STANDARD MODEL I

LECTURE - 4

E.L. $$\boxed{\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = 0}$$

$$\mathcal{L}_\varphi = \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} m^2 \varphi^2$$

φ is real

$$\frac{\partial \mathcal{L}}{\partial \varphi} = -m^2 \varphi$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} = g^{\mu\nu} \partial_\nu \varphi$$

$\xRightarrow{E.L.}$

$$-m^2\varphi - \partial_\mu g^{\mu\nu} \partial_\nu \varphi = 0$$

$$\Leftrightarrow (g^{\mu\nu}\partial_\mu \partial_\nu - m^2)\varphi = 0$$

K.G.

$$[\mathcal{L}] = M^4 \,, \quad [m^2 \varphi^2] = M^4 \Rightarrow [\varphi] = M$$

Complex scalar

$$\phi = \frac{1}{\sqrt{2}}(\varphi_1 + i\varphi_2)$$

$$\mathcal{L}_\phi = g^{\mu\nu}(\partial_\mu \phi)^* \partial_\nu \phi - m^2 \phi^* \phi$$

$$= g^{\mu\nu} \partial_\mu \left[\frac{1}{\sqrt{2}}(\varphi_1 - i\varphi_2)\right] \partial_\nu \left[\frac{1}{\sqrt{2}}(\varphi_1 + i\varphi_2)\right]$$

$$- m^2 \left[\frac{1}{\sqrt{2}}(\varphi_1 - i\varphi_2)\right] \left[\frac{1}{\sqrt{2}}(\varphi_1 + i\varphi_2)\right]$$

$$= \frac{1}{2} g^{\mu\nu} (\partial_\mu \varphi_1) \partial_\nu \varphi_1 + \frac{1}{2} g^{\mu\nu} (\partial_\mu \varphi_2) \partial_\nu \varphi_2$$

$$- \frac{1}{2} m^2 \varphi_1^2 - \frac{1}{2} m^2 \varphi_2^2$$

$$= \mathcal{L}_{\varphi_1} + \mathcal{L}_{\varphi_2}$$

$$\text{E.L.E.} \Longrightarrow \begin{cases} (\Box + m^2)\varphi_1 = 0 \\ (\Box + m^2)\varphi_2 = 0 \end{cases}$$

$$\frac{\partial \mathcal{L}_\phi}{\partial \phi^*} - \partial_\mu \frac{\partial \mathcal{L}_\phi}{\partial(\partial_\mu \phi^*)} = 0 \Longrightarrow (g^{\mu\nu} \partial_\mu \partial_\nu + m^2)\phi = 0$$

$$\frac{\partial \mathcal{L}_\phi}{\partial \phi} - \partial_\mu \frac{\partial \mathcal{L}_\phi}{\partial(\partial_\mu \phi)} = 0 \Longrightarrow (g^{\mu\nu} \partial_\mu \partial_\nu + m^2)\phi^* = 0$$

Discussed a theory for a free scalar with mass

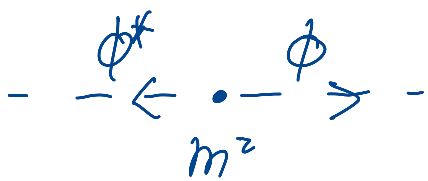
Scalar Potential $\longrightarrow V(\phi, \phi^*) = m^2 \phi^* \phi$

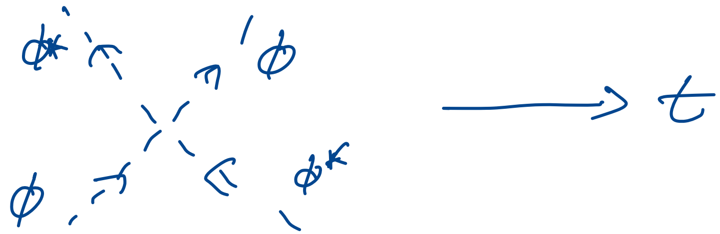
$$\mathcal{L}_\phi = g^{\mu\nu} (\partial_\mu \phi)^* \partial_\nu \phi - V(\phi, \phi^*)$$

Lets add a self-interaction (<u>quartic</u>)

$\lambda_{ijkl} \quad \phi^*_i \phi_j \phi^*_k \phi_l$

$$V(\phi, \phi^*) = m^2 \phi^* \phi + \frac{1}{4} \lambda (\phi^* \phi)^2$$





$$\frac{\partial \mathcal{L}}{\partial \phi^*} = m^2\phi + \frac{1}{2}\lambda(\phi^*\phi)\phi$$

$$\Longrightarrow$$

$$\frac{\partial \mathcal{L}}{\partial \phi} = m^2\phi^* + \frac{1}{2}\lambda(\phi^*\phi)\phi^*$$

$$\Longrightarrow \begin{cases} \left(g^{\mu\nu}\partial_\mu\partial_\nu + m^2 + \frac{\lambda}{2}\phi^*\phi\right)\phi = 0 \\ \left(g^{\mu\nu}\partial_\mu\partial_\nu + m^2 + \frac{\lambda}{2}\phi^*\phi\right)\phi^* = 0 \end{cases}$$

$$\mathcal{B} = \{\phi\} \otimes \{\phi^*\}$$

$$\Longrightarrow \begin{cases} (\Box + V'')\phi = 0 \\ (\Box + V'')\phi^* = 0 \end{cases} \qquad V'' = \frac{\partial^2 V}{\partial\phi\,\partial\phi^*}$$

$$V'' = m^2 + \frac{\lambda}{2}\phi^*\phi$$

Dirac Lagrangian

Free theory

$$(i\gamma^\mu\partial_\mu - m)\psi = 0 \iff i\gamma^0\partial_0\psi + i\gamma^k\partial_k\psi - m\psi = 0$$

Hermitian Conjugation:

$$(i\gamma^0\partial_0\psi)^\dagger + (i\gamma^k\partial_k\psi)^\dagger - (m\psi)^\dagger = 0 \iff$$

$$-i\partial_0\psi^\dagger\gamma^{0\dagger} - i\partial_k\psi^\dagger\gamma^{k\dagger} - m\psi^\dagger = 0 \iff$$

$\gamma^{0+} = \gamma^0 , \quad \gamma^{i+} = -\gamma^i$

$$-i\partial_0 \psi^+ \gamma^0 + i\partial_k \psi^+ \gamma^k - m\psi^+ = 0$$

multiply by γ^0 from the r.h.s.

$$-i\partial_0 \psi^+ \gamma^0\gamma^0 + i\partial_k \psi^+ \underbrace{\gamma^k\gamma^0}_{-\gamma^0\gamma^k} - m\psi^+\gamma^0 = 0$$

$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \Rightarrow \gamma^0\gamma^k = -\gamma^k\gamma^0$

Define $\bar{\psi} = \psi^{\dagger}\gamma^{0}$

$\bar{\psi}\psi$ is Lorentz invariant

while $\psi^{\dagger}\psi$ is not.

$$i\underbrace{\partial_0\bar{\psi}}\gamma^{0} + i\underbrace{\partial_k\bar{\psi}}\gamma^{k} + m\bar{\psi} = 0$$

$$\bar{\psi}\left(i\overleftarrow{\partial}_{\mu}\gamma^{\mu} + m\right) = 0$$

Adjoint Dirac Equation

$$\mathcal{L}_D = \bar{\psi}\left(i\gamma^\mu \overleftrightarrow{\partial}_\mu - m\right)\psi$$

$$m\bar{\psi}\psi$$

$$\underset{M^1}{M^4 = m}\,\underbrace{[\bar{\psi}]\,[\psi]}_{M^3} \Longrightarrow [\bar{\psi}] = [\psi] = M^{3/2}$$

$$\Big[\underset{6}{(\bar{\psi}\psi)^2}\ \underset{1}{\phi}\Big] = M^7$$

$$\mathcal{L}_D = \overline{\psi}(i\gamma^\mu \overleftrightarrow{\partial}_\mu - m)\psi$$

$$\frac{\partial \mathcal{L}_D}{\partial \overline{\psi}} = i\gamma^\mu \partial_\mu \psi - m\psi \quad \overset{E.L.}{\Longrightarrow} \quad (i\gamma^\mu \partial_\mu - m)\psi = 0$$

$$\frac{\partial \mathcal{L}_D}{\partial(\overrightarrow{\partial}_\mu \overline{\psi})} = 0$$

$$\frac{\partial \mathcal{L}_D}{\partial \psi} = -m\overline{\psi} \quad \overset{E.L.}{\Longrightarrow} \quad \overline{\psi}(i\overleftarrow{\partial}_\mu \gamma^\mu + m) = 0$$

$$\frac{\partial \mathcal{L}_D}{\partial(\overrightarrow{\partial}_\mu \psi)} = i\overline{\psi}\gamma^\mu$$

Add an interaction term with ϕ scalar

$$\mathcal{L}_{int} = -y\phi\bar{\psi}\psi \quad , \quad \underline{\underline{\phi \text{ is real.}}}$$

Yukawa coupling

$$\frac{\partial \mathcal{L}}{\partial \bar{\psi}} = i\gamma^\mu \partial_\mu \psi - m\psi - y\phi\psi$$

$$\overset{E.L}{\Longrightarrow} \quad (i\gamma^\mu \partial_\mu - m - y\phi)\psi = 0$$

$$(i\gamma^\mu \partial_\mu - M)\psi = 0$$

$$M = m + y\phi$$

if $m = 0 \implies M = y\phi$

$$0 = (i\gamma^\mu \partial_\mu - m_e)\psi_e \qquad m_e \to y_e\langle\phi\rangle$$

Maxwell Lagrangian

- Free massive scalar $\quad m^2 \phi^* \phi$
- Scalar self int. $\quad \frac{\lambda}{4}(\phi^*\phi)^2$
- Free massive fermion $\quad m\bar{\psi}\psi$
- Yukawa interaction $\quad y\phi\bar{\psi}\psi$

Spinors and scalars + Vectors

$$\mathcal{L}_{MAX} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

E.O.M. or E.L.

$$\frac{\partial \mathcal{L}_{MAX}}{\partial A_\nu} - \partial_\mu \frac{\partial \mathcal{L}_{MAX}}{\partial(\partial_\mu A_\nu)} = 0$$

$$\mathcal{L}_{MAX} = -\frac{1}{4} g^{\alpha\delta} g^{\beta\gamma} F_{\alpha\beta} F_{\delta\gamma}$$

$$= -\frac{1}{4} g^{\alpha\delta} g^{\beta\gamma} (\partial_\alpha A_\beta - \partial_\beta A_\alpha)(\partial_\delta A_\gamma - \partial_\gamma A_\delta)$$

$$\frac{\partial \mathcal{L}_{MAX}}{\partial A_\nu} = 0$$

$$\frac{\partial \mathcal{L}_{MAX}}{\partial(\partial_\mu A_\nu)} = -\frac{1}{4} g^{\alpha\delta} g^{\beta\gamma} \left[\frac{\partial F_{\alpha\beta}}{\partial(\partial_\mu A_\nu)} F_{\delta\gamma} + F_{\alpha\beta} \frac{\partial F_{\delta\gamma}}{\partial(\partial_\mu A_\nu)} \right]$$

Replace $\begin{cases} \alpha \leftrightarrow \delta \\ \beta \leftrightarrow \gamma \end{cases}$ on the second term

$$= -\frac{1}{4} \left[g^{\alpha\delta} g^{\beta\gamma} \frac{\partial F_{\alpha\beta}}{\partial(\partial_\mu A_\nu)} F_{\delta\gamma} + g^{\delta\alpha} g^{\gamma\beta} F_{\delta\gamma} \frac{\partial F_{\alpha\beta}}{\partial(\partial_\mu A_\nu)} \right]$$

$$= -\frac{1}{2} g^{\alpha\delta} g^{\beta\gamma} \frac{\partial F_{\alpha\beta}}{\partial(\partial_\mu A_\nu)} F_{\delta\gamma}$$

$$= -\frac{1}{2} g^{\alpha\delta} g^{\beta\gamma} \left[\frac{\partial(\partial_\alpha A_\beta)}{\partial(\partial_\mu A_\nu)} - \frac{\partial(\partial_\beta A_\alpha)}{\partial(\partial_\mu A_\nu)} \right] F_{\delta\gamma}$$

$$= -\frac{1}{2} g^{\alpha\delta} g^{\beta\gamma} \left[\delta^{\mu}_{\alpha} \delta^{\nu}_{\beta} - \delta^{\mu}_{\beta} \delta^{\nu}_{\alpha} \right] F_{\delta\gamma}$$

$$= -\frac{1}{2} \left[g^{\mu\delta} g^{\nu\gamma} - g^{\nu\delta} g^{\mu\gamma} \right] F_{\delta\gamma}$$

$$= -\frac{1}{2} \Big[\underbrace{F^{\mu\nu} - F^{\nu\mu}}_{2F^{\mu\nu}} \Big]$$

$$= - F^{\mu\nu}$$

$$\frac{\partial \mathcal{L}_{MAX}}{\partial A_\nu} - \partial_\mu \frac{\partial \mathcal{L}_{MAX}}{\partial(\partial_\mu A_\nu)} = 0 \implies$$

$$0 - \partial_\mu\left(-F^{\mu\nu}\right) = 0 \implies$$

$$\boxed{\partial_\mu F^{\mu\nu} = 0}$$

Ampere - Gauss Law with no sources

For the Faraday–Gauss law do;

$$F^{*\mu\nu} = \varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \quad \Rightarrow$$

$$\partial_\mu F^{*\mu\nu} = \varepsilon^{\mu\nu\alpha\beta} \partial_\mu \left(\partial_\alpha A_\beta - \partial_\beta A_\alpha \right)$$

$$= \underbrace{\varepsilon^{[\mu\nu\alpha]\beta} \partial_{(\mu} \partial_{\alpha)} A_\beta}_{0} - \underbrace{\varepsilon^{[\mu\nu\alpha\beta]} \partial_{(\mu} \partial_{\beta)} A_\alpha}_{0}$$

$$\boxed{\partial_\mu F^{*\mu\nu} = 0}$$

$$\mathcal{L}_{int} = -j^\nu A_\nu \quad , \quad \mathcal{L}_{tot} = \mathcal{L}_{MAX} + \mathcal{L}_{int}$$

$$\frac{\partial \mathcal{L}_{tot}}{\partial A_\nu} = -j^\nu \quad \Longrightarrow \quad \boxed{\frac{\partial \mathcal{L}_{tot}}{\partial A_\nu}} - \boxed{\partial_\mu \frac{\partial \mathcal{L}_{tot}}{\partial(\partial_\mu A_\nu)}} = 0$$

$$\frac{\partial \mathcal{L}_{tot}}{\partial(\partial_\mu A_\nu)} = -F^{\mu\nu} \quad \Longrightarrow \quad \boxed{\partial_\mu F^{\mu\nu}} = \boxed{j^\nu}$$

Ampere-Gauss Law in the presence of sources what are those sources . . .