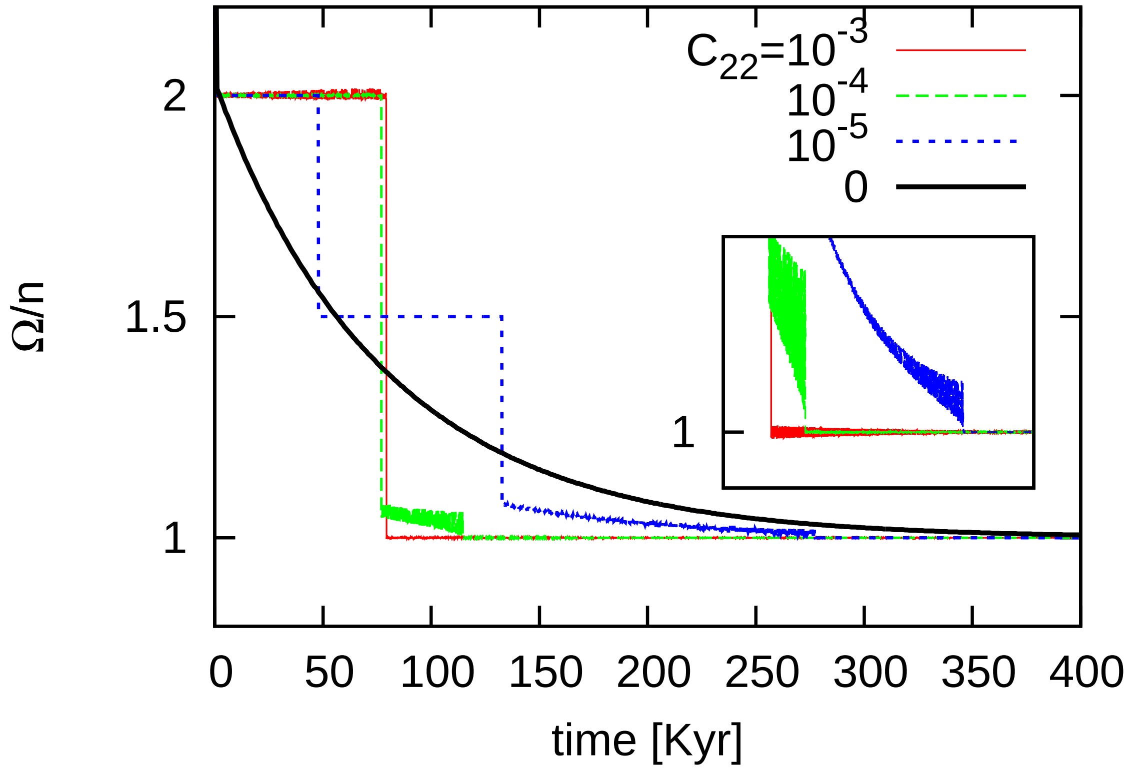
- Equations of motion

$$\ddot{\mathbf{r}} = -\frac{G(M+m)}{r^3}\mathbf{r} + \frac{(M+m)}{Mm}(\mathbf{F}_{\text{tide}} + \mathbf{F}_{\text{triax}}),$$
$$\ddot{\theta} = \dot{\Omega} = \frac{1}{C}(T_{\text{tide}} + T_{\text{triax}}).$$

- The systems

System	$M\,(\mathrm{m}_\odot)$	$m(m_\oplus)$	$R(R_\oplus)$	a_{current} (au)	e_{current}
Kepler-10[a]	0.895	4.56	1.416	0.01684	0
55 Cnc[b]	0.905	8.63	2.17	0.0156	0.057
GJ 3634[c]	0.45	7.0	-	0.0287	0.08

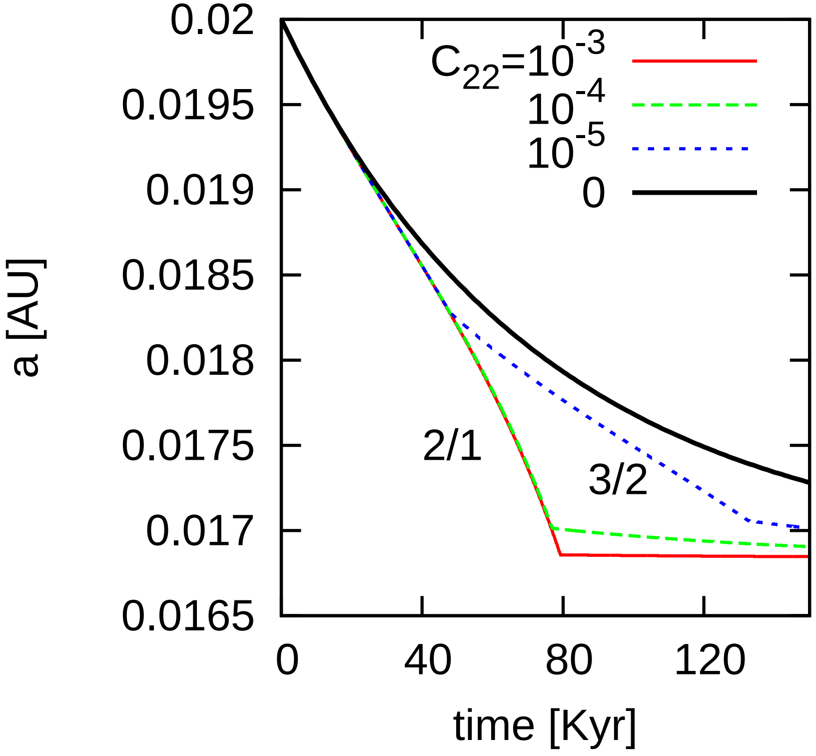
NUMERICAL SIMULATIONS

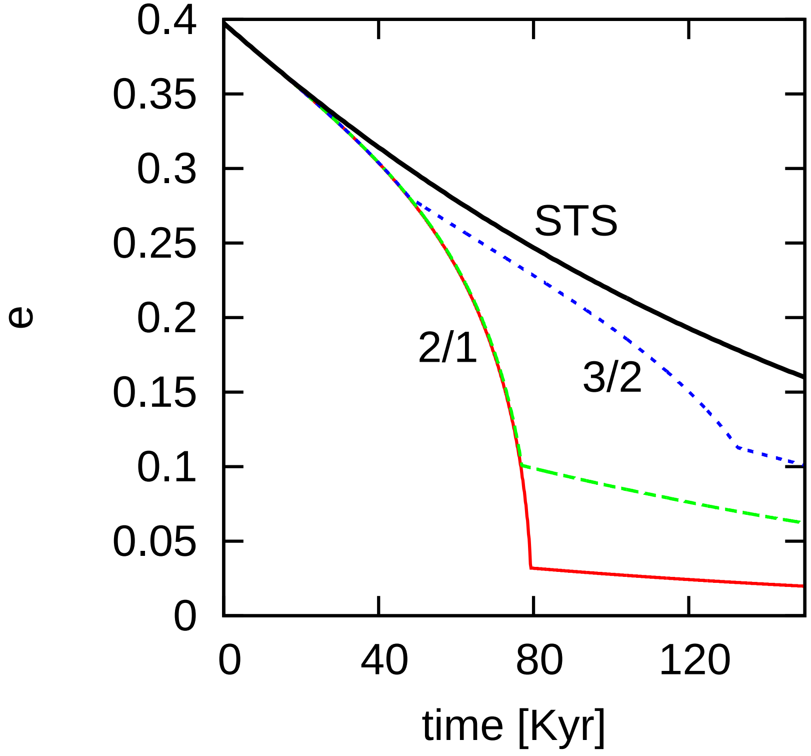


- Kepler 10b ($k_2\Delta t = 1.1$ min).
- Resonant trappings in 2/1, 3/2 and 1/1 SORs.
- Black curve ($C_{22} = 0$)$\longrightarrow$ stationary solution of the tidal problem (STS):

$\Omega_{\text{stat}}/n = 1 + 6e^2 + 3e^4/8 + 223e^6/8 + \mathcal{O}(e^7)$

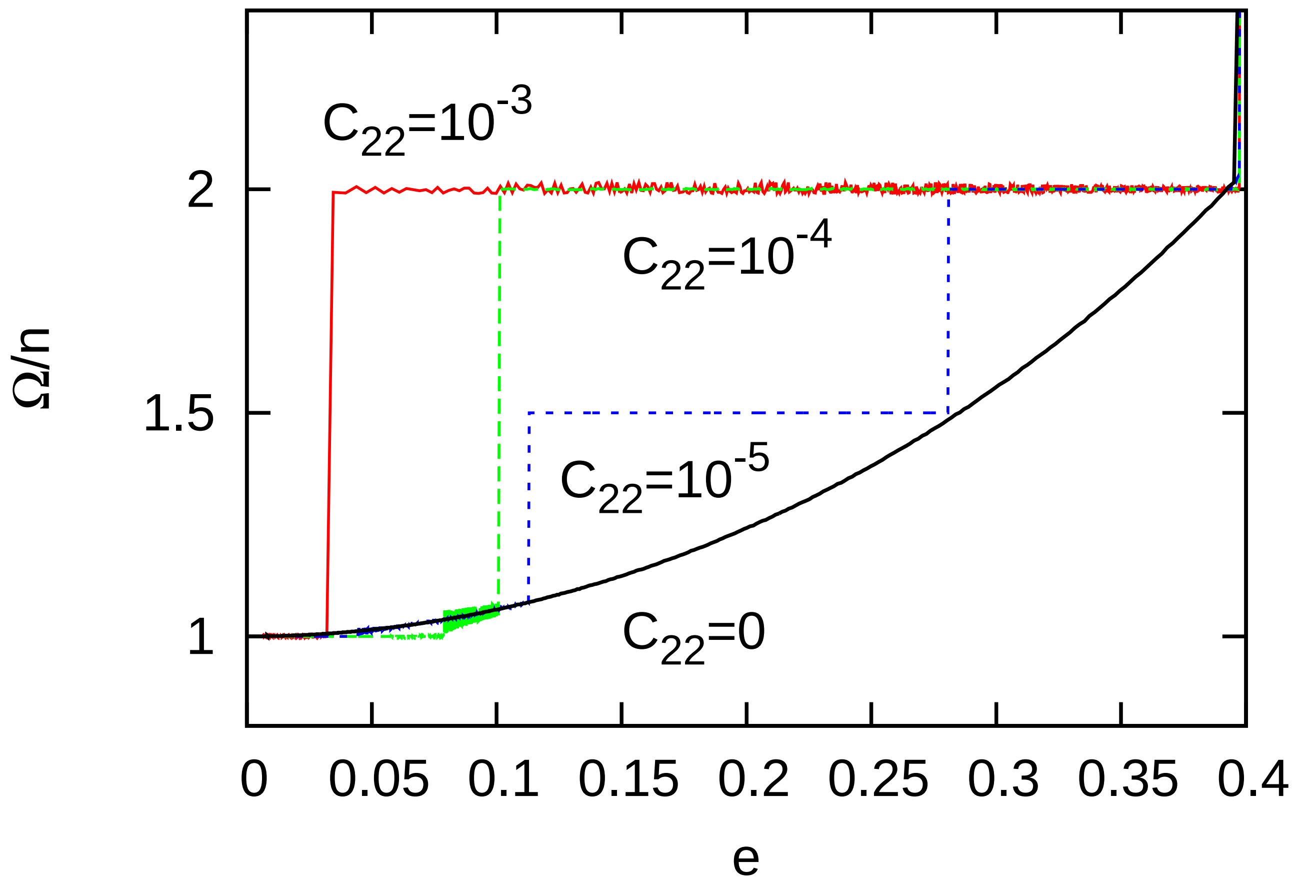
NUMERICAL SIMULATIONS





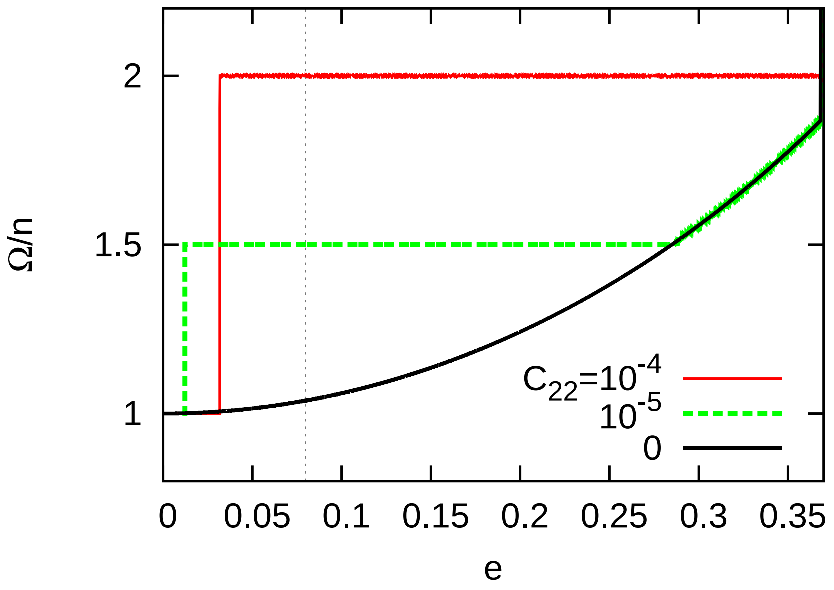
- The rates of elements variation are larger when the rotation evolves under capture in SOR.

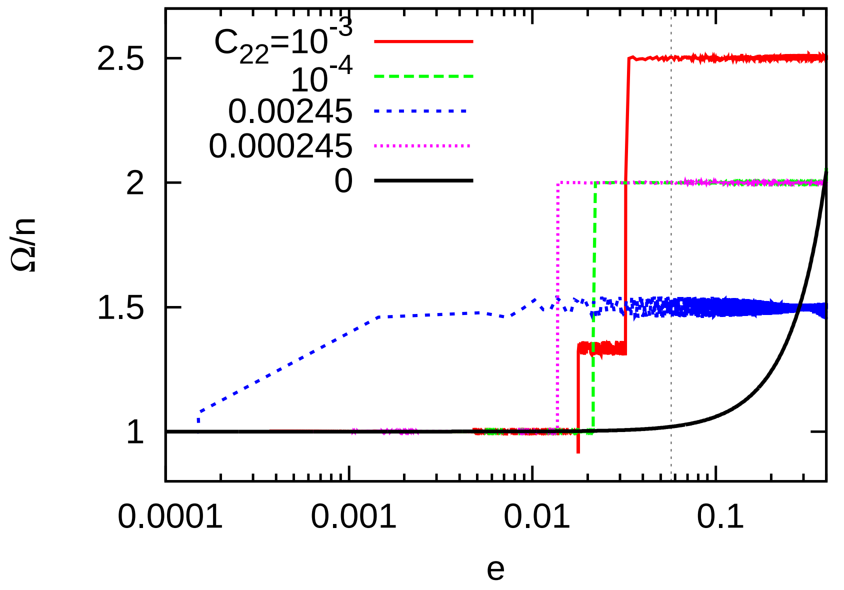
NUMERICAL SIMULATIONS



- The rotation evolves to the synchronism as the eccentricity is tidally damped and thus, the RSOs become unstable.

NUMERICAL SIMULATIONS





- GJ 3634b (left) and 55 Cnc e (right).

- Trappings in 5/2, 2/1, 4/3, 3/2 and 1/1.

- Current e–values$\longrightarrow$ non-synchronous resonant rotation may be possible, provided that the planet figure can support a permanent deformation such that $C_{22} > 0$ (as small as 10^{-5}).

AVERAGED EQUATIONS

- Resonant angle$\longrightarrow$ $\delta = \theta - p\,l$, $\dot{\theta} = \dot{\delta} + pn$, where $\dot{\theta} = \Omega \simeq pn$ and, consequently, $\dot{\delta} \simeq 0$.

$$< T_{\text{triax}} >= -6mR^2C_{22}n^2H(p,e)\sin 2\delta,$$

$$< T_{\text{tide}} >= K_{\text{tide}}\left(\frac{n}{a}\right)^3\mathcal{T}(p,e),$$

$$< T_{\text{tide}} > + < T_{\text{triax}} >= C\dot{\Omega} = 0$$

$$C_{22c} = \frac{1}{2}k_2\Delta t\, n\frac{M}{m}\left(\frac{R}{a}\right)^3\frac{\mathcal{T}(p,e)}{H(p,e)},$$

- $C_{22} > C_{22c} \longrightarrow$ stability condition for the RSO $\Omega = pn$ (strength criterion).

AVERAGED EQUATIONS

- **The 1/1 SOR**: (semi-major axis)

Replacing $p = 1$, $C_{22}\sin 2\delta$ and expanding the ratio $\frac{\mathcal{T}(p,e)}{H(p,e)}$ in powers of e

$$< \dot{a}_{\text{triax}} >= 3k_2 \Delta t \frac{M}{m}\left(\frac{R}{a}\right)^5 n^2 a \left(12e^2 + \frac{363}{4}e^4 + \frac{2911}{8}e^6\right) + \mathcal{O}(e^8).$$

$$\begin{aligned} < \dot{a}_{\text{tide}} > \quad = \quad & 3k_2 \Delta t \frac{M}{m}\left(\frac{R}{a}\right)^5 n^2 a \Big[2(-1+p) + (-46+27p)e^2 \\ & + \frac{3}{4}(-480+191p)e^4 + \frac{1}{8}(-13530+3961p)e^6\Big] + \mathcal{O}(e^8). \end{aligned}$$

$$< \dot{a}_{\text{tide}} >_{p=1} = -3k_2 \Delta t \frac{M}{m}\left(\frac{R}{a}\right)^5 n^2 a \left(19e^2 + \frac{867}{4}e^4 + \frac{9569}{8}e^6\right) + \mathcal{O}(e^8).$$

AVERAGED EQUATIONS

- **The 1/1 SOR**: (semi-major axis)

$$< \dot{a} >_{p=1} = < \dot{a}_{\text{tide}} >_{p=1} + < \dot{a}_{\text{triax}} >$$

$$< \dot{a} >_{p=1} = -3k_2 \Delta t \frac{M}{m} \left(\frac{R}{a}\right)^5 n^2 a \left(7e^2 + \frac{504}{4} e^4 + \frac{3329}{4} e^6\right) + \mathcal{O}(e^8).$$

$$\Omega_{\text{stat}}/n = 1 + 6e^2 + 3e^4/8 + 223e^6/8 + \mathcal{O}(e^7)$$

$$< \dot{a}_{\text{tide}} >_{p=p_{\text{stat}}} = -3k_2 \Delta t \frac{M}{m} \left(\frac{R}{a}\right)^5 n^2 a \left(7e^2 + 54e^4 + \frac{1133}{4} e^6\right) + \mathcal{O}(e^8).$$

$$r_a^p \equiv \frac{< \dot{a} >_p}{< \dot{a}_{\text{tide}} >_{p=p_{\text{stat}}}}.$$

- $r_a^{p=1} > 1 \longrightarrow$ the rate of semi-major axis decay is stronger when the rotation is locked in the 1/1 SOR, compared to the case when it evolves following the STS.

AVERAGED EQUATIONS

- **The 2/1 and 3/2 SORs**:

$$< \dot{a} >_{p=3/2} = -3k_2 \Delta t \frac{M}{m} \left(\frac{R}{a}\right)^5 n^2 a \left(\frac{1}{2} - \frac{5}{4}e^2 + \frac{774}{16}e^4 + \frac{8019}{16}e^6\right) + \mathcal{O}(e^8).$$

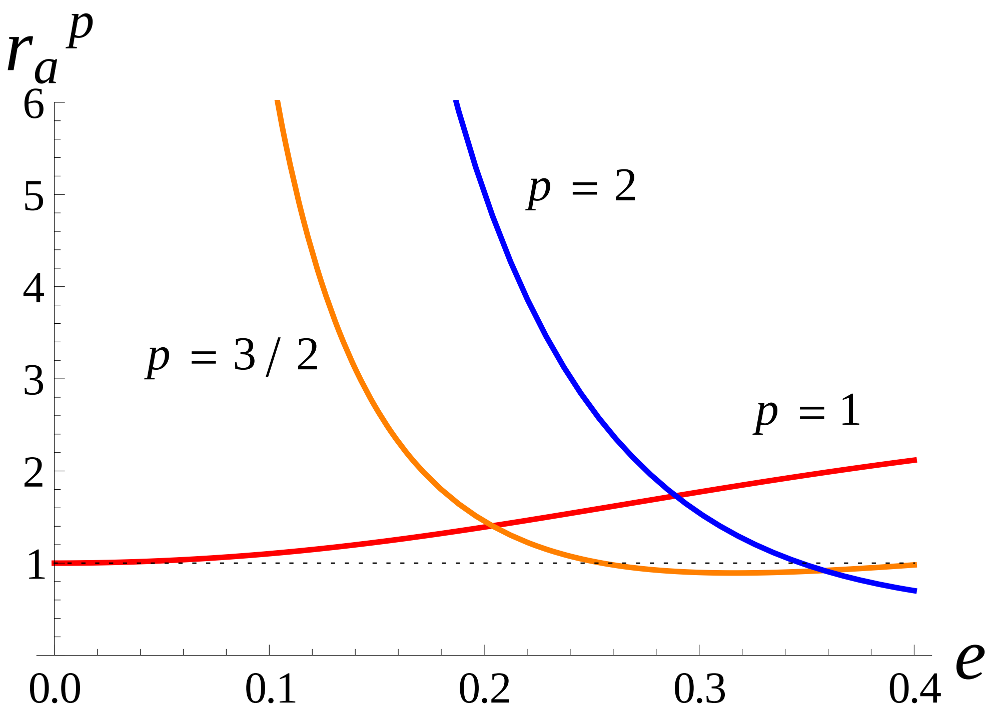
$$< \dot{a} >_{p=2} = -3k_2 \Delta t \frac{M}{m} \left(\frac{R}{a}\right)^5 n^2 a \left(2 - 2e^2 - 3e^4 + \frac{943}{4}e^6\right) + \mathcal{O}(e^8).$$

$$< \dot{a}_{\text{tide}} >_{p=p_{\text{stat}}} = -3k_2 \Delta t \frac{M}{m} \left(\frac{R}{a}\right)^5 n^2 a \left(7e^2 + 54e^4 + \frac{1133}{4}e^6\right) + \mathcal{O}(e^8).$$

$$r_a^p \equiv \frac{< \dot{a} >_p}{< \dot{a}_{\text{tide}} >_{p=p_{\text{stat}}}}.$$

- $r_a^{p=2/1}$ and $r_a^{p=3/2}$ also larger that one (for moderate e).

AVERAGED EQUATIONS



- $p = 3/2$, $p = 2 \longrightarrow r_a^p$ increase for small eccentricity, indicating that the major effect on the orbital evolution due to the resonance captures occurs as e decreases.
- $p = 1 \longrightarrow$ orbital decay is large for high e (but synchronism is attained for very low values of e...)

AVERAGED EQUATIONS

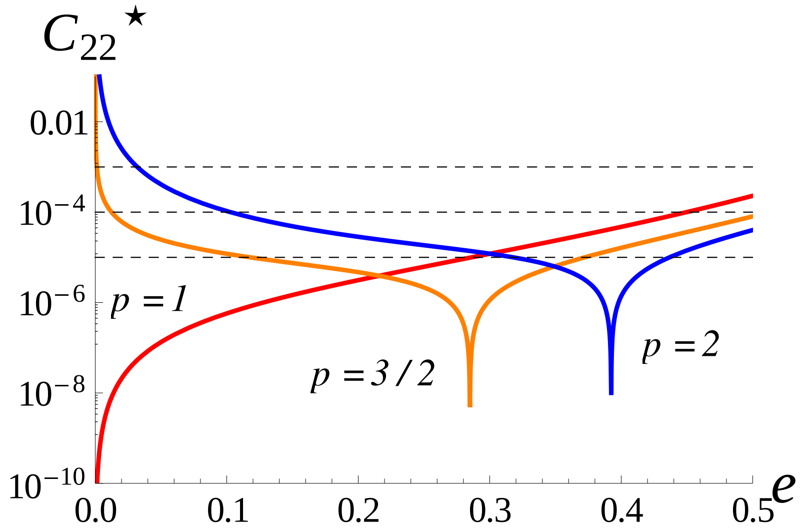
- **Critical C_{22}**:

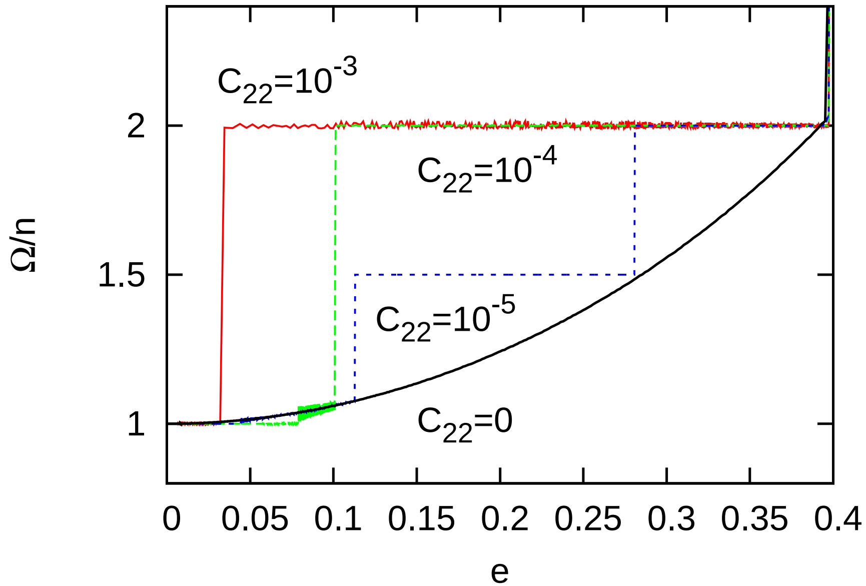
$$C_{22}^{*} = \frac{1}{2} k_2 \Delta t \, n \frac{M}{m} \left(\frac{R}{a} \right)^3 \frac{\mathcal{T}(p,e)}{H(p,e)},$$

- The stability condition of the p–SOR requires a critical value of C_{22} such that $C_{22} > C_{22}^{*}$.
The resonant motion would destabilize whenever C_{22} becomes smaller than the critical value for each SOR.

- Above equation also enable us to obtain a critical value of e for a given C_{22}.
This approach is more convenient in those cases for which the observations allow an estimation of C_{22} (see Correia & Laskar (2004) for the study of the rotation of Mercury).

AVERAGED EQUATIONS





SUMMARY

- A large variety of SORs is expected for exoplanets, depending on the values of C_{22}.

- GJ 3634b and 55 Cnc b can be examples of planet which rotates in a non-synchronous resonant state (since $e_{current} > 0$).

- The rates of variation of semi-major axis and eccentricity are fast whenever the planet rotation evolves in SOR (if compared with the rates in which the rotation evolves under the STS (i.e, pure tidal case or $C_{22} = 0$).

Reference: Rodríguez et al. (2012, accepted in MNRAS).