Can a particle detector cross a Cauchy horizon?

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Outline

Motivation

- Issues of causality (Strong cosmic censorship, Penrose [1979]).
- PDE stability problem in Reissner-Nordström (Dafermos et. al. [gr-qc/0307013, gr-qc/0309115]).
- Considerations from QFT: Direct experience of observers (Louko & Satz [arXiv:0710.5671 [gr-qc]).

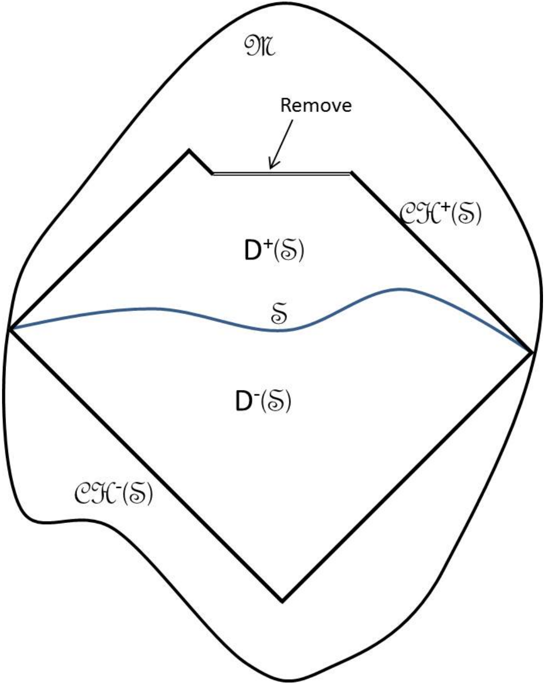
Cauchy horizons

Definition

The **future/past Cauchy development** of a closed, achronal set $\mathscr{S} \subset \mathscr{M}$ is $D^{\pm}(\mathscr{S}) = \{p \in \mathscr{M} :$ every past/future inextendible causal curve through p intersects $\mathscr{S}\}$. The **Cauchy development** is $D(\mathscr{S}) = D^{+}(\mathscr{S}) \cup D^{-}(\mathscr{S})$.

Definition

The **future/past Cauchy horizon** of $\mathscr{S} \subset \mathscr{M}$ is $\mathscr{C}\mathscr{H}^{\pm}(\mathscr{S}) = \overline{D^{\pm}(\mathscr{S})} - I^{\mp}(D^{\pm}(\mathscr{S}))$. The **Cauchy horizon** is $\mathscr{C}\mathscr{H}(\mathscr{S}) = \mathscr{C}\mathscr{H}^{+}(\mathscr{S}) \cup \mathscr{C}\mathscr{H}^{-}(\mathscr{S})$.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QFT and observers

What is a particle detector?

Physical intuition:

Two-level system coupled to a quantum field. Heuristically, can think of an atom interacting with the field by absorbing or emitting field quanta.

Mathematically:

We can characterise the transition probability at proper time τ by

$$\mathcal{F}_\tau(E) = \int d\tau' \, d\tau'' \, \chi(\tau')\chi(\tau'') \mathrm{e}^{\mathrm{i}E(\tau' - \tau'')} \mathcal{W}(\mathsf{x}(\tau'), \mathsf{x}(\tau''))$$

- $\mathcal{W}$ is the *Wightman function*.
- χ is a smooth switching function of compact support that controls physical interaction.

$1+1$ dimensional considerations

- Massless fields enjoy conformal symmetry $\Rightarrow$ Two-point functions in closed form!
- Infrared problems for the Wightman function.
- Definition of the detector must be modified:

$$\mathcal{F}_\tau(E) = \int d\tau'\, d\tau''\, \chi(\tau')\chi(\tau'')\mathrm{e}^{\mathrm{i}E(\tau'-\tau'')}\partial_\tau\partial_{\tau'}\mathcal{W}(\mathsf{x}(\tau'),\mathsf{x}(\tau''))$$

In the sharp-switching limit $\chi(\tau') \to \Theta(\tau'-\tau_0)\Theta(\tau-\tau')$, the transition rate $\dot{\mathcal{F}}_\tau(E) \doteq \partial_\tau \mathcal{F}_\tau(E)$ is finite.

Further comments on $1+1$ dimensionality

- Crossing Cauchy horizons in flat spacetime.
- Prototype: Rindler spacetime.

Static trajectory in $1+1$ dimensions: $g_2 = -\exp(2a\xi)(d\eta^2 - d\xi^2)$,

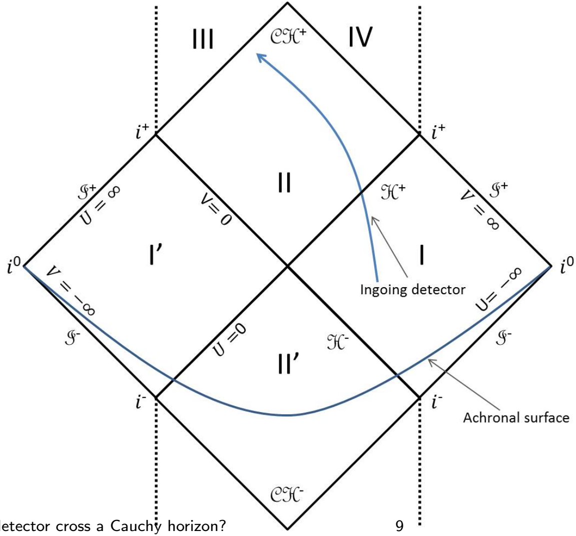
$$\dot{\mathcal{F}}(\omega, \tau, \tau_0) = -\frac{1}{4\pi}\frac{1}{\tau_h - \tau} + \mathcal{O}\Big((\tau_h - \tau)\log[a(\tau_h - \tau)]\Big)^{-1}. \quad (1)$$

Static trajectory in $3+1$ dimensions: $g = g_2 + dy^2 + dz^2$,
(Louko & Satz [arXiv:0710.5671])

$$\dot{\mathcal{F}}(\omega, \tau, \tau_0) = \frac{1}{8\pi^2\tau_h}\log\left(1 - \frac{\tau}{\tau_h}\right) + \mathcal{O}\left(\log\left[-\log\left(1 - \frac{\tau}{\tau_h}\right)\right]\right). \quad (2)$$

$1+1$ Reissner-Nordstström, $M > |Q|$

Exterior: $g = -F(r)du\,dv, \quad F(r) = (r - r_+)(r - r_-)/r^2$ (3)



$1+1$ Reissner-Nordstström HHI

Crossing $\mathscr{C}\mathscr{H}^+$ in the **Hartle-Hawking-Israel** state,

$$\mathcal{W}_{\mathrm{H}}(\mathsf{x},\mathsf{x}') = -\frac{1}{4\pi}\log\left[(\varepsilon + \mathrm{i}\Delta U)(\varepsilon + \mathrm{i}\Delta V)\right], \tag{4}$$

along the integral curves of $\dot{\gamma}(\tau)$:

$$\dot{t} = \frac{E}{F(r)}, \qquad \dot{r} = -\sqrt{E^2 - F(r)}, \tag{5}$$

one finds that the transition rate diverges as $\tau \to \tau_h$ like

$$\dot{\mathcal{F}}_{\mathrm{H}}(\omega,\tau,\tau_0) = \frac{1}{4\pi}\left[-1 + \frac{\kappa_+}{\kappa_-}\left[\frac{3}{2} - \cos(\omega\Delta\tau)\right]\right]\frac{1}{\tau_h - \tau} + o(\tau_h - \tau)^{-1}. \tag{6}$$

- This result is in fact independent of the details of $F(r)$!

$1+1$ Reissner-Nordstström Unruh

This result holds for the **Unruh state**,

$$\mathcal{W}_{\mathrm{U}}(\mathsf{x},\mathsf{x}') = -\frac{1}{4\pi}\log\left[\left(\varepsilon + \mathrm{i}\Delta U\right)\left(\varepsilon + \mathrm{i}\Delta v\right)\right], \quad (7)$$

since in $1+1$ dimensions, the left-moving and right-moving modes decouple. The right-moving modes along $\xi = \partial_v$ contribute as $\mathcal{O}(1)$ and the left-moving modes diverge as in the HHI state.

Future directions

Stress-energy tensor:

- Infrared convergent in $1+1$ dimensions!
- In $1+1$ dimensions HHI state is conformal vacuum $\Rightarrow \langle \mathcal{T} \rangle_{\mathrm{H}}^{\mathrm{ren}}$ in closed form.
- Right and left moving decoupling $\Rightarrow \langle \mathcal{T} \rangle_{\mathrm{U}}^{\mathrm{ren}}$ in closed form.
- One can compute the experience of stress-energy along the trajectory (*à la* QEI, see e.g. Fewster [gr-qc/9910060]).

$$\mathcal{I}_{\mathrm{H/U}} = \int d\tau' \, \chi(\tau') \dot{\gamma}(\tau')^a \dot{\gamma}(\tau')^b \langle \mathcal{T}(\tau')_{ab} \rangle_{\mathrm{H/U}}^{\mathrm{ren}}. \tag{8}$$

Calculation in progress!!

Thanks for your attention!

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