Acceleration of particles by black hole horizons: kinematic properties nad near-horizon geometry

Oleg B. Zaslavskii
Kharkov V.N. Karazin National University,
Kharkov, Ukraine

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Plan of talk

Energy in the centre of mass frame and its growth

From particular metrics to generic rotating and charged black holes

Strong and weak versions of effect

Kinematic explanation

Geometric explanation

Unification. General nature of effect and

Collisions near inner horizon

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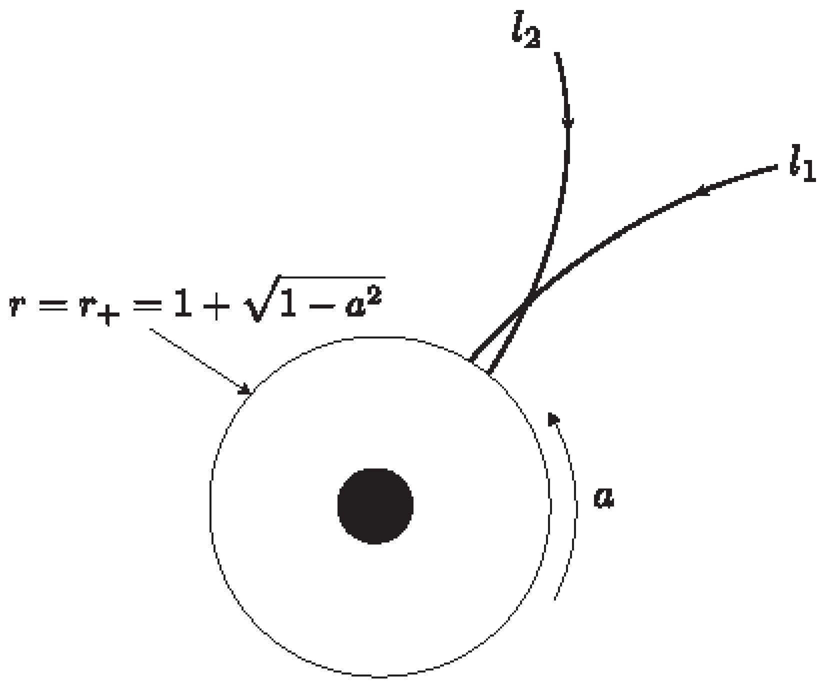

$$-2(1 + \sqrt{1 + a}) < l < 2(1 + \sqrt{1 - a})$$

FIG. 1. Schematic picture of two particles falling into a black hole with angular momentum a (per unit black hole mass) and colliding near the horizon. The allowed range of l for geodesics falling into the black hole is also given.

Banados, Silk, West (BSW effect)

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Energy in centre of mass frame

$$(E_{\text{c.m.}}, 0, 0, 0) = m u^i_{(1)} + m u^i_{(2)},$$

$$u^i u_i = 1$$

$$E_{\text{c.m.}} = m\sqrt{2}\sqrt{1 + u^i_{(1)} u_{(2)i}}$$

$$E^2_{cm} = \left| P_m P^m \right|$$

Total momentum $$P_m = p^{(1)}{}_m + p^{(2)}{}_m$$

Infinite growth near horizon

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Schwarzschild metric (a=0)

Schwarzschild

$E_{cm} = 2m\sqrt{5}$ Baushev 2008 No effect

Kerr (rotation)

Reissner-Nordstrom (charge)

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BSW Kerr metric

Kerr

$$(E_{\rm c.m.}^{\rm Kerr})^2 = \frac{2m_0^2}{r(r^2 - 2r + a^2)}[2a^2(1 + r) - 2a(l_2 + l_1) - l_2 l_1(-2 + r) + 2(-1 + r)r^2 - \sqrt{2(a - l_2)^2 - l_2^2 r + 2r^2}\sqrt{2(a - l_1)^2 - l_1^2 r + 2r^2}].$$

On horizon both numerator and denominator=0

Extremal case: a=1

$$E_{\rm c.m.}^{\rm Kerr}(r \to r_+) = \sqrt{2}m_0\sqrt{\frac{l_2 - 2}{l_1 - 2} + \frac{l_1 - 2}{l_2 - 2}}$$

critical value $l = 2$

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BSW: Extremal Kerr

Grib and Pavlov: also nonextremal

O. Z. generic rotating BH, charged BH (even for radial motion)

BSW effect: both particles move toward horizons

needs fine-tuning between energy and momentum

2009

PS effect: 1 of particles moves away from horizon
Always exists

T. Piran and J. Shanam,

1977

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Acceleration of particles as universal property of rotating black holes

O. Z., PRD 2010

Role of horizon

Universality of black hole physics

Unified approach to nonextremal versus extremal black holes

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For geodesic motion

$$\frac{E_{cm}^2}{2m^2} = c + 1 - Y,\ c = \frac{X}{N^2}$$

$$X = X_1 X_2 - Z_1 Z_2,$$

$$X_i \equiv E_i - \omega L_i,$$

$$Z_i = \sqrt{(E_i - \omega L_i)^2 - N^2 b_i},\ b_i = 1 + \frac{L_i^2}{g_{\phi\phi}},$$

$$Y = \frac{L_1 L_2}{g_{\phi\phi}}.$$

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Let, for generic L_i, one approaches the horizon, so $N \to 0$.

$$\left(\frac{E_{cm}^2}{2m^2}\right)_H = 1 + \frac{b_{1(H)}(L_{2(H)} - L_2)}{2(L_{1H} - L_1)} + \frac{b_{2(H)}(L_{(1)H} - L_1)}{2(L_{2(H)} - L_2)} - \frac{L_1 L_2}{(g_{\phi\phi})_H}, \; L_{i(H)} \equiv \frac{E_i}{\omega_H}$$

Critical value:

$$L_1 = L_{1(H)}(1 - \varepsilon), \; \varepsilon \ll 1, \; L_2 \neq L_{2(H)}$$

$$(X_1)_H = E_1 - w_H L_1 = 0$$

$$\left(\frac{E_{cm}^2}{2m^2}\right)_H \approx \frac{b_{1(H)}(L_{2(H)} - L_2)}{2L_{1(H)}\varepsilon}.$$

$$\lim_{L_1 \to L_{1(H)}} \lim_{N \to 0} E_{cm} = \infty.$$

Unbound growth

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Strong and weak versions

Extremal

$$\lim_{H} E_{c.m.}(1critical) \rightarrow \infty$$

Proper time

$$\tau \sim \int \frac{dlN}{Z} \sim l \rightarrow \infty$$

Actual collision with infinite E does NOT occur

Near horizon energy is finite but can be made as large as one likes (weak version)

$X_1 : N^2$

Cannot approach horzion

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Nonextremal case

$$Z_1 = \sqrt{X^2_1 - N^2 b} \qquad X_1 > \sqrt{b} N$$

Critical

$$X_1 = E_1 - w L_1 = E_1 (1 - \frac{w}{w_H})$$

near horizon

$$\omega = \omega_H + BN^2 + ...$$

cannot reach horizon

However, if $X_1 : d : N$ $\qquad E^2_{c.m.} : d$

Energy is finite but can be made as large as one likes (weak version)

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Kinematic censorship

only weak version can be realized

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Kinematic explanation

$$E_{\mathrm{c.m.}}^2 = -(p_1^\mu + p_2^\mu)(p_{1\mu} + p_{2\mu}) = m_1^2 + m_2^2 - 2m_1 m_2 u_1^\mu u_{2\mu}.$$

$$\gamma = -u_1^\mu u_{2\mu} = \frac{1}{\sqrt{1-w^2}}$$

The effect of unbound energies occurs if $w \longrightarrow 1$, so $\gamma \longrightarrow \infty$.

$$w^2 = 1 - \frac{(1-v_1^2)(1-v_2^2)}{[1 - v_1 v_2(\vec{n}_1 \vec{n}_2)]^2}.$$

Infinite grow if particle 1 is critical, particle 2 – is usual

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Conserved quantity

$$E - \omega L = \frac{N}{\sqrt{1 - v^2}},$$

V measured by ZAMO

Usual particle: $X_H = E - w_H L \neq 0 \quad N \to 0, v \to 1$

critical: $X = E - w L \to 0 \quad N \to 0, v \to v_H < 1$

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Extension to collision of photons and electrons

For photons: interplay between gravitational blue shift and Doppler effect

BSW effect

Either critical photon and usual electron

or critical electron and usual photon

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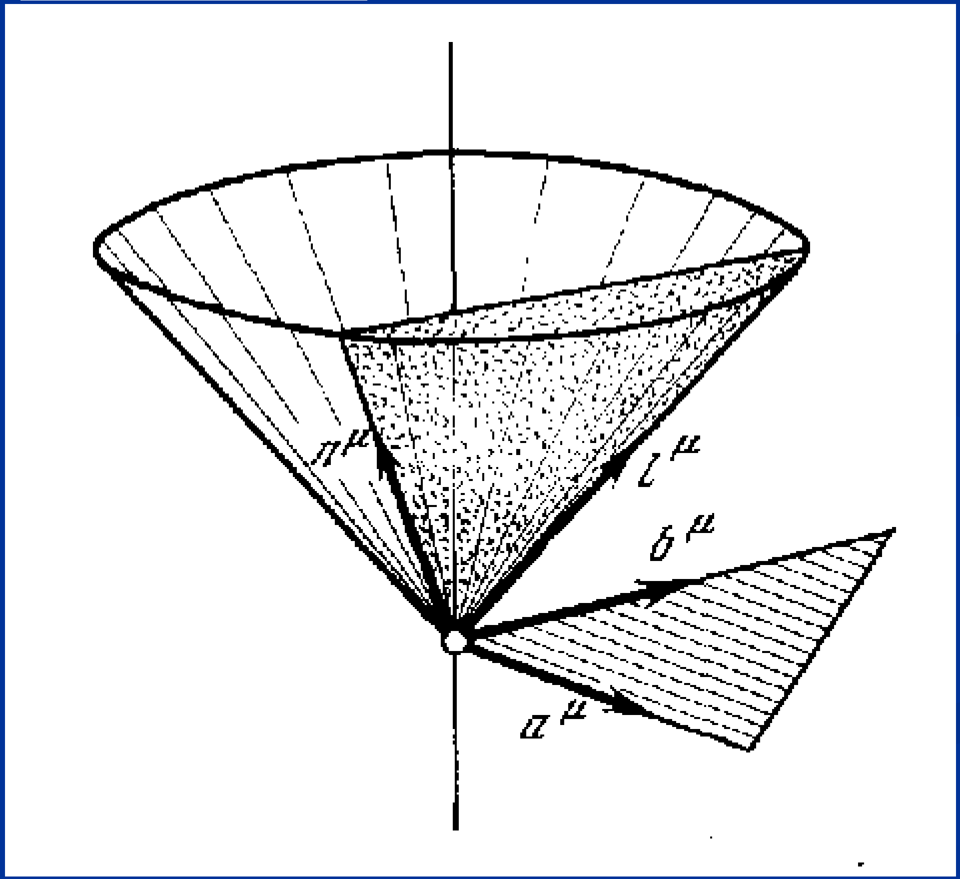
General geometric explanation

$$g_{\alpha\beta} = -l_\alpha N_\beta - l_\beta N_\alpha + \sigma_{\alpha\beta}$$

lightlike vectors l^μ, N^μ

spacelike vectors a^μ, b^μ orthogonal to them



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Four-velocity

$$u_i^\mu = \frac{l^\mu}{2\alpha_i} + \alpha_i N^\mu + s_i^\mu,\ s_i^\mu = A_i a^\mu + B_i b^\mu$$

$$-(u_1 u_2) = \frac{1}{2}(\frac{\alpha_1}{\alpha_2} + \frac{\alpha_2}{\alpha_1}) - (s_1 s_2).$$

$a = 0$

Case 1
always

For head-on collision means that particle cannot cross horizon

$$E_{c.m.}^2 = m_1^2 + m_2^2 + m_1 m_2[\frac{\alpha_1}{\alpha_2} + \frac{\alpha_2}{\alpha_1} - 2(s_1 s_2)].$$

Case 2
Special condition

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Case 2

$\alpha_1 \to 0$

Now, special condition

$$E^2_{c.m.} \to \infty.$$

Kruskal-like coordinates

$$ds^2 = -CdXdY + \gamma_{ab}dx^a dx^b$$

$$Cu^X u^Y = 1$$

$u^X \sim \alpha \to 0.$

$\tau \sim -\ln X \to \infty$

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Combined explanation

General, not necessarily geodesic motion near horizon

$$ds^2 = -N^2 dt^2 + g_{\phi\phi}(d\phi - \omega dt)^2 + dl^2 + g_{zz} dz^2.$$

$$h_{(0)\mu} = -N(1,0,0,0),$$

$$h_{(1)\mu} = (0,1,0,0)$$

$$h_{(2)\mu} = \sqrt{g_{zz}}(0,0,1,0)$$

$$h_{(3)\mu} = \sqrt{g_{\phi\phi}}(-\omega,0,0,1)$$

ZAMO

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$$u^\mu = \frac{l^\mu}{2\alpha} + \beta N^\mu + s^\mu,\; s^\mu = Aa^\mu + Bb^\mu,$$

$$v^{(1)} = \frac{N^2 - \alpha\beta}{N^2 + \alpha\beta},$$

$$v^{(2)} = 2(h_{(2)}s)\frac{N\alpha}{\alpha\beta + N^2},$$

$$v^{(3)} = \frac{2LN}{\sqrt{g_{\phi\phi}}}\frac{\alpha}{N^2 + \alpha\beta}.$$

Horizon limit: $v^{(1)} \to -1,\; v^{(2)} \to 0,\; v^{(3)} \to 0,\; v \to 1$

for usual particle

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$$v^{(i)} = v_{(i)} = \frac{u^\mu h_{\mu(i)}}{-u^\mu h_{\mu(0)}}.$$

$$u^\mu = \frac{l^\mu}{2\alpha} + \beta N^\mu + s^\mu,\ s^\mu = Aa^\mu + Bb^\mu,$$

Critical particle: $\beta \approx c_1 N$ $\alpha \approx c_2 N$

Then, v remains finite and previous explanation applies.

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Universality:

1) horizon and
2) presence of a critical particle

Gravitational radiation, backreaction

E. Berti, V. Cardoso, L. Gualtieri, F. Pretorius, and U. Sperhake, Phys. Rev. Lett. **103**, 239001 (2009).

T. Jacobson and T. P. Sotiriou, Phys. Rev. Lett. **104**, 021101 (2010).

Complicates picture but does not abolish effect

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Collision on inner horizon

$$ds^2 = -\frac{dT^2}{g(T)} + g(T)dy^2 + T^2 d\omega^2.$$

$$\frac{E_{cm}^2}{2m^2} = 1 + \frac{Z_1 Z_2 - X_1 X_2}{gm^2}.$$

$$Z_i = \sqrt{X_i^2 + m^2 g},$$

$X_1 X_2 > 0$ BSW effect

$X_1 X_2 < 0$ PS effect

$X_1 X_2 = 0$ intermediate

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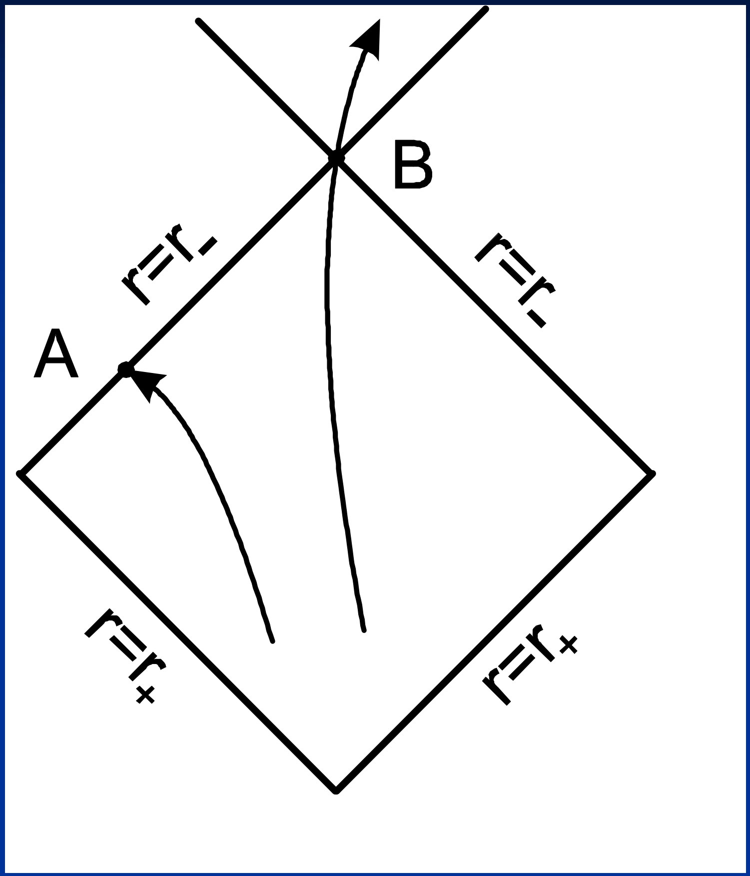
Collisions on inner horizon



Critical particle
From T region inside to
T region outside
passes through bifurcation point

Figure 1. Impossibility of the strong version of the BSW effect. The critical particle 1 passes through the bifurcation point whereas a usual one 2 hits the left horizon.

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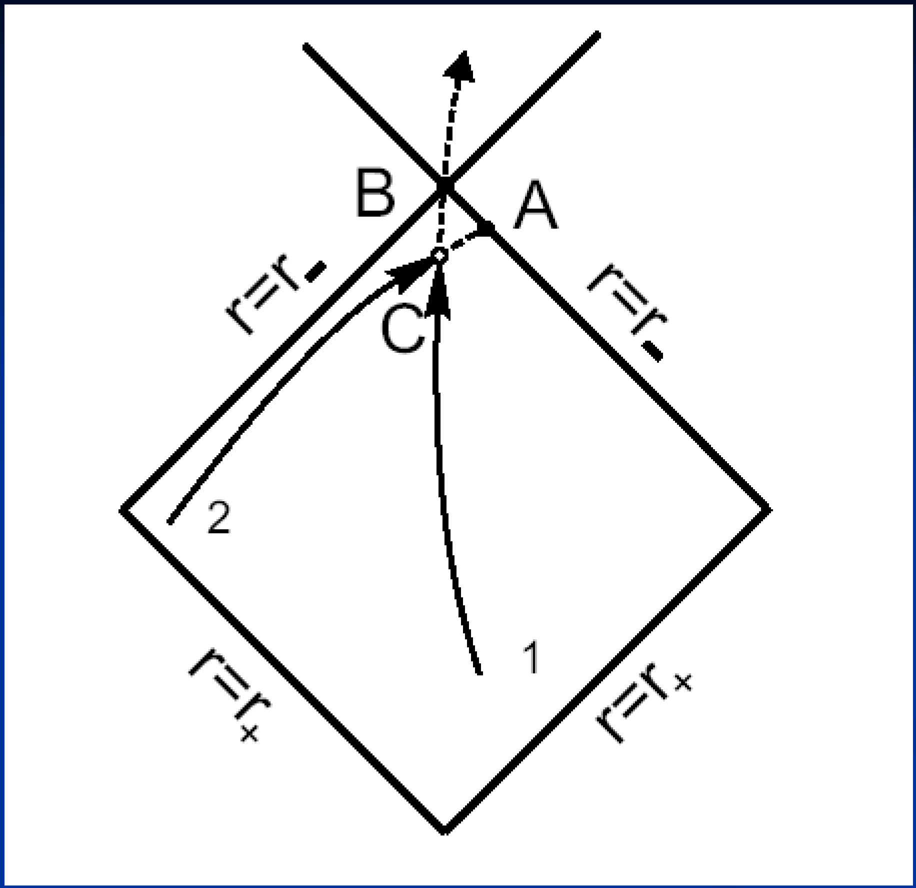

Figure 2. The weak version of the BSW effect. Near-horizon collision between critical particle 1 and usual one 2.

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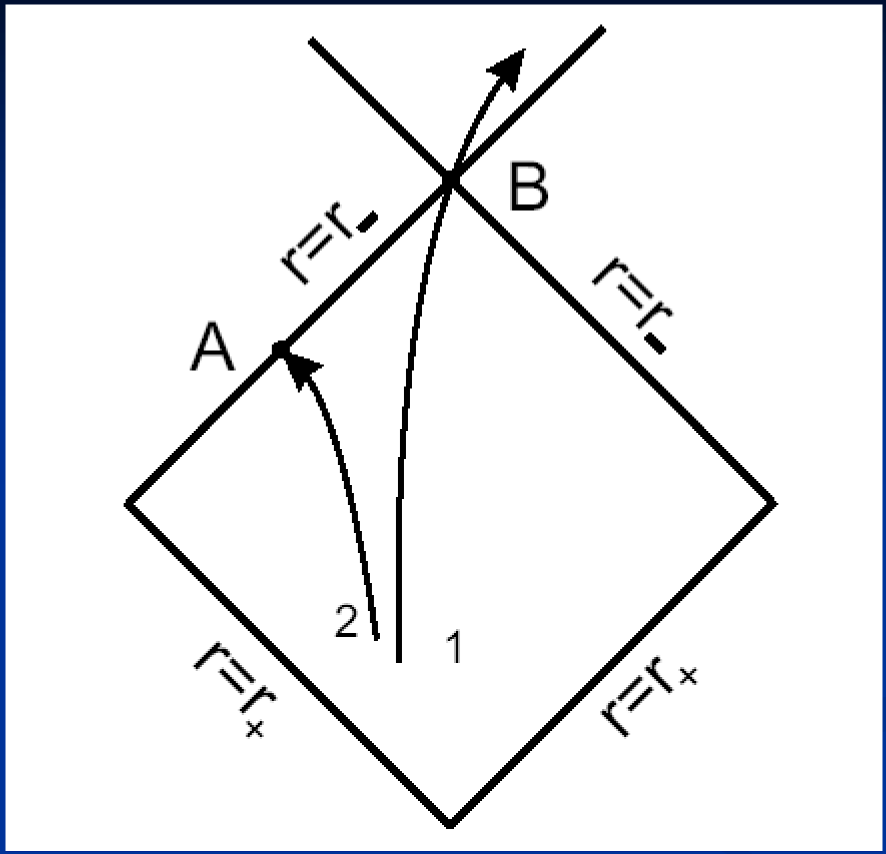

Figure 3. Impossibility of the strong version of the PS effect. The critical particle 1 passes through the bifurcation point whereas a usual one 2 hits the left horizon.

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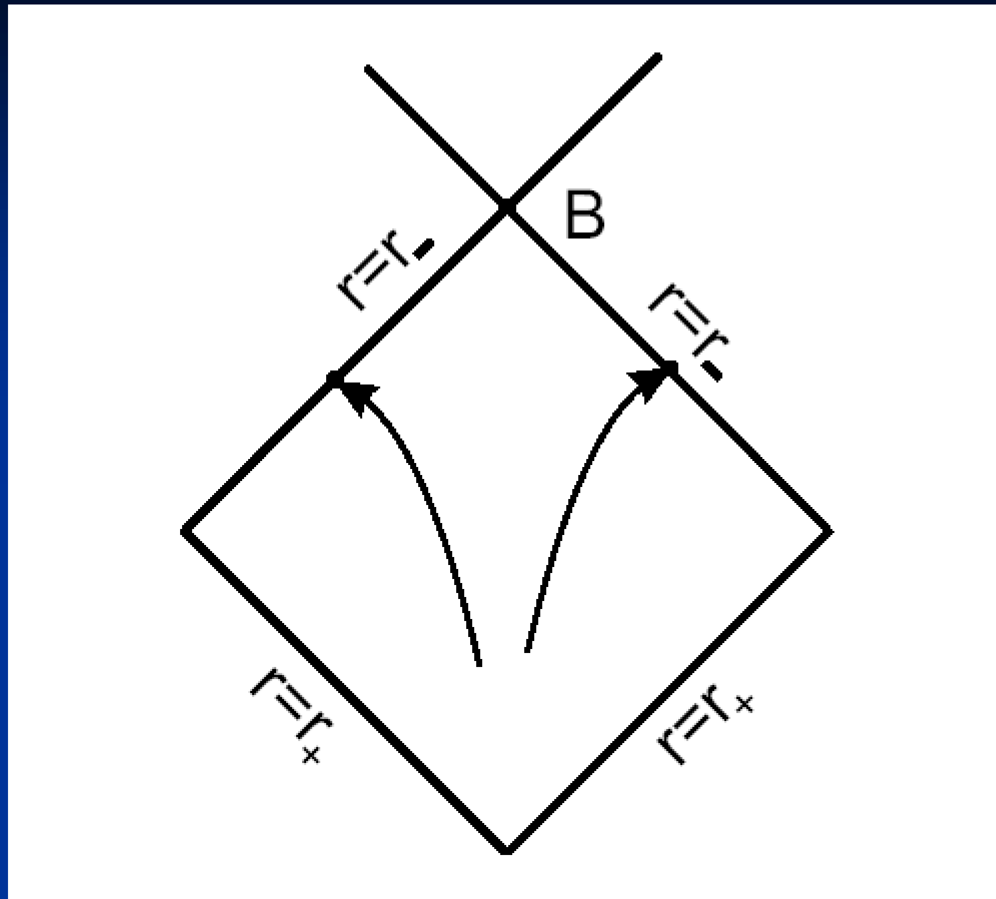

Figure 4. Impossibility of the strong version of the PS effect. Two usual particles hit different branches of the horizon.

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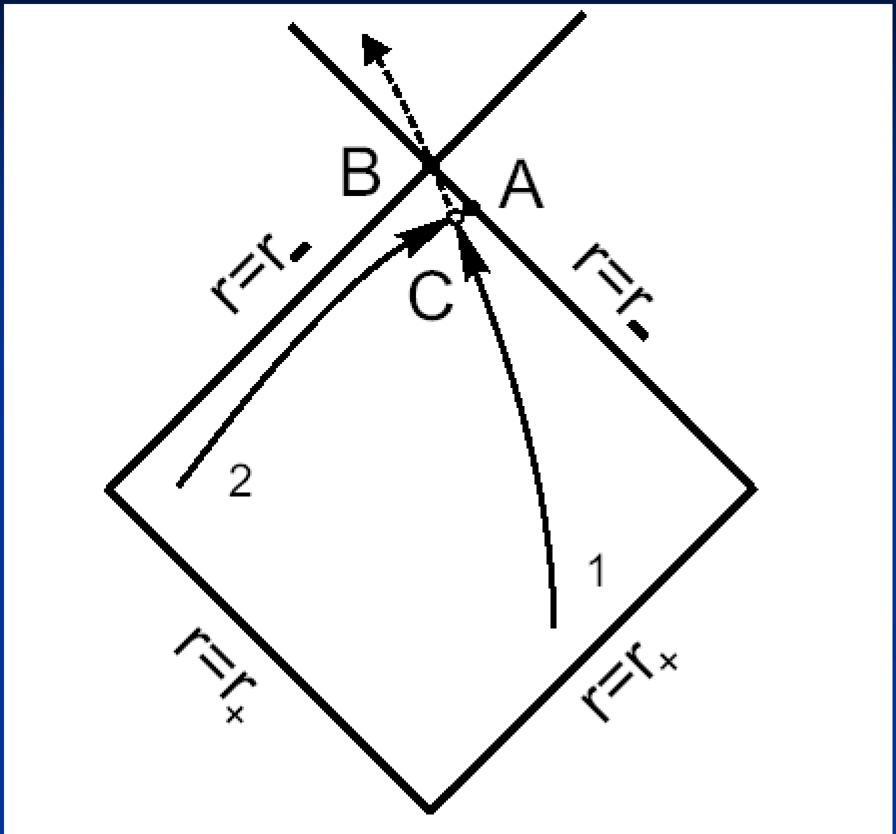

Figure 5. The weak version of the PS effect. Near-horizon collision between critical particle 1 and usual one 2.

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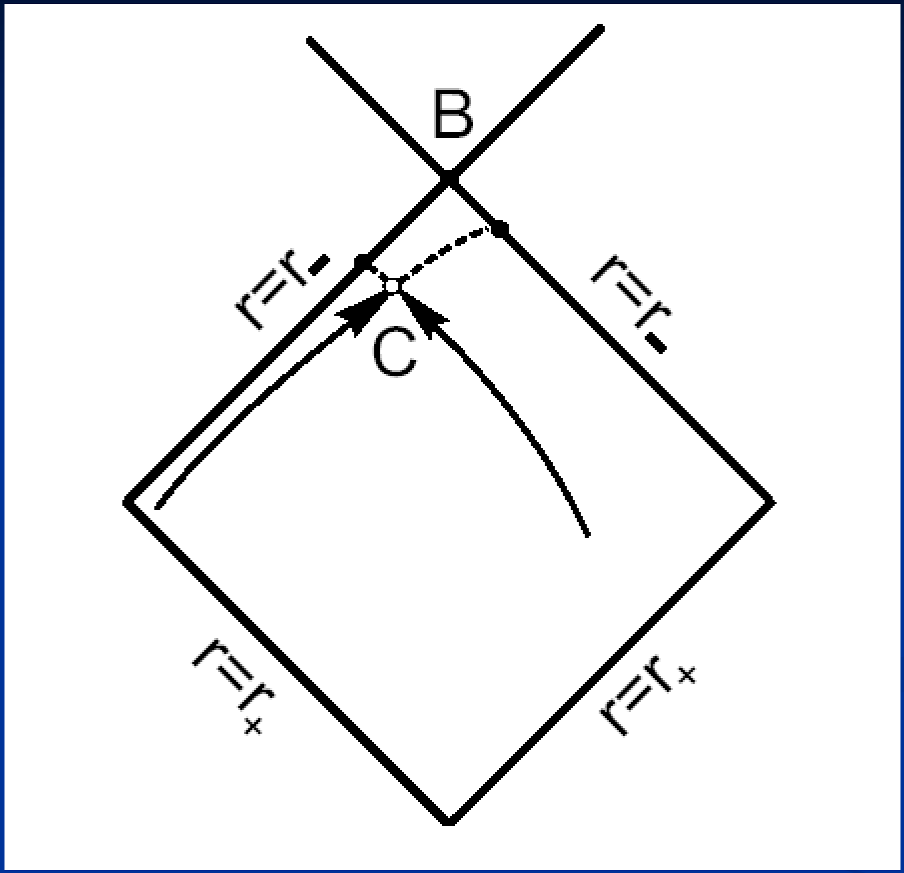

Figure 6. The weak version of the PS effect. Collision between two usual particles near the left horizon.

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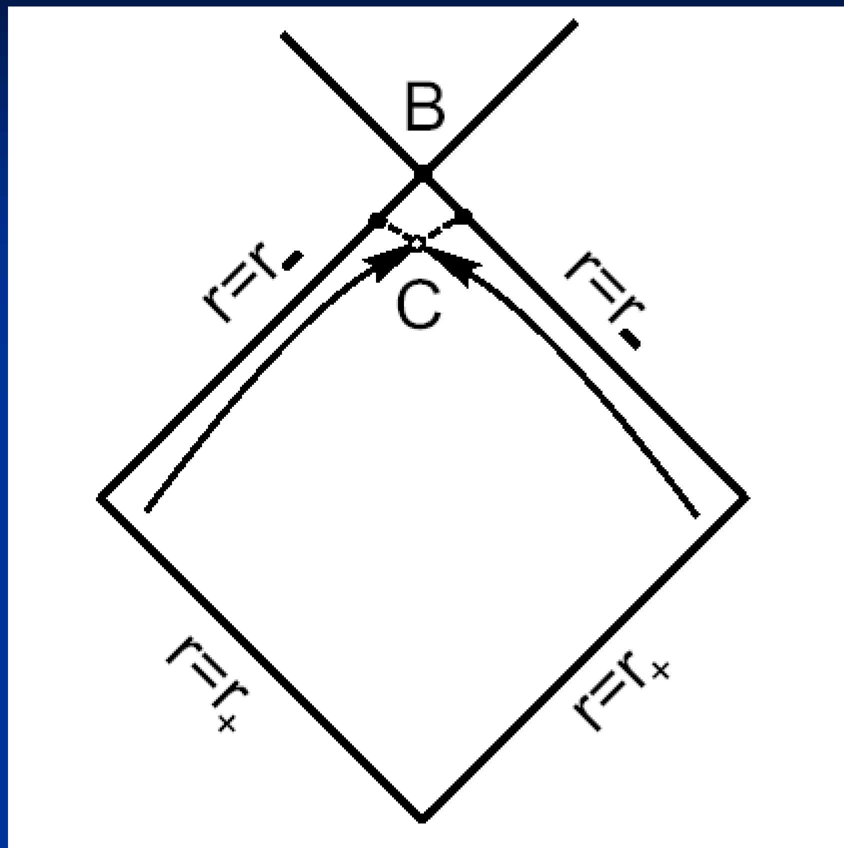

Figure 7. The weak version of the PS effect. Collision between two usual particles near the bifurcation point.

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$Lim(SO) \neq FO$

Summary

- BSW effect – generic feature of rotating and charged BH
- Key property: presence of event horizon plus critical partcile
- Similar effect for inner horzion (BSW and PS on equal footing)
- Only weak version manifests itself in nature
- Explanation based on kinematics and geometry – universality
- Role of gravitation and backreaction not quite cear but these factors seem not to destroy effect itself

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Thank you!

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