VII BLACK HOLES WORKSHOP
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Chiral Gap Effect in Curved Space*#

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Outline

- Introduction
- Thermodynamical Potential in Curved Space
- Similarities and Dissimilarities with thermal mass
- Thermal excitation and quark deconfinement
- Conclusions

Quantum field theory in curved space has been very successful

Even though it only provides a limited description of quantum phenomena in the presence of gravity, it has produced remarkable results

Particle production in expanding universe
Hawking radiation
Unruh effect
Casimir effect
...

All manifestations of
a deep connection

Gravity

QFT in curved space

Thermodynamics *Quantum Field Theory*

Phenomenological applications to nuclear physics and condensed matter are being developed

“Testing” Quantum field theory in curved space

In the “lab”

- Phenomenological applications of QFT in curved space are **generically difficult** in the sense that gravity is weak
- So unless the curvature scale is large effects of curved space will be negligible
- In QCD, for instance, **gravitational effects are usually negligible** as compared to the typical scale Λ_{QCD}

“Testing” Quantum field theory in curved space

In the “Universe”

Black holes as lab for high energy physics

- **Black holes** offer a set-up where curvature effects may become comparable to Λ_{QCD}
- In the vicinity of a black hole, the Standard Model will still be valid under non-negligible gravitational corrections
- In **QCD** we have additional complications in relation to the **complex vacuum structure** of the theory

QCD & Black Holes

In order to accommodate non-perturbative phenomena (eg, dynamical mass generation, or confinement of quarks and gluons), it is conceivable to think of the **black hole** as an **extended object surrounded by a media of QCD matter**

This set-up can be naturally associated with QCD phase transitions

[AF, PRD **R**88 2103, AF & Tanaka, PRD **R**84 2011]

Classically, we may think that nothing happens. However, in QCD, the hadron wave function in a boosted frame are different from those at rest

The QCD vacuum filled with quantum fluctuations should change drastically near a black hole

Other places of relevance

In relativistic heavy ion collisions **QGP** is created and undergoes through an expansion at the speed of light. In this context, speculative scenarios that relate particle production in QCD to the Hawking temperature already exist (Castorina, Kharzeev, and Satz, EPJ C52 2007)

Similarities can also be found in **cold atomic systems**, where Hawking radiation from black hole analogues is also a topic of interest

Effects of curvature on massless Dirac fermions emerge also in strongly correlated systems, like **graphene** or **TI**

OBSERVATION (Chiral Gap Effect):

Dirac fermions can have a chiral invariant mass gap due to the curvature

Thermodynamic Potential in Curved Space

In fermionic systems, the **effective mass** M_{eff} with interaction clouds can differ from the bare one.

In the chiral limit M_{eff} should be **proportional to the scalar chiral condensate**

$$M_{eff} = G \langle \bar{\psi}\psi \rangle$$

G is coupling constant

The effective mass should solve the gap equation, i.e. should minimize the Grand Thermodynamic Potential

$$\Omega[M_{eff}] = \Omega_{tree}[M_{eff}] + \Omega_{loop}[M_{eff}]$$

Thermodynamic Potential in Curved Space

$$\beta\,\Omega_{\mathrm{loop}}[M_{eff}] = -\,\nu \ln \mathrm{Det}\,(i\,\partial\!\!\!/ - M_{eff})$$

β : inverse temperature

ν : number of fermionic dof

Thermodynamic Potential in Curved Space

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Consider $g_{\tau\tau} = 1$ (ultrastatic case)

$$\beta\,\Omega_{loop}[M_{eff}] = -\,\frac{\nu}{2}\ln \mathrm{Det}\,(\Box - M_{eff}^2 + \frac{R}{4})$$

and express the propagator as

$$\mathbf{Tr}_{space}\,\mathbf{e}^{-t\,(\,-\,\partial_\tau^2 - \Delta\, +\, M_{eff}^2 + \frac{R}{4}\,)} = (4\,\pi\,t)^{-2}\,\mathbf{e}^{-t\,(\,-\,\partial_\tau^2 - \Delta\, +\, M_{eff}^2 + \frac{R}{12}\,)} \times$$

$$\times \sum_k \mathbf{tr}\, a_k\, t^k$$

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For illustration, let me consider the case of a maximally symmetric spacetime and take D to be large. Then the dominant contribution comes from the k = 0 term

$$\mathrm{Tr}_{\text{space}}\, e^{-t\left(-\partial_\tau^2 - \Delta + M_{eff}^2 + \frac{R}{4}\right)} \sim (4\pi t)^{-2}\, e^{-t\left(-\partial_\tau^2 - \Delta + M_{eff}^2 + \frac{R}{12}\right)}$$

$$M_{eff}^2 \longrightarrow M_{eff}^2 + \frac{R}{12}$$

Chiral Gap in Curved Space

Simple picture

The effect of the scalar curvature is to shift the effective mass

$$M_{eff}^2 \longrightarrow M_{eff}^2 + \frac{R}{12}$$

In the chirally symmetric phase, we have $M_{eff}^2 = 0$, but fermions are still gapped due to curvature effect

(Dis)Similarities with m_T

Similar to what happens at finite T (thermal mass)

$$m_T^2 = (g^2/6)\, T^2$$

The critical temperature lowers with increasing m_T

[Hikida & Kitazawa, PRD **75** 011091 2007]

With Curvature:

$$T_c^* = T_c - \alpha \;\; R / G^2$$

- R is an independent quantity, while m_T is not. (Quantum phase transition at finite curvature)
- If R is negative, then T_c should increase. If $R<0$ and large, the chiral symmetry breaking and deconfinement could become completely distinct

Thermal excitations and quark deconfinement

Intuitive description of the simultaneous crossover of deconfinement and chiral phase transition (as observed on the lattice) **[**Fukushima, PLB **591** 277 2004**]**

At finite T the quark deconfinement can be characterized by the Polyakov loop

$$\Phi = \frac{1}{N_c}\,\mathrm{tr}\,L$$

For pure YM, this can be rigorously defined as an order parameter that breaks center symmetry

In QCD the transition turns out to be smooth due to fermion interactions

The chiral phase transition controls the fermion mass, and the transition to the deconfined phase is more favored with lighter quarks after the chiral phase transition.

Thermal excitations and quark deconfinement

In flat space thermally excited fermions on the gluon background generate terms that break center symmetry

In curved space we can characterize the same physics

$$\beta\,\Omega_{loop}[M_{eff}] = \sum_{i=1}^{N_c}\left[-\nu \ln \mathrm{Det}\,(i\,\partial\!\!\!/ - M_{eff} + i\phi_i\,\gamma^t)\right]$$

Similar to what we consider before (+ trace over colors)

$$L = \mathrm{diag}\,(e^{i\phi_i}) \qquad \text{PL matrix}$$

The potential can be evaluatd also in this more general case

$$\Omega_{loop}[M_{eff}] = \Omega^{T=0}_{loop}[M_{eff}] + \Omega^{T\neq 0}_{loop}[M_{eff}]$$

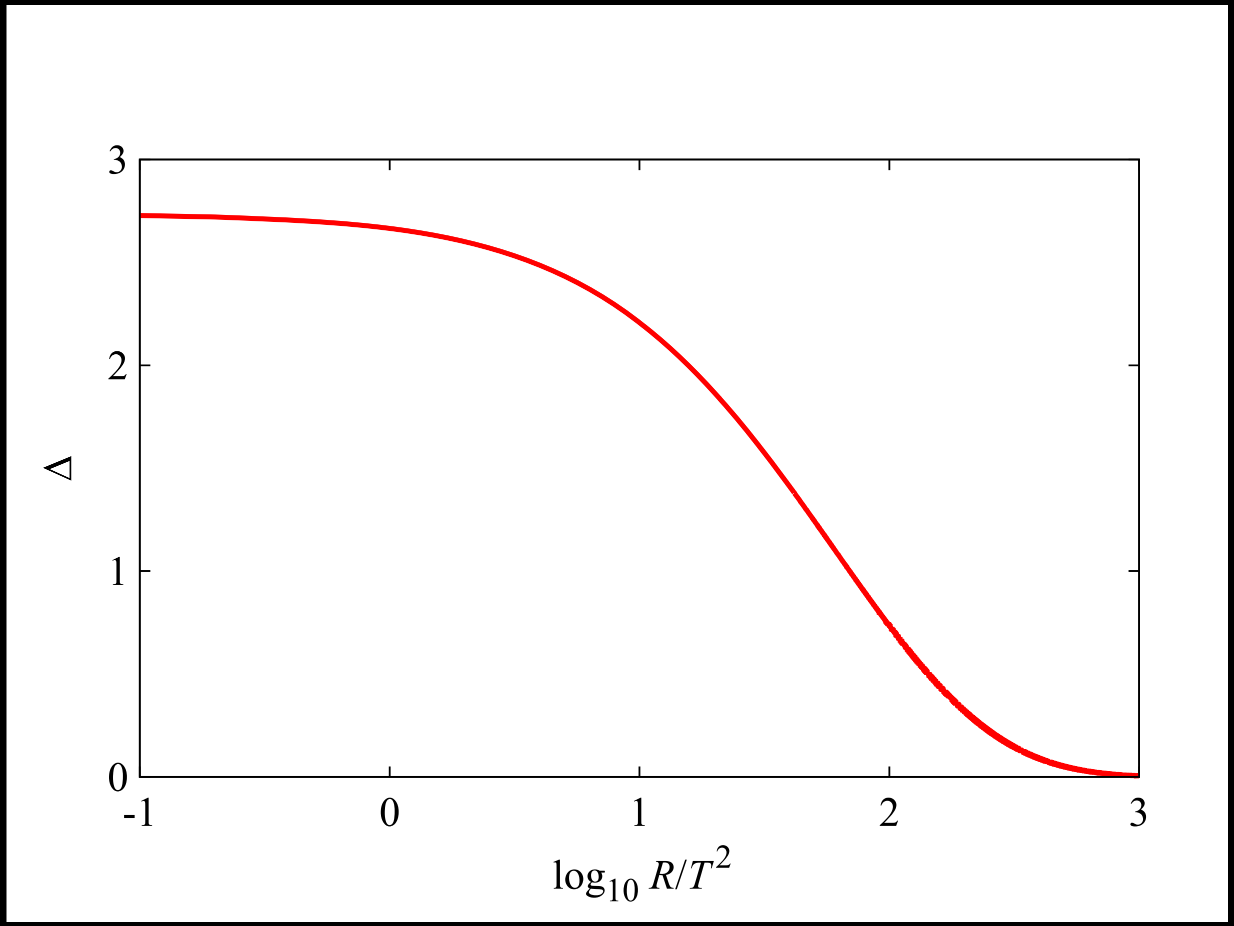
Thermal excitations and quark deconfinement

$$\beta\, \Omega^{T\neq 0}_{loop}[M_{eff}] = -2\, N_f V \int \frac{d^3p}{(2\pi)^3}\, \mathbf{tr} \Big[\ln [\, 1 + L\, e^{-\beta(\varepsilon_p - \mu)}] + \ln [\, 1 + L^\dagger e^{-\beta(\varepsilon_p + \mu)}] \Big]$$

$$\varepsilon_p = (p^2 + M^2_{eff} + R/12)^{1/2}$$

- In flat space M^2_{eff} controls the explicit breaking of center symmetry
- As soon as non zero R is switched on, thermally excited fermions are suppressed not only by M^2_{eff} by also by R
- Therefore, even in the chiral limit, if $R > T$, fermion excitations are almost absent and center symmetry can be an approximate symmetry

Dimensionless center symmetry breaking parameter



$$\Delta \equiv (\beta^4/V)(\Omega^T_{\rm loop}[\Phi=1]-\Omega^T_{\rm loop}[\Phi=-1])$$

R must be hundreds of times larger than T^2 in order to realise decoupling

Near black holes:

- Once the decoupling happens, the gluon sector should behave as pure Yang-Mills leading to a quark deconfinement transition of 1st order

Main Conclusions

- The predominant effect on fermions in curved space is the appearance of a chirally symmetric mass gap due to the scalar curvature
- The problem can be consistently formulated in term of a resummed expansion of the propagator
- The chiral mass gap gives an intuitive explanation of the nature of the chiral phase transition in curved space
- Chiral symmetry tends to get restored with large R, while the chiral condensate and the critical temperature become larger with $R < 0$
- Effects of curvature suggest a decoupling between the chiral dynamics and the deconfinement leading to a first order transition (pure YM) near a strongly gravitating source

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THANK YOU!