Scattering by acoustic holes

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Outline

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Historic Background

The first results based on the black hole scattering study are dated from the 60's, but the history shows it got much attention since Hawking discovered that black holes can evaporate.
This subject is related to many interesting phenomena that occurs around black holes as superradiance, electromagnetic to gravitational (and vice-versa) conversion, quasinormal ringing, glory effect etc.
On 70's, a special decade to the black hole scattering research, most works were done analytically, and restricted to low- and high-frequency limits. Recently, however, this subject has been mainly treated numerically, so that most of these results were obtained for arbitrary values of the incident wave frequency.

Base Works

This presentation is based on the following works:

- L. C. B. Crispino, E. S. O., and G. E. A. Matsas, *Absorption cross section of canonical acoustic holes*, PRD **76**, 107502 (2007);
- S. R. Dolan, E. S. O., and L. C. B. Crispino, *Scattering of sound waves by a canonical acoustic hole*, PRD **79**, 064014 (2009);
- E. S. O., S. R. Dolan, and L. C. B. Crispino, *Absorption of planar waves in a draining bathtub*, PRD **81**, 124013 (2010);
- S. R. Dolan, E. S. O., and L. C. B. Crispino, *Aharonov-Bohm effect in a draining bathtub vortex*, PLB **701**, 485 (2011).

Outline

Perturbation Behavior

The Main Idea of Analogues

If the fluid is inviscid barotropic and the flux is locally irrotational, a perturbation behaves like the massless scalar field in an effective spacetime [Unruh (1981)].

Klein-Gordon Equation

$$\frac{1}{\sqrt{|g|}}\partial_\mu(\sqrt{|g|}g^{\mu\nu}\partial_\nu\psi) = 0.$$

Then, the perturbation behavior can be determined if the metric $g_{\mu\nu}$ of the effective spacetime is known. Here we will work with two different effective spacetimes [Visser (1998)]:

- the acoustic hole formed in a spherically symmetric fluid flow;
- the (2+1)-dimensional rotating acoustic hole formed in a "draining bathtub".

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The Spacetimes

The Canonical Acoustic Hole

The ‘canonical’ acoustic hole is formed in a fluid which velocity is radial and has intensity $v = cr_h^2/r^2$ (demanded by the continuity equation).

Spherically Symmetric Flux Effective Spacetime

$$ds^2 = \left(1 - \frac{r_h^4}{r^4}\right) c^2 dt^2 - \left(1 - \frac{r_h^4}{r^4}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2).$$

The characteristics of this spacetime are:

- It is static;
- It is asymptotically flat;
- With one event horizon, it is analogue to the Schwarzschild black hole;
- The sound speed is constant.

The Spacetimes

The (2+1)-dimensional Rotating Acoustic Hole

The (2+1)-dimensional rotating acoustic hole is formed in a fluid which flux velocity is $\mathbf{v} = -(D/r)\hat{r} + (C/r)\hat{\phi}'$.

Draining Bathtub Effective Spacetime

$$ds^2 = -\left(1 - \frac{D^2 + C^2}{c^2 r^2}\right) c^2 dt^2 + \left(1 - \frac{D^2}{c^2 r^2}\right)^{-1} dr^2 - 2C d\phi dt + r^2 d\phi^2.$$

The characteristics of this spacetime are:

- It is stationary;
- It possess an event horizon of radius $r_h = D/c$ and an ergoregion of radius $r_e = \sqrt{D^2 + C^2}/c$;
- It presents supperradiance and is very similar to Kerr black holes;
- The sound speed is constant.

Perturbation Equation

Canonical Acoustic Hole

The treatment of the Klein-Gordon equation around the canonical acoustic hole is very similar to the treatment in the Schwarzschild spacetime. Initially, a separation of variables is done $\psi = (u(r)/r)Y_{lm}(\theta,\phi)e^{-i\omega t}$. The radial part of each partial wave obeys the following (Schrödinger like) equation:

$$\frac{d^2}{dr_*^2}u(r) + \left[\omega^2 - V(r)\right]u(r) = 0,$$

where the effective potential is

$$V(r) = g_{tt}\left(\frac{g'_{tt}}{r} + \frac{l(l+1)}{r^2}\right),$$

and the tortoise coordinate is defined as

$$\frac{d}{dr_*} \equiv g_{tt}\frac{d}{dr}.$$

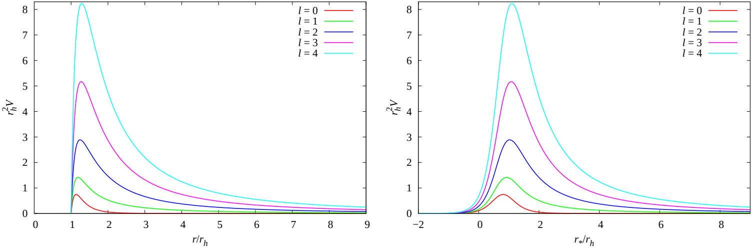

Figure: Effective potential behavior in terms of r (left) and r_* (right).

Knowing the potential behavior it is straightforward to determine asymptotic forms for the radial part of the perturbation. They are

$$u(r) \sim \begin{cases} A_{\rm tr} e^{-i\omega r_*} & (r_* \to -\infty), \\ A_{\rm in} e^{-i\omega r_*} + A_{\rm out} e^{i\omega r_*} & (r_* \to \infty). \end{cases}$$

This asymptotic form is very important, because the phase shifts are defined through it:

$$e^{2i\delta_l(\omega)} = (-1)^{l+1} A_{\rm out}/A_{\rm in}.$$

Perturbation Equation

Rotating Acoustic Hole

For the draining bathtub, the perturbation can be separated as $\psi = (G_{\omega m}(r)/\sqrt{r})e^{im\phi}e^{-i\omega t}$. The radial part obeys the equation

$$\frac{d^2}{dr_*^2}G_{\omega m} + \left\{\left(\omega - \frac{Cm}{r^2}\right)^2 - \frac{1}{g_{rr}}\left[\frac{1}{r^2}\left(m^2 - \frac{1}{4}\right) + \frac{5D^2}{4r^4}\right]\right\}G_{\omega l} = 0, \tag{1}$$

where the tortoise coordinate is now defined as

$$\frac{d}{dr_*} \equiv \frac{1}{g_{rr}}\frac{d}{dr}.$$

Asymptotically, Eq. (1) can be written as:

$$\frac{d^2}{dr_*^2}G_{\omega m} + \left\{\omega^2 - \frac{1}{r_*^2}\left(m^2 + 2mC\omega - \frac{1}{4}\right)\right\}G_{\omega m} = 0.$$

The asymptotic forms of $G_{\omega m}$ are then

$$G_{\omega m} \sim \begin{cases} A_{\text{tr}} \exp\left[-i\left(\omega - \frac{mC}{D^2}\right) r_*\right] & (r_* \to -\infty); \\ A_{\text{in}} e^{-i\omega r_*} + A_{\text{out}} e^{i\omega r_*} & (r_* \to \infty). \end{cases}$$

It is interesting to note that if $\omega < mC/D^2$, energy flux is going out from the acoustic hole.

On these (2+1)-dimensional spacetime, the phase shift is defined as

$$e^{2i\delta_m(\omega)} = i(-1)^m \frac{A_{\text{out}}}{A_{\text{in}}}.$$

Cross Sections

The scattering amplitude is defined on the asymptotic wave behavior:

$$\psi(r) \sim e^{i\omega z} + f_\omega(\theta)\frac{e^{i\omega r}}{r}.$$

In terms of the phase shifts, the scattering amplitude is

$$f_\omega(\theta) = \frac{1}{2i\omega}\sum_{l=0}^{\infty}(2l+1)\left[e^{2i\delta_l(\omega)} - 1\right]P_l(\cos\theta),$$

Scattering Cross Section

$$\frac{d\sigma_{\mathrm{sc}}}{d\Omega} = |f_\omega(\theta)|^2.$$

Absorption Cross Section

$$\sigma_{\mathrm{abs}} = \sum_{l=0}^{\infty}\sigma_{\mathrm{abs}}^{(l)} = \frac{\pi}{\omega^2}\sum_{l=0}^{\infty}(2l+1)\left(1 - \left|e^{2i\delta_l(\omega)}\right|^2\right).$$

For the (2+1)-dimensional scattering, we have

$$\psi \sim e^{i\omega x} + f_\omega(\phi)\frac{e^{i\omega r}}{\sqrt{r}}$$

with the scattering amplitude given by

$$f_\omega(\phi) = \frac{1}{\sqrt{2i\pi\omega}} \sum_{m=-\infty}^{\infty} \left[e^{2i\delta_m(\omega)} - 1\right] e^{im\phi}.$$

Scattering Length

$$\frac{d\sigma_{\text{sc}}}{d\phi} = |f_\omega(\phi)|^2.$$

Absorption Length

$$\sigma_{\text{abs}} = \sum_{m=-\infty}^{\infty} \sigma_{\text{abs}}^{(l)} = \frac{1}{\omega} \sum_{m=-\infty}^{\infty} \left(1 - \left|e^{2i\delta_m(\omega)}\right|^2\right)$$

Methods of Computation

An disadvantage of using the partial wave method to compute the scattering cross section is that a huge number of phase shifts has to be computed. Even if the sum converges, a small numerical imprecision on the phase shifts computation can lead to a wrong scattering cross section form.

- For the canonical acoustic hole, the effective potential behaves like $1/r^4$, so that phase shifts found with the Born approximation can be used to find a convergent result;
- The absorption cross sections follow directly from the numerical phase shifts.

Approximations

It is possible to obtain the following approximations:

- Low-frequency absorption cross section; for both canonical and (2+1)-dimensional rotating acoustic holes, this result obeys the general result of [Higuchi (2001)]: the low-frequency absorption equals the black hole area.
- High-frequency absorption cross section can be obtained through the geodesic approach, as well the weak field deflection angle.
- For the canonical acoustic hole, the glory approximation formula [Matzner et al. (1985)] can be applied:

$$\frac{d\sigma_{\rm sc}}{d\Omega} \approx 2\pi\omega b_g^2 \left|\frac{db}{d\theta}\right|_{\theta=\pi} \left[J_{2s}(b_g\omega\sin\theta)\right]^2 .$$

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Geodesic Analysis

The geodesic analysis can provide us with the high-frequency absorption cross section and weak field deflection. Also, the parameters from the glory formula are found via geodesic study. The geodesic equations are

Canonical Acoustic Hole

$$\left(\frac{du}{d\phi}\right)^2 = \frac{1}{b^2} - u^2 + r_h^4 u^6.$$

Rotating Acoustic Hole

$$\left(\frac{du}{d\phi}\right)^2 = \frac{1}{g_{rr}^2(C + \tilde{b} g_{tt})}\left[1 - \tilde{b}(\tilde{b} + 2C)u^2 + \tilde{b}^2(D^2 + C^2)u^4\right]$$

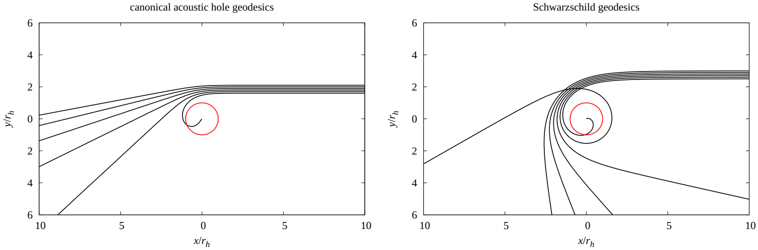

Figure: Geodesics in the canonical acoustic hole spacetime (left) and the Schwarzschild spacetime (right).

	Canonical Acoustic Hole	**Schwarzschild**
r_c/r_h	1.3161	1.5000
b_c/r_h	1.6119	2.5981
$\sigma^{hf}_{\text{abs}}/\pi r_h^2$	2.5981	6.7500
$\sigma^{lf}_{\text{abs}}/\pi r_h^2$	4.0000	4.0000

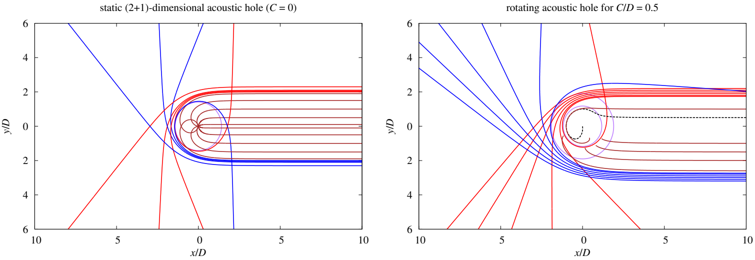

Figure: Geodesics around a (2+1)-dimensional acoustic hole.

For the (2+1)-dimensional rotating acoustic hole, the 'natural' impact parameter is $\tilde{b} = L/E$. The impact parameter is $b = \tilde{b} + C$. 'Radial' geodesics is this spacetime happen when $b = C$ (the same of the equatorial slice of Kerr black holes [Chandrasekhar (1983)]). In high frequency limits

$$\tilde{b}^{\pm} = -2C \pm 2\sqrt{D^2 + C^2}.$$

The total absorption length is $\sigma_{\mathrm{abs}}^{hf} = 4r_e$. $\tilde{b}^{+}$ ($\tilde{b}^{-}$) is the absorption length for corrotating (counterrotating) waves.

Glory

Strong Field Scattering

The glory approximation formula can be applied to find the differential scattering cross section in the limit $\theta \approx 180°$.

Canonical Acoustic Hole

$$r_h^{-2}(d\sigma_{\text{sc}}/d\Omega) = 0.0198\omega r_h \left[J_0(1.61\omega r_h \sin\theta)\right]^2$$

Schwarzschild

$$r_h^{-2}(d\sigma_{\text{sc}}/d\Omega) = 3.38\omega r_h \left[J_0(2.67\omega r_h \sin\theta)\right]^2$$

These two formulas reveal that the glory effect in the canonical acoustic hole is around 170 times smaller then in Schwarzschild black holes. The interference fringes for the acoustic hole are 1.66 wider then for the black holes.

Born Approximation

Canonical Acoustic Hole

It is possible to find analytic phase shifts form through Born Approximation in the canonical acoustic hole case. For this, we write the radial equation asymptotically as:

$$\frac{d^2X}{dr^2} + \left[\omega^2 - \frac{l(l+1)}{r^2} + U(r)\right]X = 0, \qquad \left[X(r) \equiv g_{tt}^{1/2}u(r)\right]$$

where

$$U(r) = \frac{2r_h^2\omega^2}{r^4} - \frac{[l(l+1)-6]r_h^4}{r^6} + \frac{3r_h^8\omega^2}{r^8} + \mathcal{O}(r^{-10}).$$

We can apply now the well known Born formula to find the phase shifts in the weak filed limit:

$$\delta_l(\omega) = -\omega\int_0^\infty r^2[j_l(\omega r)]^2U(r)dr.$$

The phase shifts obtained through Born approximation are

$$\delta_l(\omega) \approx \frac{5\pi(\omega r_h)^4}{32(l+1/2)^3}.$$

We have to note that these phase shifts behave like $(l+1/2)^{-3}$ instead $\ln(l+1/2)$ [Sanchez (1976)] as phase shifts in Schwarzschild spacetime do. This happens because Schwarzschild spacetime behaves as a *Coulombian* scatter on this limit. Also, the phase shifts above say that the differential scattering cross section will be finite even for $\theta \to 0$.
It is also possible to find the deflection angle using the Ford and Wheeler semiclassical description $b = (l+1/2)/\omega$:

$$\Theta = \frac{15\pi}{16}\frac{r_h^4}{b^4},$$

what gives a divergent differential scattering cross section in the limit $\theta \to 0$.

Born Approximation

Rotating Acoustic Hole

We can find the phase shifts in the low-frequency limit by doing $G_{\omega m} = g_{rr}^{1/2} X(r)$ and expanding the equation for $X(r)$ in $1/r$:

$$\frac{d^2X}{dr^2} + \left[\omega^2 - \frac{\bar{m}^2 - 1/4}{r^2} + U(r)\right] X = 0,$$

where $\bar{m}^2 = m^2 + 2\alpha m - 2\beta^2$ and

$$U(r) = \frac{(\alpha^2 - \beta^2)m^2 - 4\beta^2\alpha m + 2\beta^2 + 3\beta^4}{\omega^2 r^4} + \frac{\beta^2(2\alpha^2 - \beta^2)m^2 - 6\beta^4\alpha m + 3\beta^4 + 4\beta^6}{\omega^4 r^6} + O\left(\omega^{-6} r^{-8}\right),$$

with $\alpha = C\omega$ and $\beta = D\omega$.

In order to find the corresponding phase shifts from these equations, we apply the "Born" approximation:

$$\delta_m \approx \frac{\pi}{2}(m-\bar{m}) + \frac{\pi}{2}\int_0^\infty r[J_{\bar{m}}(\omega r)]^2 U(r)dr. \qquad (2)$$

The phase shifts found through this method are

$$\delta_m \cong -\frac{\alpha\pi}{2}\frac{m}{|m|} + \frac{3\pi(\alpha^2+\beta^2)}{8|m|} - \frac{5\alpha\pi(\alpha^2+\beta^2)}{8m^2}\frac{m}{|m|}. \qquad (3)$$

To understand what each term means, it is useful to use the semiclassical approach to the deflection angle. In this case, using that $\Theta = -d(2\delta_m)/dm$ and that $\tilde{b} = m/\omega = b - C$, we find:

$$\Theta = \frac{3\pi}{4b^2}\left(C^2+D^2\right) - \frac{\pi C}{b^3}\left(C^2+D^2\right). \qquad (4)$$

Phase Shift Analysis

What does each term on the phase shift mean?

- The **second term** is related to a symmetric deflection of geodesics. Until second order of $1/b$, geodesics that pass by opposite sides of the black hole are deflected by the same amount.
- The **third term** is related to an asymmetric correction in the deflection angle. Geodesics that pass by opposite sides of the acoustic hole are deflected by different angles.
- The first term is not related to a geodesic deflection, since it does not depend on m and b. Rather, geodesics that pass by opposite sides of the acoustic hole are "time shifted" by $|\Delta t| = 2\pi C$. What role does this term play on the interference pattern?

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The Aharonov-Bohm Effect

The Aharonov-Bohm effect comes from the significance that electromagnetic potentials have on Quantum Mechanics. Classically, a charged particle cannot be affected in a region of null electric $\mathbf{E}$ and magnetic $\mathbf{B}$ fields. These fields can be related to a vector $\mathbf{A}$ and scalar φ potential as:

$$\mathbf{E} = -\nabla\varphi - \frac{\partial \mathbf{A}}{\partial t}, \qquad \mathbf{B} = \nabla \times \mathbf{A}. \tag{5}$$

- It is possible that the fields $\mathbf{E}$ and $\mathbf{B}$ are zero in a region where the potentials $\mathbf{A}$ and φ are not.
- Hamiltonian is written in terms of the potentials:

$$H = \frac{1}{2m}\left(\frac{\hbar}{i}\nabla - q\mathbf{A}\right)^2 + q\varphi. \tag{6}$$

Scattering by a Solenoid

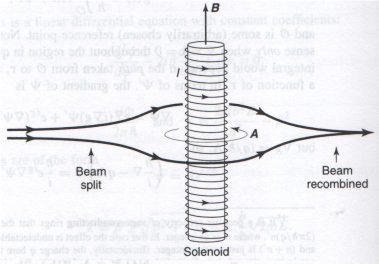

Figure: Schematic scattering by a solenoid [Griffiths (2005)].

For such configuration, it is possible to show that the beams that pass by opposite sides of the solenoid have phases $\eta_{\pm} = \frac{q}{\hbar} \int \mathbf{A} \cdot d\mathbf{r}$. The phase difference between the two beams is

$$\Delta\eta = 2\pi\alpha = \frac{q}{\hbar} \oint \mathbf{A} \cdot d\mathbf{r} = \frac{q\Phi}{\hbar}. \tag{7}$$

In order to determine the scattering length, we need still find the phase shift for the mode $m = 0$, since our analytical results to the phase shift are only valid in the regime $\alpha \ll m$. This mode has phase shift (exact value):

$$\delta_0 = \frac{1}{2} i\pi\beta. \tag{8}$$

We have to make the following considerations:

- This phase shift does not depend on the acoustic hole rotation;
- It is imaginary what means that this mode is absorbed by the acoustic hole;
- In the low-frequency regime, this is the mode that most interacts with the event horizon.

Scattering Length

Considering the most important term of our previous analytical phase shifts in the very low-frequency regime and the $m = 0$ mode, we find that the scattering length is

$$\frac{d\sigma_{\alpha\beta}}{d\phi} = |f_\omega(\phi)|^2 \cong \frac{\pi}{2\omega} \frac{[\alpha\cos(\phi/2) - \beta\sin(\phi/2)]^2}{\sin^2(\phi/2)}. \tag{9}$$

For the case which $\beta = 0$ (non-draining vortex), this result becomes:

$$\frac{d\sigma_{\text{vortex}}}{d\phi} = \frac{\pi\alpha^2}{2\omega}\cot^2(\phi/2), \tag{10}$$

that is symmetric and resembles the Aharonov-Bohm scattering length:

$$\frac{d\sigma_{AB}}{d\phi} = \frac{1}{2\pi\omega}\frac{\sin^2(\pi\alpha)}{\sin^2(\phi/2)}. \tag{11}$$

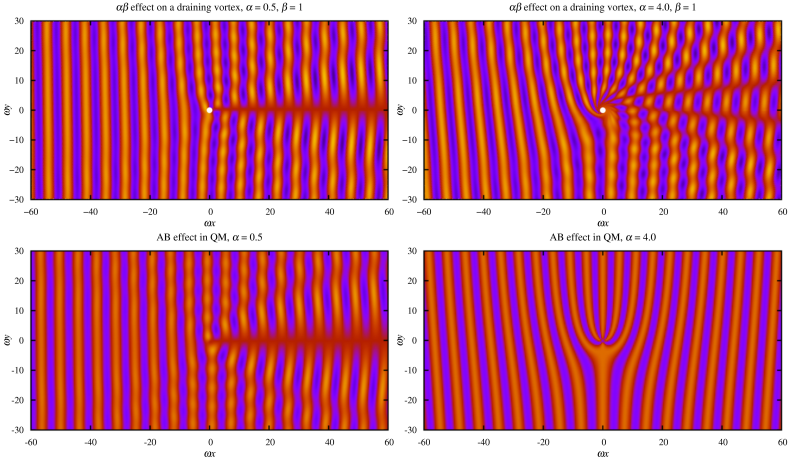

Figure: Wave scheme of the scattering in a draining vortex and by a solenoid. As the effect strengths α increase, the effects behave in different ways. For the Aharonov-Bohm effect, it is possible to have zero scattering length if α is integer. For the draining bathtub there is no zero scattering length, since there is deflection.

Outline

Numerical Computations

It is not possible to find analytic solutions to the radial equations that can be written in terms of special functions which properties are well known. Then,

- to compute the absorption cross section (length), we solve the radial equation numerically and match this solution asymptotically to find the phase shifts.
- to compute the differential scattering cross section of the canonical acoustic hole we use both the numerical and analytic phase shifts together to obtain convergent results. (For Schwarzschild black hole, using the partial wave method, a convergence method is necessary.)
- to check the precision of our results, we compare them, when is possible, with the analytic results.

Canonical Acoustic Hole

Absorption Cross Section

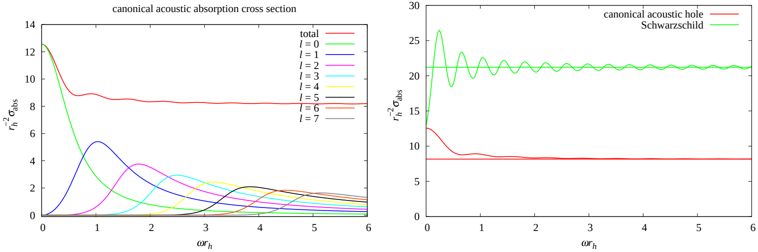

Figure: Absorption for the canonical acoustic hole (left) compared with the Schwarzschild absorption (right).

As expected from the geodesic analysis, the high-frequency absorption cross section is bigger for Schwarzschild black holes. In the low-frequency limit, both results tend to the hole area.

Canonical Acoustic Hole

Differential Scattering Cross Section

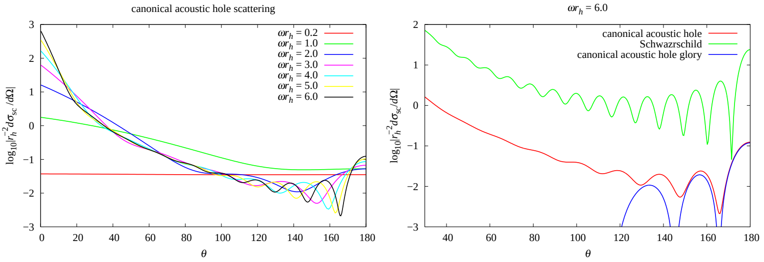

Figure: Canonical acoustic hole differential scattering cross section (left) compared with Schwarzschild and glory results (right). (Note the log scale.)

The scattered flux is much bigger for Schwarzschild black holes. We see the excellent agreement between the numerical result and glory approximation as $\theta \to 180°$. Also, the differential scattering cross section for the canonical acoustic hole is finite.

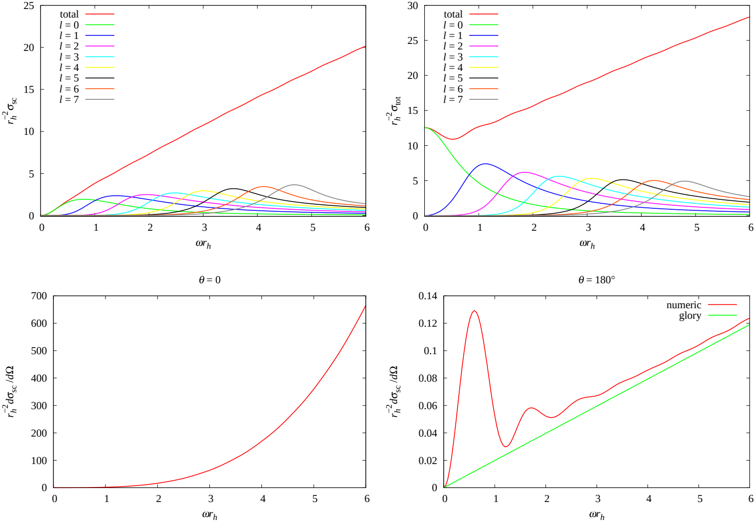

Figure: *Top-left*: Total scattering cross section; *Top-right*: Total cross section; *Bottom-left*: Forward differential scattering cross section; *Bottom-left*: Backward differential scattering cross section.

Rotating Acoustic Hole

Absorption Length

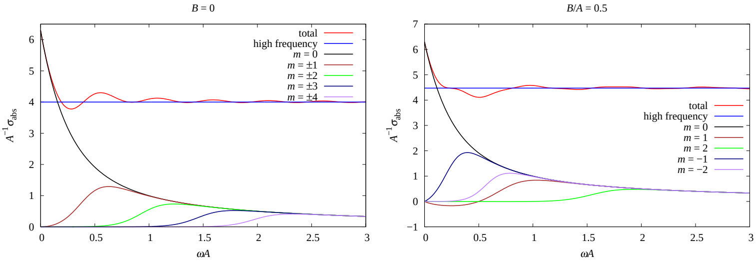

Figure: Absorption length for a (2+1)-dimensional rotating acoustic hole: *Left*: static case ($B = 0$); *Right*: for $B/A = 0.5$.

The absorption length here behaves very similar to the observed from the black holes absorption. However, superradiance may occur, what makes the absorption have negative values for determined values of frequency.

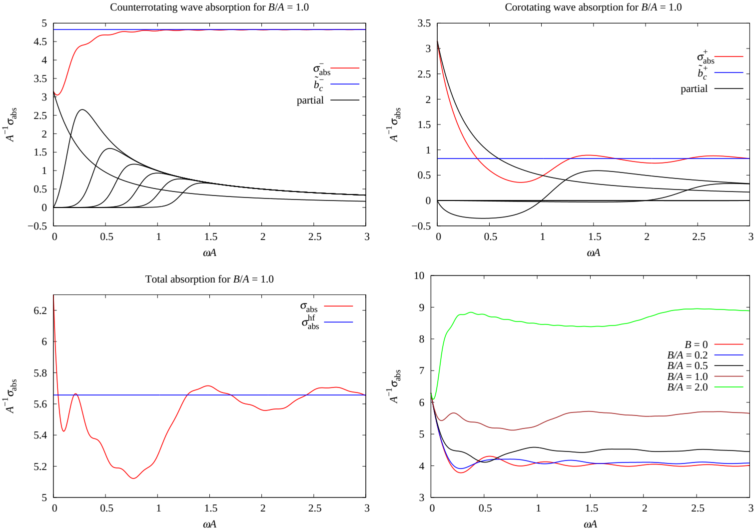

Figure: We can split the absorption length in the absorption of corrotating and counterrotating waves. The corrotating modes will present superradiance (i. e. $|R_{\omega m}|^2 > 1$). The absorption of counterrotating modes will be bigger.

Superradiance

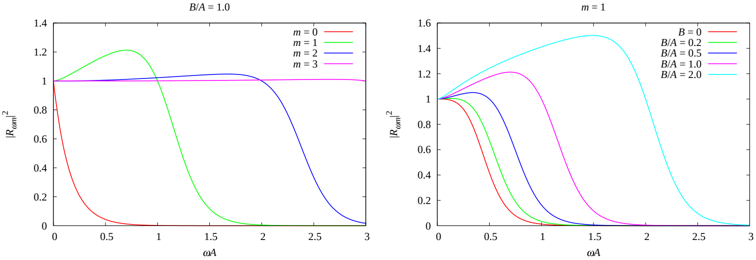

Figure: Numeric results for the reflection coefficient that presents superradiance [Berti et al. (2004)].

Summary

- Here we have showed the scattering properties of acoustic holes using the main tools of General Relativity. The original purpose of analogues was to shed light on black holes study. The study presented here is an excellent example that the opposite can happen.
- In contrast to Schwarzschild black hole scattering, the scattering cross section of the canonical acoustic hole is finite. This happens because the canonical acoustic hole potential falls off with r^{-4}, while for Schwarzschild black holes the potential goes with r^{-1}.
- For the draining bathtub we showed that (i) counterrotating modes are preferentially absorbed by the hole; (ii) superradiance occurs in corotating modes, and is most significant for $m = 1$; (iii) superradiance is insufficient to lead to a negative total absorption length.

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Outlook

- The scattering properties of the draining bathtub is being analyzed for arbitrary values of the wave frequency.
- We hope extend these studies for more realistic models, expecting some they our results can be observed in laboratory.
- At high frequencies, when the wavelength becomes comparable to the interatomic distance in the fluid, new behavior may be observed in the scattering properties of acoustic holes.

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