# Black Hole collisions

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7th May 2012, Group Seminar, Universidade de Aveiro

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4. Black holes in de Sitter
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5. Further topics
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   - Conclusions
   - Future work

# Outline

# Why numerical relativity

**Study of systems with strong and dynamical gravitational fields**

- Gravitational radiation
  - Astrophysics, gravitational wave astronomy
- Mathematical and theoretical Physics:
  - Cosmic censorship
  - Instabilities (Black hole interior, Myers-Perry)
- High-energy particle systems:
  - AdS/CFT correspondence;
  - Black hole production at the LHC;

# Gravitational waves

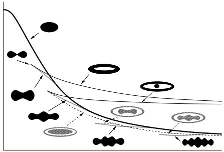
- Accelerated bodies emit gravitational radiation
- Detected indirectly by measurements of the Hulse-Taylor binary system (1993 Nobel Prize)
- Interact weakly with matter $\Rightarrow$ carry unique information about astronomical phenomena
  - $\Rightarrow$ New window to the universe

- Observations suggest we live in an approximately de Sitter Universe;
  - can we make numerical relativity in de Sitter?

# Mathematical and theoretical Physics

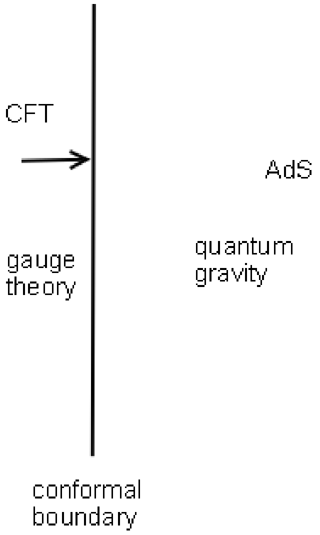
- Cosmic censorship hypothesis:
  - does it hold under extreme conditions?
    Sperhake, Cardoso, Pretorius, Berti, Gonzales, 2008
- No no-hair theorem for $D > 4 \Rightarrow$ black hole solutions with non-spherical topology.
  - (Non-)Linear stability of higher-dimensional black objects:
    - Myers-Perry black hole
    - Black ring
    - ...

Choptuik, Lehner, Olabarrieta, Petryk, Pretorius, Villegas, 2003
Shibata & Yoshino, 2010



Emparan & Reall, 2008

# AdS/CFT duality




**Issues with numerical simulations in AdS:**

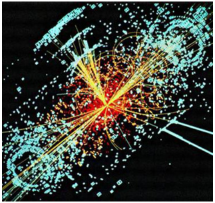
- AdS is not globally hyperbolic
- The boundary plays an active role

# High-energy particle systems

- Large extra dimensions scenarios:
  - fundamental Planck scale could be as low as the TeV: $\Rightarrow$ at the LHC particles collide at centre of mass energies above the fundamental Planck scale.
- *Matter does not matter*: for energies above Planck scale, $E = 2\gamma m_0 c^2 > E_{\rm Planck}$
  - gravity is the dominant force;
  - internal structure of particle not important for understanding of process.

$\Rightarrow$ high energy particle collisions should be well described by black hole collisions – **classical** general relativity.

# High-energy particle systems



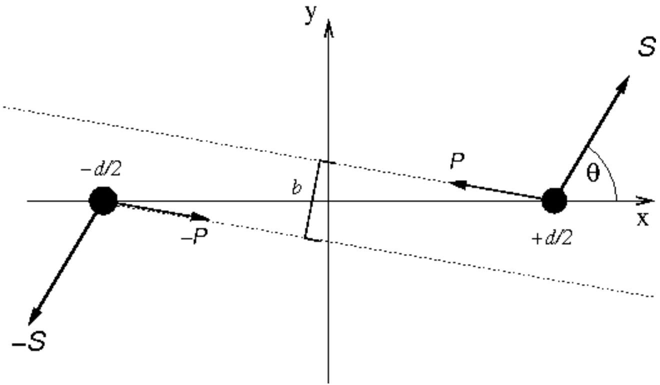
- If quantum gravity scale is near the TeV:
  $\Rightarrow$ black hole production at the LHC
  $\Rightarrow$ evidences for extra dimensions

- High-energy collision of two black holes can model a high-energy collision of relativistic objects;
- Amount of collision energy lost in gravitational radiation as a function of space-time dimension;
- Need cross section for black hole production;

# Outline

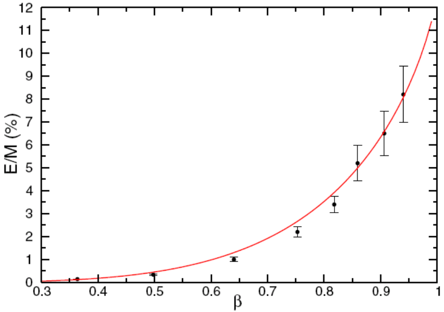
# Black hole collisions in $D = 4$

- two black holes
  Total rest mass: $M_0 = M_\mathrm{A} + M_\mathrm{B}$
  Initial position: $\pm x_0$
  Linear momentum: $\mp P[\cos\alpha, \ \sin\alpha, \ 0]$
- Impact parameter: $b \equiv \frac{L}{P}$



# Head-on collisions: $b = 0, \quad \vec{S} = 0$

- Total radiated energy: $14 \pm 3$ % for $v \to 1$
  Sperhake *et al.* '08



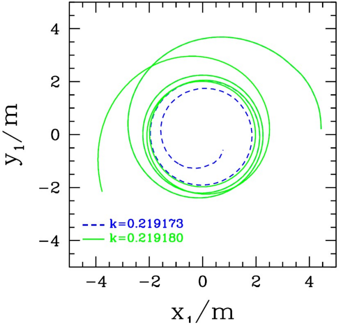

- Agreement with approximative methods
  *e.g.* Berti *et al.* '10

# Zoom whirl orbits: $b \neq 0$

1-parameter family of initial data: impact parameter

Pretorius & Khurana '07

- Fine tune parameter $\Rightarrow$"Threshold of immediate merger"
- Reminiscent of "Critical Phenomena"



# Recoil in grazing collisions

- 2 sequences merging: $b = 3.34\ M$ scattering: $b = 3.25\ M$
- $v_{\rm max,s} = 12\,200\ {\rm km/s}$
  $v_{\rm max,m} = 14\,900\ {\rm km/s}$
- Large recoils for merger and scattering!
- $v_{\rm max} \propto E_{\rm rad}$

# Outline

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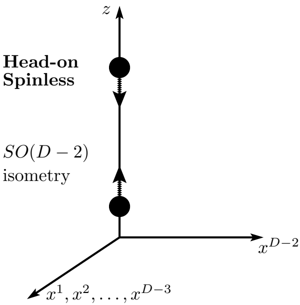
# Axial symmetry $SO(D-2)$ and $SO(D-3)$




- Highly symmetric systems;
- Can be reduced to effective $3+1$ systems;
  $\Rightarrow$ We can use existing numerical codes (with adaptations);

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# Formalism

Most general metric element

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + e^{2\phi} d\Omega^2_{D-4}$$

$\mu = 0, 1, 2, 3.$
$D$-dimensional vacuum Einstein equations imply

$$e^{2\phi}\left((D-4)\partial^\alpha\phi\partial_\alpha\phi + \nabla^\alpha\partial_\alpha\phi\right) = D-5$$
$$R_{\mu\nu} = (D-4)\left(\nabla_\nu\partial_\mu\phi + \partial_\mu\phi\partial_\nu\phi\right)$$

# Formalism

The resulting system is

$$
\begin{aligned}
(\partial_t - \mathcal{L}_\beta)\,\gamma_{ij} &= -2\alpha K_{ij}, \\
(\partial_t - \mathcal{L}_\beta)\,K_{ij} &= -D_i\partial_j\alpha + \alpha\left({}^{(3)}R_{ij} + KK_{ij} - 2K_{ik}K^k{}_j\right) \\
&\quad - \alpha(D-4)\left(D_i\partial_j\phi - K_{ij}K_\phi + \partial_i\phi\partial_j\phi\right), \\
(\partial_t - \mathcal{L}_\beta)\,\phi &= -\alpha K_\phi, \\
(\partial_t - \mathcal{L}_\beta)\,K_\phi &= \alpha\Big[(D-5)e^{-2\phi} - (D-4)\partial_i\phi\partial^i\phi - D^i\partial_i\phi \\
&\quad + (D-4)K_\phi^2 + KK_\phi\Big] - \partial^i\alpha\partial_i\phi,
\end{aligned}
$$

$\rightarrow$ effective $3+1$ system with source terms

# Formalism

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\end{aligned}
$$

$\rightarrow$ effective $3+1$ system with source terms

# Formalism

## Constraints

$$\mathcal{H} \equiv R + K^2 - K_{ij}K^{ij} - 16\pi E = 0$$

$$\mathcal{M}^i \equiv \nabla_j \left( K^{ij} - \gamma^{ij} K \right) - 8\pi p^i = 0$$

# Electromagnetic analogy

## Evolution equations

$$-\partial_t \vec{E} + \nabla \times \vec{H} = 4\pi \vec{j}$$
$$-\partial_t \vec{H} + \nabla \times \vec{E} = 0$$

## Constraints

$$\nabla \cdot \vec{E} = 4\pi \rho$$
$$\nabla \cdot \vec{H} = 0$$

# Initial data

## Example: Brill-Lindquist initial data

$$ds^2 = \psi^{\frac{4}{D-3}} \left( dz^2 + d\rho^2 + \rho^2 d\xi^2 \right) + \psi^{\frac{4}{D-3}} \rho^2 \sin^2 \xi^2 d\Omega^2_{D-4},$$

$$\psi = 1 + \frac{\mu_1}{4\left[(z-z_1)^2 + \rho^2\right]^{\frac{D-3}{2}}} + \frac{\mu_2}{4\left[(z-z_2)^2 + \rho^2\right]^{\frac{D-3}{2}}}$$

## To match with

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + e^{2\phi} d\Omega^2_{D-4}$$

# Wave extraction

Generalization of Regge-Wheeler-Zerilli to higher-dimensions.

## Kodama-Ishibashi formalism

$$g_{AB} = g^{(0)}_{AB} + h_{AB}$$

$$(ds^{(0)})^2 = -\left(1 - \frac{r_S^{D-3}}{r^{D-3}}\right) dt^2 + \left(1 - \frac{r_S^{D-3}}{r^{D-3}}\right)^{-1} dr^2 + r^2 d\Omega_{D-2}$$

Kodama & Ishibashi 2003

Decomposition of perturbation into harmonic functions:

- scalar harmonics, $\Box\mathcal{S} = -k^2\mathcal{S}$, $k^2 = l(l+D-3)$
  - in D = 4: "polar" or even perturbation
- vector harmonics, $\Box\mathcal{V}_k = -k_V^2\mathcal{V}_k$
  - in D = 4: "axial" or odd perturbation
- tensor harmonics, $\Box\mathcal{T}_{kl} = -k_T^2\mathcal{T}_{kl}$
  - vanish in D = 4

# Wave extraction

## Emitted gravitational waves described by master function $\Phi$

$$(\Box - V)\,\Phi_l = 0$$

## Energy flux

$$\frac{dE_l}{dt} = \frac{1}{32\pi}\frac{D-3}{D-2}k^2(k^2-D+2)(\Phi^l_{,t})^2$$

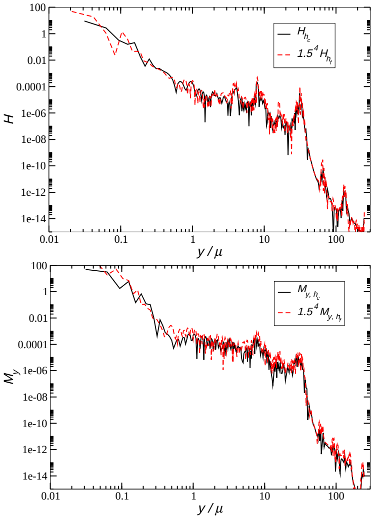
$$E = \sum_{l=2}^{\infty}\int_{-\infty}^{+\infty} dt\,\frac{dE_l}{dt}$$

## Procedure

1. Compute master function $\Phi$
2. $\rightarrow$ Energy flux

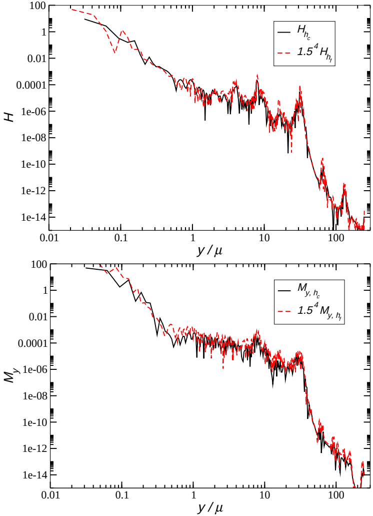
# Outline

# Constraints




- The evolution is stable and the constraints are preserved

# Constraints

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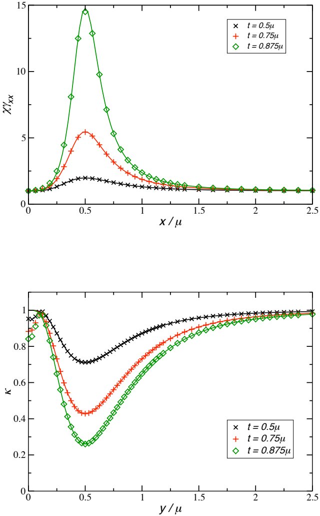
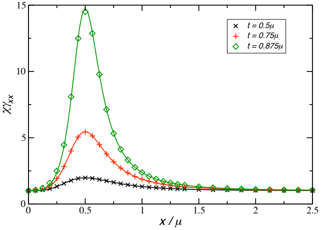
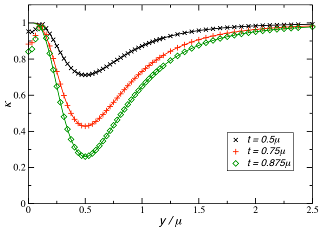
# Geodesic slicing




- Excellent agreement with the analytical solution

# Geodesic slicing

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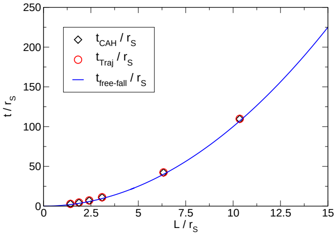
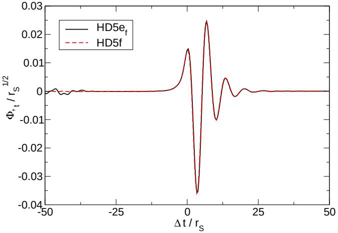
# Collision from rest




- Estimates for time it takes for BHs to collide in $D = 5$.

# Wave forms

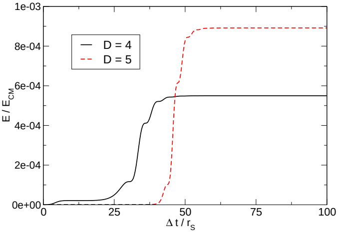



- $l = 2$ mode of $\Phi_{,t}$
  $d = 6.37\, r_S$ (black)
  $d = 10.37\, r_S$ (red)

characteristic ringdown frequency:

$$r_S\omega = 0.955 \pm 0.005 - i(0.255 \pm 0.005)$$
$$(r_S\omega = 0.9477 - i0.2561, \text{ e.g., Berti et al. 2009})$$

# Radiated energy




| $D$ | $E^{\rm rad}/M(\%)$ |
|---|---|
| 4 | 0.055 |
| 5 | 0.089 |

# Outline

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# Formalism

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 0, \quad \Lambda = 3H^2$$

## Evolution equations

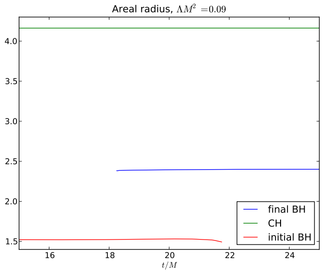
$$(\partial_t - \mathcal{L}_\beta)\, \gamma_{ij} = -2\alpha K_{ij}$$
$$(\partial_t - \mathcal{L}_\beta)\, K_{ij} = -D_i \partial_j \alpha + \alpha \left( {}^{(3)}R_{ij} - 2 K_i{}^k K_{jk} + K_{ij} K - \Lambda \gamma_{ij} \right)$$

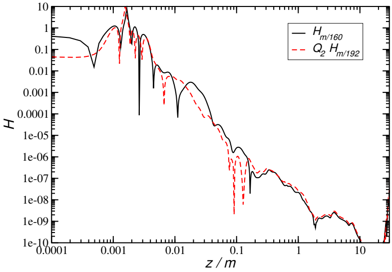
## Initial data

$$ds^2 = -\left( \frac{1-\xi}{1+\xi} \right)^2 dt^2 + a(t)^2 (1+\xi)^4 (dr^2 + r^2 d\Omega_2)$$
$$K^i_j = -H \delta^i_j, \qquad a(t) = \exp(Ht), \quad \xi \equiv \frac{m}{2a(t)r}$$

# Outline

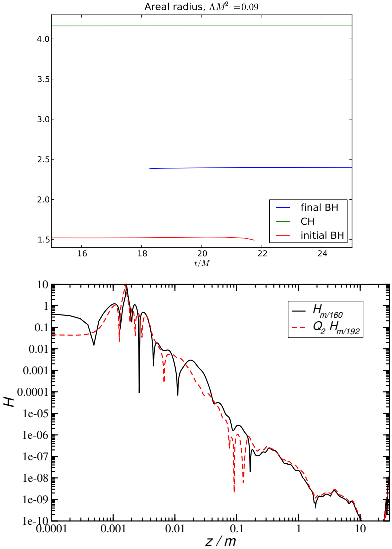
# Results




- Evolution is stable and the constraints are preserved;
- Successfully monitor the apparent horizons;

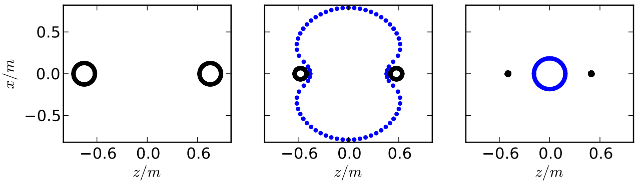
# Results




- Evolution is stable and the constraints are preserved;
- Successfully monitor the apparent horizons;

# Black holes in de Sitter

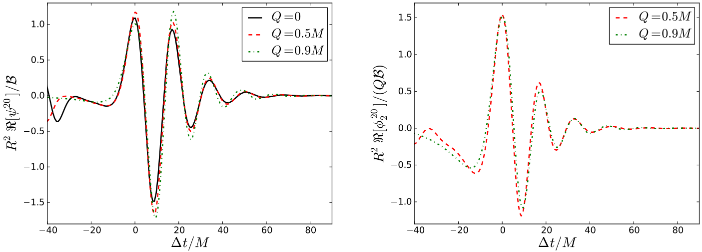
- Two parameters: $H$, $d$
- Initial data: McVittie type binaries McVittie '33
- "Small BHs": $d < d_{crit} \Rightarrow$ merger
  $d > d_{crit} \Rightarrow$ no common AH
- "Large" holes at small $d$: Cosmic Censorship holds



# Outline

# Collisions of charged BHs in $D = 4$

- Electro-vacuum Einstein-Maxwell Moesta et al. '10
- Brill-Lindquist construction for equal mass, charge BHs
- Wave extraction: $\Phi_2 \equiv F_{\mu\nu}\bar{m}^\mu k^\nu$, $\Psi_4 \equiv C_{\alpha\beta\gamma\delta}k^\alpha \bar{m}^\beta k^\gamma \bar{m}^\delta$




$(\mathcal{B} \equiv 1 - Q^2/M^2)$

# Outline

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# Conclusions

1. Numerically solving Einstein's equations in dynamical situations has the potential to answer important questions.
2. "3+1" numerical framework can be modified for higher $D$: head-on collision of non-spinning black holes in any dimension can be reduced to an effective $3+1$ system with matter.
3. Enables us to study higher-dimensional systems (with enough symmetry) using the existing numerical codes.
   Stability not yet as robust as in $D = 4$
4. We successfully evolved a black hole collision (from rest) in a five-dimensional space-time.
5. BH in de Sitter
   - Evolved head-on collision of BHs in asymptotically de Sitter spacetime and monitored apparent horizons;
   - Studied formation of common apparent horizon as function of initial separation;

# Outline

# To do

1. Boosted black hole collisions in $D \geq 5$;
2. Collisions of spinning black holes;
3. Stability properties of higher dimensional black objects;
4. Collisions of charged BH; does charge play a role in high speed collisions?