Banados-Silk-West effect with nongeodesic particles

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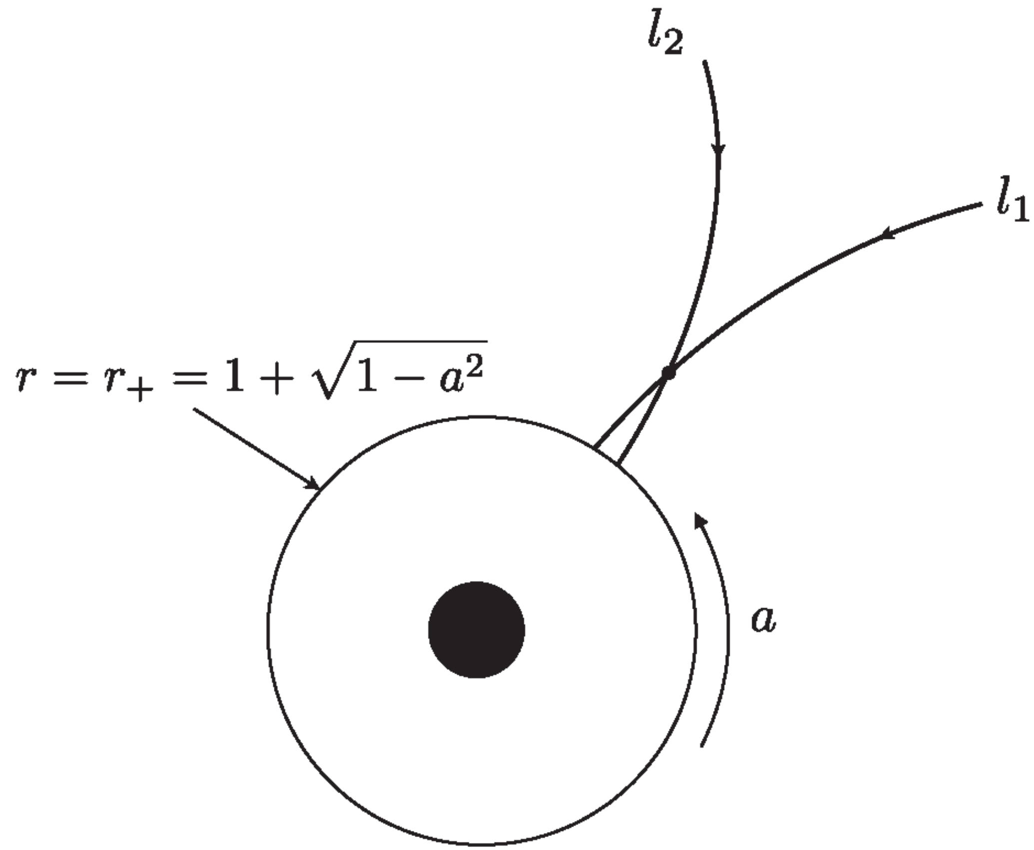
M. Banados, J. Silk, and S. M. West PRL 2009



$$-2(1 + \sqrt{1 + a}) < l < 2(1 + \sqrt{1 - a})$$

Both particles experience blue shift, centre of mass frame is in free fall.

Role of gravitation radiation - ?

E. Berti, V. Cardoso, L. Gualtieri, F. Pretorius, and U.Sperhake PRL, 2009

T. Jacobson and T. P. Sotiriou PRL 2010

I. V. Tanatarov and O. Z.

PRD 2013, 2014

Partial answer in favor of BSW

Main ingredients of effect

Horizon

Critical trajectories (fine-tuned parameters)

If horizon is given, question remains: whether or not critical trajectories exist

Collision of 1 critical particle and 1 usual one give unbounded energy In CM frame

$$ds^2 = -N^2 dt^2 + g_\phi (d\phi - \omega dt)^2 + \frac{dr^2}{A} + g_\theta d\theta^2,$$

Metric does not depend on t, ϕ

Two integrals of motion $E = -p_\mu \xi^{(t)\mu}$

Angular momentum $L = p_\mu \xi^{(\phi)\mu}$

Killing vectors responsible for time translations and rotation

$$\xi^{(t)\mu} = (1, 0, 0, 0) \qquad \xi^{(\phi)\mu} = (0, 1, 0, 0)$$

$$E_{c.m}^2 = -(p_1^\mu + p_2^\mu)(p_{1\mu} + p_{2\mu}) = m_1^2 + m_2^2 + 2m_1 m_2 \gamma_{c.m},$$

$$\gamma_{c.m.} = -u_{1\mu} u_2^\mu$$

$$m_1 m_2 \gamma_{c.m.} = \frac{X_1 X_2 - Z_1 Z_2}{N^2} - \frac{L_1 L_2}{g_\phi} - g_z p_1^z p_2^z.$$

without force or with force

$$X = E - \omega L.$$

$$Z^2 = X^2 - N^2 \left[\frac{L^2}{g_\phi} + m^2 \right].$$

Equatorial motion

Two particle move towards black hole and collide near horizon

$E_{c.m.}$ grows unbound, if critical particle collides with usual one

BSW effect

Critical $(X)_H = 0$ $X = E - \omega L.$

Usual $(X)_H \neq 0$

$$X = \frac{mN}{\sqrt{1 - V^2}}.$$

Horizon limit: $N \to 0$

Usual particle: $V \to 1$

Critical: $X \sim N$ $V < 1$

Influence of dynamics in terms of geometry: whether critical trajectories still exist or not.

Energy and angular momentum

$$\frac{d}{d\tau}(\xi^{\mu}u_{\mu}) = \xi^{\mu}a_{\mu}.$$

Killing vector

$$\frac{1}{m}\frac{dE}{d\tau} = Na_{o}^{(t)} + \omega\sqrt{g_{\phi}}\ a_{o}^{(\phi)};$$
$$\frac{1}{m}\frac{dL}{d\tau} = \sqrt{g_{\phi}}\ a_{o}^{(\phi)}.$$

Two frames OZAMO and FZAMO

Usual particle: singular transformation

Since V close to c

Conditions on force: finite in FZAMO frame

$$a_f^{(t)} = (a_f^{(t)})_0 + (a_f^{(t)})_1 N + O(N^2);$$

$$a_f^{(r)} = (a_f^{(r)})_0 + (a_f^{(r)})_1 N + O(N^2);$$

$$a_o^{(t)} = +\frac{(a_f^{(t)})_0 - (a_f^{(r)})_0}{N} + (a_f^{(t)})_1 - (a_f^{(r)})_1 + O(N);$$

$$a_o^{(r)} = -\frac{(a_f^{(t)})_0 - (a_f^{(r)})_0}{N} - (a_f^{(t)})_1 - (a_f^{(r)})_1 + O(N),$$

$$a_f^{(\phi)} = a_o^{(\phi)} = O(1);$$

$$a_f^{(z)} = a_o^{(z)} = O(1).$$

Critical particles

V<C, force finite in both frames E, L finite

Two mutually complimentary cases

O frame (r) and (t) components of acceleration diverge for usual particles and stay finite for the critical ones

F frame, situation opposite: finite for usual particles and diverge for critical ones.

For critical particles, additionally $a_o^{(\phi)} = O(N)$

Are aforementioned conditions compatible with dynamics?

Results

If r tetrad component of force is finite and

$$a_o^{(\phi)} = O(r - r_H).$$

BSW effect exist

Critical trajectoris possible, althougth form of fine-tuning slightly changes

Example: pure azimuthal dissipative force

$$N^2 = \nu_2\xi^2 + \nu_3\xi^3 + \ldots,$$
$$\omega = \omega_H - \omega_1\xi + \omega_2\xi^2 + \ldots,$$
$$g_\phi = g_H + g_1\xi + g_2\xi^2 + \ldots,$$
$$X = x_1\xi + x_2\xi^2 + \ldots,$$
$$L = l_H + l_1\xi + l_2\xi^2 + \ldots,$$

$$a_o^{(t)} = a_1\xi + a_2\xi^2 + \ldots,$$

$$a_o^{(t)} \approx -\frac{2Z}{\omega_1\sqrt{\nu_2}}\frac{x_1\omega_2 + x_2\omega_1}{1 + g_H\omega_1^2/\nu_2} \sim Z \sim \xi.$$

Self-consistent algebraic systems on coefficients of expansion

Conclusions

Under weak physically reasonable condition BSW effect survives
Action of force.
More viable than one could expect.

Although dissipative forces in flat spacetime generically bound the values of energy peaks from above, in the strong gravitational field regime near the horizon the geometry dominates over the influence of dissipative forces on the system.

Nonextremal horizons: main conclusions are the same

Thank you!