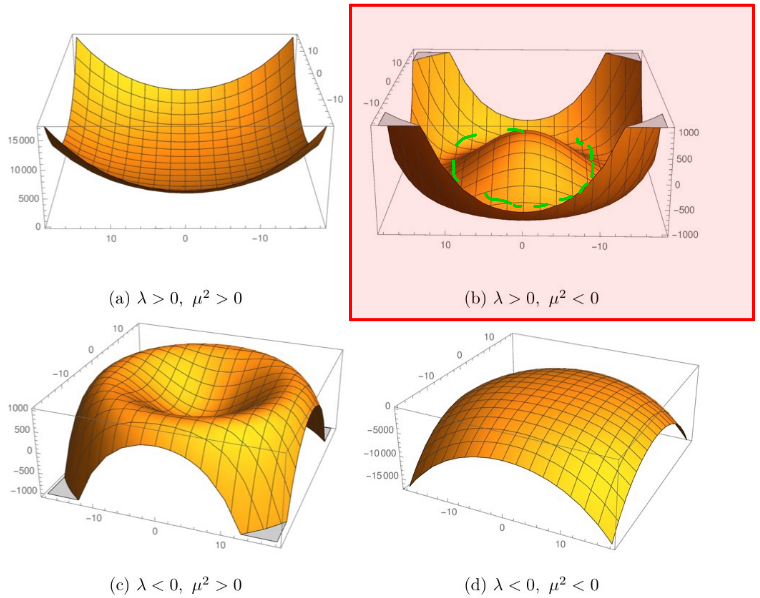
STANDAR MODEL II

LECTURE 7

$$V(\phi, \phi^*) = \mu^2 \phi^* \phi + \frac{1}{4} \lambda (\phi^* \phi)^2$$

$$\phi = \frac{1}{\sqrt{2}} (\varphi_1 + i \varphi_2)$$



(a) $\lambda > 0,\ \mu^2 > 0$ (b) $\lambda > 0,\ \mu^2 < 0$

(c) $\lambda < 0,\ \mu^2 > 0$ (d) $\lambda < 0,\ \mu^2 < 0$

$$m_h^2 = \frac{1}{2} \lambda v^2$$

$$\boxed{m_\sigma^2 = 0}$$

$$v^2 = \varphi_1^2 + \varphi_2^2$$

$$U(1) \longrightarrow \mathbb{Z}_2$$

GOLDSTONE THEOREM

Whenever a continuous symmetry group G of $V(\varphi_1, \dots, \varphi_n)$ is broken to a smaller group $H \subset G$, the final scalar potential $V(h_1, \dots h_k, G_1, \dots, G_m)$ will contain ONE MASSLESS scalar for EACH symmetry generator T^a that is broken.

If G has n generators, T^a with $a = 1, \dots, n$, then if $H \subset G$ has $n-m$, $n > m$, unbroken generators, there will be m Goldstone bosons. $k = n-m$ and is the number of physical scalars / Higgs bosons.

Demonstration

Any complex representation can always be transformed in a real one by doubling the dimension of the corresponding vector space.

- $\Phi = \frac{1}{\sqrt{2}}(\varphi_1 + i\varphi_2)$: $\Phi \longrightarrow (\varphi_1, \varphi_2)$
- $\Phi_1, \dots, \Phi_k \longrightarrow (\varphi_1, \varphi_2, \dots, \varphi_{2k}) \equiv \phi$ Real $O(n)$

$$\mathcal{L} = \underbrace{\frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi)}_{\text{Invariant under } G \text{ (also } O(n))}$$

G has n generators $T^{a=1,\dots,n}$

Infinitesimal transformations:

$$\delta\phi = i\,\omega_a T^a \phi$$

$$(i\,q\,\alpha\,\phi)$$

$V(\phi)$ is invariant under $G \Longrightarrow V(\phi) = V(\phi')$

$$0 = \delta V = \sum_i \frac{\partial V}{\partial \phi^i}\,\delta\phi^i = i \sum_{i,j} \frac{\partial V}{\partial \phi^i}\,\omega_a T^a_{ij}\,\phi^j$$

Since ω_a are arbitrary we obtain

n equations:

$$\boxed{\sum_{i,j} \frac{\partial V}{\partial \phi^i} T^a_{ij} \phi^j = 0 \quad , \quad a = 1, \dots, n}$$

$V(\phi^1, \phi^2, \dots, \phi^n)$

Let's now drop the sums → All repeated indices are summing!

n equations: $\frac{\partial V}{\partial \phi_i} T^a_{ij} \phi^j = 0$

Let's differentiate these equations:

$$\frac{\partial^2 V}{\partial \phi_i \partial \phi_k} T^a_{ij} \phi^j + \frac{\partial V}{\partial \phi_i} T^a_{ij} \underbrace{\frac{\partial \phi^j}{\partial \phi_k}}_{= \delta^{jk}} = 0 \quad \Longleftrightarrow$$

$$\frac{\partial^2 V}{\partial \phi_i \partial \phi_k} T^a_{ij} \phi^j + \frac{\partial V}{\partial \phi_i} T_{ik} = 0$$

what happens in the minimum?

$$\boxed{\left.\frac{\partial^2 V}{\partial \phi_i \partial \phi_k}\right|_{\langle \phi_i \rangle = v_i}} T^a_{ij} \phi^j + \underbrace{\left.\frac{\partial V}{\partial \phi_i}\right|_{\langle \phi_i \rangle = v_i} T^a_{ik}}_{=0} = 0 \Longrightarrow$$

$$\boxed{(M^2)^{ik} T^a_{ij} v^j = 0} \quad (*)$$

$\delta\phi = i\omega_a T^a \phi$

Consider a generic orthogonal matrix Q acting on the vacuum v^i

$$Q_{ij} v^j = \left(\delta_{ij} - i\omega_a T^a_{ij} \right) v^j \neq v^i$$

if $T^a_{ij} v^j \neq 0$

For at least one $T^a_{ij} v^j \neq 0$ then the symmetry is spontaneously broken, as the vacuum is no longer preserved.

Consider $H \subset G$ a reamnant symmetry in the vaccum v^i such that H contains $m < n$ generators.

For the preserved/unbroken generators the condition $G_{ij} v^j = v^i$ must hold

$\Longrightarrow$ $\boxed{T^a_{ij} v^j = 0}$ $\longrightarrow$ Unbroken generators

Two possibilities are left:

a) $T^a_{ij} v^j = 0$ and nothing can be concluded about $(M^2)^{ik}$

$$\boxed{(M^2)^{ik} T^a_{ij} v^j = 0}$$

b) $T^a_{ij} v^j \neq 0$ then $(M^2)^{ik}$ has a zero eigenvalue for each of the generators that break the symmetry

These are the Goldstone bosons!

$\boxtimes$

$V(h, \sigma)$ $\phi(x) = \frac{1}{\sqrt{2}}\left[v + h(x) + \sigma(x)\right]$

$$\boxed{\phi(x) = \frac{1}{\sqrt{2}}\left(v + h(x)\right) e^{i\frac{\theta(x)}{v}}}$$

to first order in θ $\longrightarrow$ $\phi(x) \simeq \frac{1}{\sqrt{2}}\left(v + h(x) + i\,\theta(x)\right)$

$$V(\phi^*, \phi) = \mu^2 \phi^* \phi + \frac{1}{4}\lambda (\phi^* \phi)^2$$

$$V(h) = \frac{1}{16}\lambda h^4 + \frac{1}{4}\lambda v h^3 + \underbrace{\frac{1}{4}\lambda v^2}_{\text{mass}} h^2 - \frac{1}{16}\lambda v^4$$

In this example we have U(1) global

$$\mathcal{L} = (\partial\phi)^* \partial\phi - V(\phi^*, \phi)$$

$$\underbrace{(\partial_\mu \phi)^*(\partial^\mu \phi)}_{|\partial_\mu \phi|^2} = \frac{1}{2}\left| \partial_\mu h\, \cancel{e^{i\theta/v}} + (v+h)\frac{i}{v}\partial_\mu \theta\, \cancel{e^{i\theta/h}} \right|^2$$

$$= \frac{1}{2}\left[(\partial h)^2 + (\partial \theta)^2 \frac{(v+h)^2}{v} \right]$$

$$= \frac{1}{2}\left[(\partial h)^2 + (\partial \theta)^2 \right] + \frac{1}{v}(\partial \theta)^2 h + \frac{1}{2v^2}(\partial \theta)^2 h^2$$

$(\partial \theta)^2 = \partial_\mu \theta\, \partial^\mu \theta$, $(\partial h)^2 = \partial_\mu h\, \partial^\mu h$

Let's look to the cubic term $(\partial \theta)^2 h$

$$\partial_\mu \left[\theta (\partial^\mu \theta) h \right] = 0 \longrightarrow \text{surface term}$$

SC:

$$0 = (\partial_\mu \theta)(\partial^\mu \theta) h + \underbrace{\cancel{\theta\, \partial_\mu \partial^\mu \theta\, h}}_{\theta \Box \theta h} + \theta (\partial^\mu \theta)(\partial_\mu h)$$

K.G. for θ is $\Box \theta = 0$, $m_\theta^2 = 0$

$$(\partial_\mu \theta)(\partial^\mu \theta) h = -\theta (\partial^\mu \theta)(\partial_\mu h) = -\frac{1}{2} \partial^\mu (\theta^2)(\partial_\mu h)$$

$$\partial^\mu (\theta^2) = 2\theta (\partial^\mu \theta) \Longrightarrow \theta (\partial^\mu \theta) = \frac{1}{2} \partial^\mu (\theta^2)$$

Use again a surface term:

$$\partial^\mu\left[\theta^2(\partial_\mu h)\right]=0 \Rightarrow \partial^\mu(\theta^2)(\partial_\mu h)+\theta^2\underbrace{\Box h}_{-m_h^2 h}=0$$

E.O.M. $(\Box+m_h^2)h=0$

$$\Rightarrow \partial^\mu(\theta^2)(\partial_\mu h)=m_h^2\theta^2 h$$

$\circledast$

$$(\partial_\mu\theta)(\partial^\mu\theta)h \simeq (\partial\theta)^2 h = -\frac{1}{2}m_h^2\theta^2 h$$

Similarly:

$$(\partial\theta)^2 h^2 = -m_h^2\theta^2 h^2$$

Finally:

$$(\partial_\mu \phi^*)(\partial^\mu \phi) = \frac{1}{2}\left[(\partial \theta)^2 + (\partial h)^2\right] - \frac{m_h^2}{2v}\, \theta^2 h$$

$$- \frac{m_h^2}{2v^2}\, \theta^2 h^2$$

The Goldstone boson θ is a physical/propagating d.o.f having interactions and kinetic terms.

Explicit symmetry breaking

- Possibility in order to give mass to global Goldstone bosons
- The "problem" is the $U(1)$ symmetry, or any continuous global symmetry in general!

$$V(\phi^*,\phi) \longrightarrow V(\phi^*,\phi) + \underbrace{\mu_1^2\left[\phi^2 + (\phi^*)^2\right]}_{\text{Explicitly breaks } U(1)} \left(+\phi + \phi^3 + c.c.\right)$$

$$\phi \longrightarrow \phi' = \phi e^{-iq\alpha}$$

$$\phi^* \longrightarrow \phi'^* = \phi' e^{iq\alpha}$$

$$\phi^2 \longrightarrow \phi^2 e^{-2i\alpha} \neq \phi^2$$

$$\phi^{*2} \longrightarrow \phi^{*2} e^{2i\alpha} \neq \phi^{*2}$$

The Lagrangian is no longer invariant under U(1)

With the μ_1 terms the new model contains a $Z_2^A \times Z_2^B$ symmetry. $\mu_1^2 = \phi^2 + (\phi^*)^2$

$$Z_2^A \left\{ \phi \longrightarrow -\phi^* \right.$$

$$Z_2^B \left\{ \phi \longrightarrow \phi^* \right.$$

$$\phi^2 \longrightarrow (-\phi^*)^2 = (\phi^*)^2$$

$$(\phi^*)^2 \longrightarrow [(-\phi^*)^*]^2 = \phi^2$$

$$\phi^2 \longrightarrow (\phi^*)^2$$

$$(\phi^*)^2 \longrightarrow [(\phi^*)^*]^2 = \phi^2$$

$$V_{new} = \underbrace{V(\phi^*, \phi)}_{\text{U(1) preserving part}} + \underbrace{\mu_1^2 \left[\phi^2 + (\phi^*)^2 \right]}_{\text{U(1) breaking part}}$$

stationarity points

a) $\phi_1 = 0 \qquad \phi_2 = \pm \dfrac{\sqrt{4\mu_1^2 - 2\mu^2}}{\sqrt{\lambda}}$

b) $\phi_1 = \phi_2 = 0$

c) $\phi_1 = \pm \dfrac{\sqrt{-4\mu_1^2 - 2\mu^2}}{\sqrt{\lambda}} \quad , \quad \phi_2 = 0$

$0 < 2\mu_1^2 < -\mu^2 \quad , \quad \mu^2 < 0$

a) $$m^2 = \begin{pmatrix} \overbrace{4\mu_1^2}^{>0} & 0 \\ 0 & \underbrace{4\mu_1^2 - 2\mu^2}_{>0} \end{pmatrix}$$

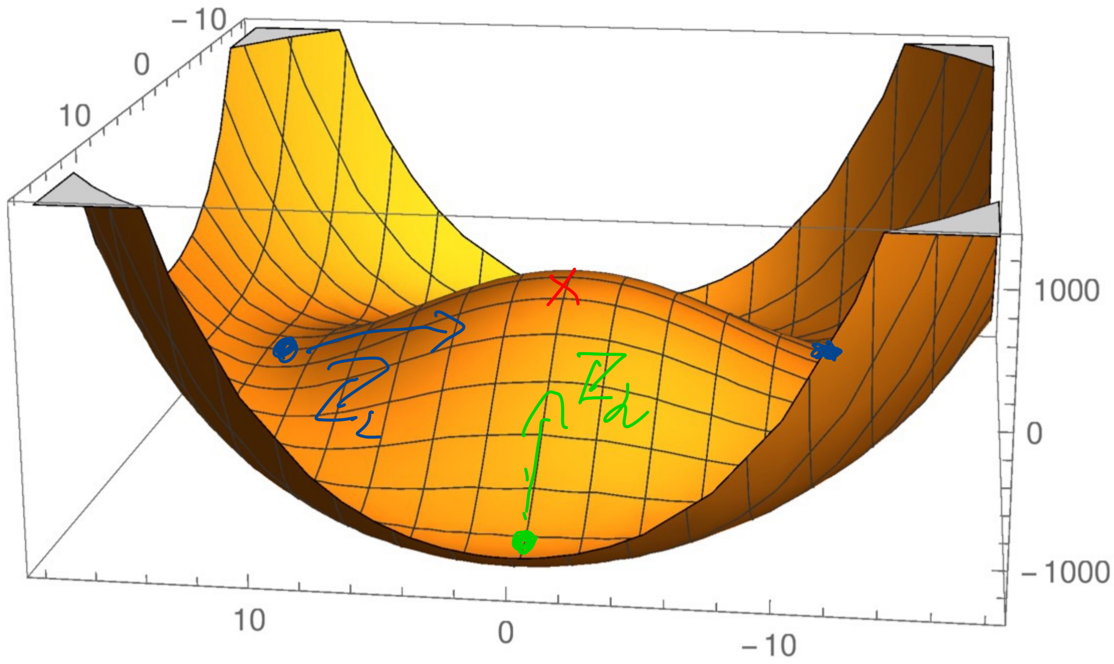
MINIMUM

NO Goldstone boson

if $|\mu_1^2| << |\mu^2|$ then $m_\phi^2 = 4\mu_1^2$ is a PSEUDO-GOLDSTONE

c) $$m^2 = \begin{pmatrix} \overbrace{-4\mu_1^2 - 2\mu^2}^{>0} & 0 \\ 0 & \underbrace{-4\mu_1^2}_{<0} \end{pmatrix}$$

SADDLE POINT



(a) $\lambda > 0,\ \mu^2 < 0$

THE ABELIAN HIGGS MECHANISM

— Goal : Vector boson mass generation

|| Abelian means $U(1)$ local or gauge symmetry

$$\phi \longrightarrow \phi' = \phi\, e^{-iq\alpha(x)}$$

$$\phi^* \longrightarrow \phi'^* = \phi^*\, e^{iq\alpha(x)}$$

Local symmetry implies $\partial_\mu \longrightarrow \mathcal{D}_\mu$

$$\mathcal{L} = [\mathcal{D}^\mu \phi]^* [\mathcal{D}_\mu \phi] - \overbrace{V(\phi^*, \phi)}^{\mu^2 \phi^* \phi + \frac{1}{4}\lambda(\phi^*\phi)^2} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

This is exactly the same

use $\phi = \frac{1}{\sqrt{2}}\left(v + h + i\sigma\right)$

by minimization and rotation to the physical basis

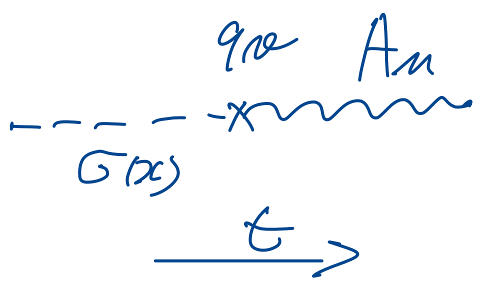
$$\begin{cases} m_h^2 = \frac{1}{2}\lambda v^2 \\ m_\sigma^2 = 0 \\ \mu^2 = \frac{1}{4}\lambda v^2 \end{cases}$$

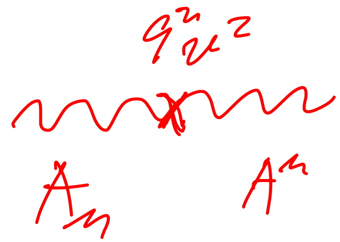
$$(\partial^\mu\phi)^*(\partial_\mu\phi) = \frac{1}{2}\left|(\partial_\mu + iqA_\mu)(v + h(x) + i\sigma(x))\right|^2$$

$$= \frac{1}{2}(\partial h)^2 + \boxed{\frac{1}{2}(\partial\sigma)^2} + \overbrace{\boxed{\frac{1}{2}q^2v^2A^\mu A_\mu}}^{m_A^2\,?} + \overbrace{\boxed{qv(\partial^\mu\sigma)A_\mu}}^{\text{offending}}$$

+ terms with higher powers on the fields (power 3 and 4)

σ and A_μ can transform one into another, thus, we cannot promptly interpret the $A^\mu A_\mu$ term as a mass for the gauge field A_μ !!!





$$\phi(x) = \frac{1}{\sqrt{2}}\left(v + h(x)\right) e^{i\frac{G(x)}{v}}$$

Goldstone as a phase

$$\phi \longrightarrow \phi\, e^{-iq\alpha(x)}$$

$$A_\mu \longrightarrow A_\mu + \partial_\mu \alpha(x)$$

We have gauge freedom to choose $\alpha(x)$

Let's choose our gauge to: $\boxed{\alpha(x) = \frac{G(x)}{qv}}$

gauge choice

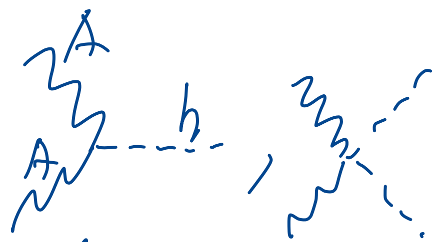
$$\phi = \frac{1}{\sqrt{2}}(v+h)\, e^{i\frac{G(x)}{v}} \longrightarrow \phi' = \frac{1}{\sqrt{2}}(v+h)\, e^{i\frac{G(x)}{v}}\, e^{-iq\overbrace{\frac{G(x)}{qv}}^{\alpha(x)}}$$

$$\Longrightarrow \boxed{\phi' = \frac{1}{\sqrt{2}}(v+h)}$$

$$\boxed{A'_\mu = A_\mu + \frac{1}{qv}\partial_\mu \sigma(x)}$$

Because of gauge invariance we do have

$$\mathcal{L}(\phi, A_\mu) = \mathcal{L}(\phi', A'_\mu)$$



$$(\partial_\mu \phi')^*(\partial^\mu \phi') = \frac{1}{2}(\partial h)^2 + \underbrace{\boxed{\frac{1}{2} q^2 v^2 A'_\mu A'^\mu}}_{\frac{1}{2} m^2_{A'} A'_\mu A'^\mu} + \text{interaction terms}$$

We no longer have a term mixing σ with A_μ

$$\frac{1}{2} q^2 v^2 A'^\mu A'_\mu = \boxed{\frac{1}{2} q^2 v^2 A^\mu A_\mu} + \boxed{\frac{1}{2} (\partial\sigma)^2}$$

$$+ \boxed{q v (\partial^\mu \sigma) A_\mu}$$

Before the breaking : $\text{d.o.f} \longrightarrow \underbrace{2}_{h \quad \sigma} + \overset{A_\mu}{\underbrace{2}_{\text{transverse polarizations of a massless gauge boson.}}}$

After the breaking: $\text{d.o.f} \longrightarrow 1 + 3$ (A^1_μ)

$1 \to h$, ~~σ~~

$3 \to$ transverse and longitudinal polarizations of massive vector field

$V(\phi'^*, \phi') =$ only terms in h

THERE IS NO σ IN THE THEORY ANY LONGER!

σ was "EATEN" by the gauge bosons

Fermion mass generation

$$\underbrace{\text{Abelian Higgs mechanism with chiral fermions}}_{\text{The abelian standard Model}}$$

	$\alpha(x)$ $U(1)_B$ local	$\beta(x)$ $U(1)_C$ local
ϕ	1	1
ψ	1	0
χ	0	-1

$$\phi \longrightarrow \phi\, e^{i(g\alpha(x) + g'\beta(x))}$$

$$\psi \longrightarrow \psi\, e^{ig\alpha(x)}$$

$$\chi \longrightarrow \chi\, e^{-ig'\beta(x)}$$

g and g' are the $U(1)_B$ and $U(1)_C$ gauge couplings respectively

the sizes of g and g' set the

Strenght of the corresponding gauge interactions.

B_μ is the $U(1)_B$ gauge field

C_μ is the $U(1)_C$ gauge field

$F^{\mu\nu} \longrightarrow B^{\mu\nu}$ and $C^{\mu\nu}$

$$\mathcal{L} = [\partial^\mu \phi]^* [\partial_\mu \phi] - V(\phi^*, \phi) \longrightarrow \text{Scalar sector}$$

$$-\frac{1}{4} B^{\mu\nu} B_{\mu\nu} - \frac{1}{4} C^{\mu\nu} C_{\mu\nu} \longrightarrow \text{Maxwell-like sector}$$

$$+ i \bar{\Psi} \gamma^\mu d_\mu \Psi + i \bar{\chi} \gamma^\mu D_\mu \chi \longrightarrow \text{kinetic terms for fermions}$$

$$y \bar{\Psi} \phi \chi + c.c. \longrightarrow \text{Yukawa interaction}$$

	$\bar{\Psi}$	ϕ	χ	
$U(1)_B$	-1	1	0	✓
$U(1)_L$	0	-1	1	✓

$$m \bar{\Psi} \chi + c.c. \Big\} \text{ mass terms are forbidden!!!}$$

$\bar{\Psi}$	χ	
-1	0	✗
0	1	✗

ψ and χ are chiral fermions

LH Weyl spinor

$$\psi_D = \begin{pmatrix} \psi_L \\ \bar{\chi}_L \end{pmatrix}$$

$$\psi = P_L \psi_D = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix}, \quad \chi = P_R \psi_D = \begin{pmatrix} 0 \\ \bar{\chi}_L \end{pmatrix}$$

$$\bar{\psi}\psi = \bar{\chi}\chi = 0$$

$$\mathcal{D}_\mu = \partial_\mu + i g B_\mu + i g' C_\mu$$

$$d_\mu = \partial_\mu + i g B_\mu$$

$$D_\mu = \partial_\mu - i g' C_\mu$$

What happens to the vector bosons masses?

B_μ and C_μ !

$$|\partial_\mu \phi|^2 = \frac{1}{2}\left|\left(\partial_\mu + i g B_\mu + i g' C_\mu\right)(v + h + i\sigma)\right|^2$$

$$= \frac{1}{2} v^2 \left(g^2 B^2 + g'^2 C^2 + g g' B C + g' g C B \right)$$

$$+ g v (\partial^\mu \sigma) B_\mu + g' v (\partial^\mu \sigma) C_\mu$$

Choosing the gauge $\frac{\sigma(x)}{v} = g\,\alpha(x) + g'\,\beta(x)$

for simplicity and using gauge freedom set $\alpha(x) = \beta(x)$

$$\Rightarrow \quad \alpha(x) = \frac{G(x)}{(g+g')v}$$

$$|\partial_\mu \phi'|^2 = \frac{1}{2}\left|\left(\partial_\mu + i g B'_\mu + i g' C'_\mu\right)(v+h)\right|^2$$

$$= \frac{1}{2} v^2 \Big(\underbrace{g^2 B'^2 + g'^2 C'^2 + g g' B'C' + g' g C' B'}_{\text{mass terms for gauge bosons!}} \Big)$$

$\{B, C\}$

$$M^2 = v^2 \begin{pmatrix} g^2 & g g' \\ g g' & g'^2 \end{pmatrix}, \quad O = \begin{pmatrix} \sin\theta & \cos\theta \\ \cos\theta & -\sin\theta \end{pmatrix}$$

Diagonalize M^2 such that

photon-like

$$m^2 = O^T M^2 O = v^2 \begin{pmatrix} 0 & 0 \\ 0 & \boxed{g^2 + g'^2} \end{pmatrix}$$

only 1 massive gauge boson

Z-like boson

$$m_A^2 = 0 \quad , \quad m_Z^2 = v^2 (g^2 + g'^2)$$

$$U(1)_B \times U(1)_C \longrightarrow U(1)_A$$

$$A_\mu = \sin\theta \, B_\mu + \cos\theta \, C_\mu$$

$$Z_\mu = \cos\theta \, B_\mu - \sin\theta \, C_\mu$$

$$g = \frac{e}{\sin\theta}$$

$$g' = \frac{e}{\cos\theta}$$

Fermion masses:

$$\underbrace{y\,\bar{\Psi}\phi\chi}\,, \qquad \phi = \frac{1}{\sqrt{2}}(h+v)$$

$$\frac{y}{\sqrt{2}}\bar{\Psi}h\chi + \underbrace{\frac{y}{\sqrt{2}}v}_{m_f}\,\bar{\Psi}\chi \ldots$$

Let's find the gauge interactions of the fermions in the $U(1)_A$ phase

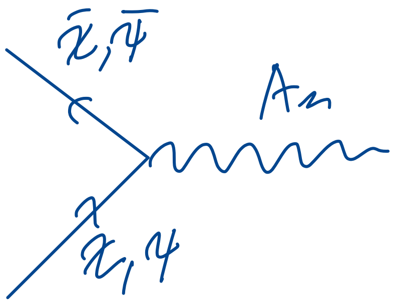
$$i\bar\Psi\gamma^\mu \partial_\mu \Psi + i\bar\chi\gamma^\mu D_\mu \chi =$$

write B_μ and C_μ in terms of A_μ and Z_μ

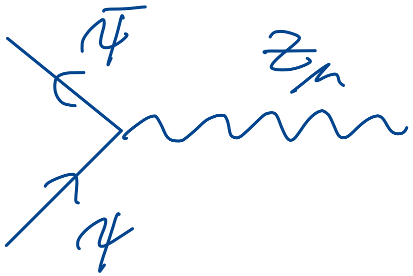
$$= i\bar\Psi\gamma^\mu\partial_\mu\Psi + i\bar\chi\gamma^\mu\partial_\mu\chi + \boxed{e\gamma^\mu A_\mu \bar\chi\chi}$$

$$+ \boxed{e\gamma^\mu A_\mu\bar\Psi\Psi} - \boxed{e\cot\theta\,\gamma^\mu Z_\mu\bar\Psi\Psi}$$

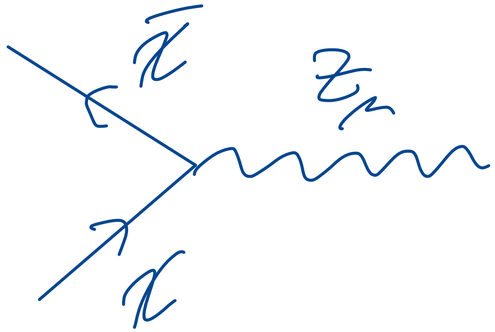
$$+ \boxed{e\tan\theta\,\gamma^\mu Z_\mu\bar\chi\chi}$$



$= -ie\gamma^\mu$,



$= -ie\cot\theta\,\gamma^\mu$



$$= e \tan\theta \, \gamma^\mu$$