Extremal rotating black holes in 5D Einstein-Maxwell-Chern-Simons theory:

Radially excited solutions, non-uniqueness and near horizon geometry

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RESEARCH TRAINING GROUP
Models of Gravity

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Extremal rotating black holes in 5D Einstein-Maxwell-Chern-Simons theory:

Radially excited solutions, non-uniqueness and near horizon geometry

1. Introduction: Ansatz and general properties

2. Near-horizon formalism

3. Numerical results

4. Future work: negative cosmological constant

1. Introduction:
Ansatz and general properties

|| 1. Introduction ||

We will consider black holes in 5 dimensions in
Einstein-Maxwell-Chern-Simons theory

Black holes in higher dimensions have some special properties

- Topologies of stationary black holes can be non-spherical
  For example black ring solution (Emparan 2002)

- More than one independent plane of rotation
  In D dimensions there are N = [(D-1)/2] planes of rotation

  N independent angular momenta

|| 1. Introduction ||

Einstein theory in odd D-dimensions

$$I = \int d^D x \sqrt{-g} \Big[R + \mathcal{L}_M \Big]$$

$$16\pi G_D = 1$$

R = curvature scalar

$$\mathcal{L}_M = -F^2 - \frac{2\lambda}{3\sqrt{3}} \epsilon^{\alpha\beta\mu\nu\rho} A_\alpha F_{\beta\mu} F_{\nu\rho}$$

Einstein-Maxwell-Chern-Simons
Gravity coupled to a U(1)
electro-magnetic potential A_μ
and a Chern-Simons term (D=5)

|| 1. Introduction ||

We are interested in the higher dimensional generalization of the Kerr-Newman black holes:

Axisymmetric and stationary,
Spherical topology of the horizon,
Asymptotically flat
Electrically charged

All angular momenta of the same magnitude:

$$|J|=|J_1|=|J_2|=\ldots=|J_N|$$

We have enhanced *U(N)* symmetry

Not even with these constraints uniqueness is granted

We use numerical methods to obtain
global solutions with these properties.

We also make use of the near-horizon formalism.

|| 1. Introduction ||

Ansatz for the metric:

$$ds^2 = -f dt^2 + \frac{m}{f}\left[dr^2 + r^2 \sum_{i=1}^{N-1}\left(\prod_{j=0}^{i-1}\cos^2\theta_j\right) d\theta_i^2\right] + \frac{n}{f} r^2 \sum_{i=1}^{N}\left(\prod_{j=0}^{i-1}\cos^2\theta_j\right)\sin^2\theta_i \left(\varepsilon_i d\varphi_i - \frac{\omega}{r}dt\right)^2$$
$$+\frac{m-n}{f} r^2 \left\{\sum_{i=1}^{N}\left(\prod_{j=0}^{i-1}\cos^2\theta_j\right)\sin^2\theta_i d\varphi_i^2 - \left[\sum_{i=1}^{N}\left(\prod_{j=0}^{i-1}\cos^2\theta_j\right)\sin^2\theta_i \varepsilon_i d\varphi_i\right]^2\right\}$$

$$\theta_0 \equiv 0,\ \theta_i \in [0, \pi/2] \text{ for } i = 1, \ldots, N-1$$

$$\theta_N \equiv \pi/2,\ \varphi_k \in [0, 2\pi] \text{ for } k = 1, \ldots, N$$

$$\varepsilon_k = \pm 1$$

Lewis-Papapetrou coordinates. The radial coordinate **r** is isotropic.

Ansatz for the gauge field:

$$A_\mu dx^\mu = a_0 dt + a_\varphi \sum_{i=1}^{N}\left(\prod_{j=0}^{i-1}\cos^2\theta_j\right)\sin^2\theta_i \varepsilon_i d\varphi_i\ .$$

|| 1. Introduction ||

Extremal black holes present non integer exponents in their horizon expansion.

✓ EM case

✓ EMCS case

Special parametrization of the functions in order to numerically solve the problem.

$$f = f_4 r^4 + f_\alpha r^\alpha + o(r^6)$$
$$m = m_2 r^2 + m_\beta r^\beta + o(r^4)$$
$$n = n_2 r^2 + n_\gamma r^\gamma + o(r^4)$$
$$\omega = \omega_1 r + \omega_2 r^2 + o(r^3)$$
$$a_0 = a_{0,0} + a_{0,\lambda} r^\lambda + o(r^2)$$
$$a_\phi = a_{\phi,0} + a_{\phi,\mu} r^\mu + o(r^2)$$
$$\phi = \phi_0 + \phi_\nu r^\nu + o(r^2)$$

$4 < \alpha < 6$	$2 < \beta < 4$	$2 < \gamma < 4$
$0 < \lambda < 2$	$0 < \mu < 2$	$0 < \nu < 2$

2. Near-horizon formalism

|| 2. Near Horizon Formalism ||

The space-time outside the event horizon of extremal black holes can be divided in two different regions:

- Near-horizon geometry
- Bulk geometry

Extracting the NHG from a known analytical solution by a coordinate transformation:

1. Move to a frame comoving with the event horizon
2. Center the radial coordinate on the event horizon
3. Scale parameter Λ in the new radial and temporal coordinates.
4. Series expansion for small Λ

$$r = (\Lambda y + a)$$

$$dt = \frac{a_0}{\Lambda} dT + (a_2/r^2 + a_1/r)dr$$

First term is scale independent: near-horizon geometry

|| 2. Near Horizon Formalism ||

Properties of the near-horizon geometry of extremal black holes.

H. K. Kunduri and J. Lucietti, Living Reviews in Relativity 16 (2013)

- Extremal black holes with spherical topology: near-horizon geometry is the product of two independent spaces.

$$AdS_2 \times S^{D-2}$$

Isometries: $SO(2,1) \times SO(D-1)$ static case (sphere)

$SO(2,1) \times U(1)^N$ rotation (squashed sphere)

This factorization is obtained for all the known examples of topologically spherical black holes

|| 2. Near Horizon Formalism ||

Hence we can assume such factorization in our black holes (extremal case)

Metric:

$$\begin{aligned} ds^2 \;&=\; v_1\left(\frac{dr^2}{r^2}-r^2dt^2\right)+v_2\sum_{i=1}^{N-1}\left(\prod_{j=0}^{i-1}\cos^2\theta_j\right)d\theta_i^2 \\ &\quad +v_2v_3\sum_{i=1}^{N}\left(\prod_{j=0}^{i-1}\cos^2\theta_j\right)\sin^2\theta_i\left(\varepsilon_i d\varphi_i-krdt\right)^2 \\ &\quad +v_2(1-v_3)\left\{\sum_{i=1}^{N}\left(\prod_{j=0}^{i-1}\cos^2\theta_j\right)\sin^2\theta_i d\varphi_i^2\right. \\ &\quad \left.-\left[\sum_{i=1}^{N}\left(\prod_{j=0}^{i-1}\cos^2\theta_j\right)\sin^2\theta_i\varepsilon_i d\varphi_i\right]^2\right\} \end{aligned}$$

Gauge potential:

$$A_\mu dx^\mu=(q_1-q_2k)rdt+q_2\sum_{i=1}^{N}\left(\prod_{j=0}^{i-1}\cos^2\theta_j\right)\sin^2\theta_i\varepsilon_i d\varphi_i$$

|| 2. Near Horizon Formalism ||

Near-horizon geometry:

- Field equations + Ansatz: algebraic relations for the Ansatz parameters
- Alternatively: Extremal of entropy functional
- Global charges can be calculated: **(J, Q)**
- Horizon charges: area, horizon angular momentum

Parameters related to the asymptotical structure of the global solution cannot be calculated:

Mass, angular velocity

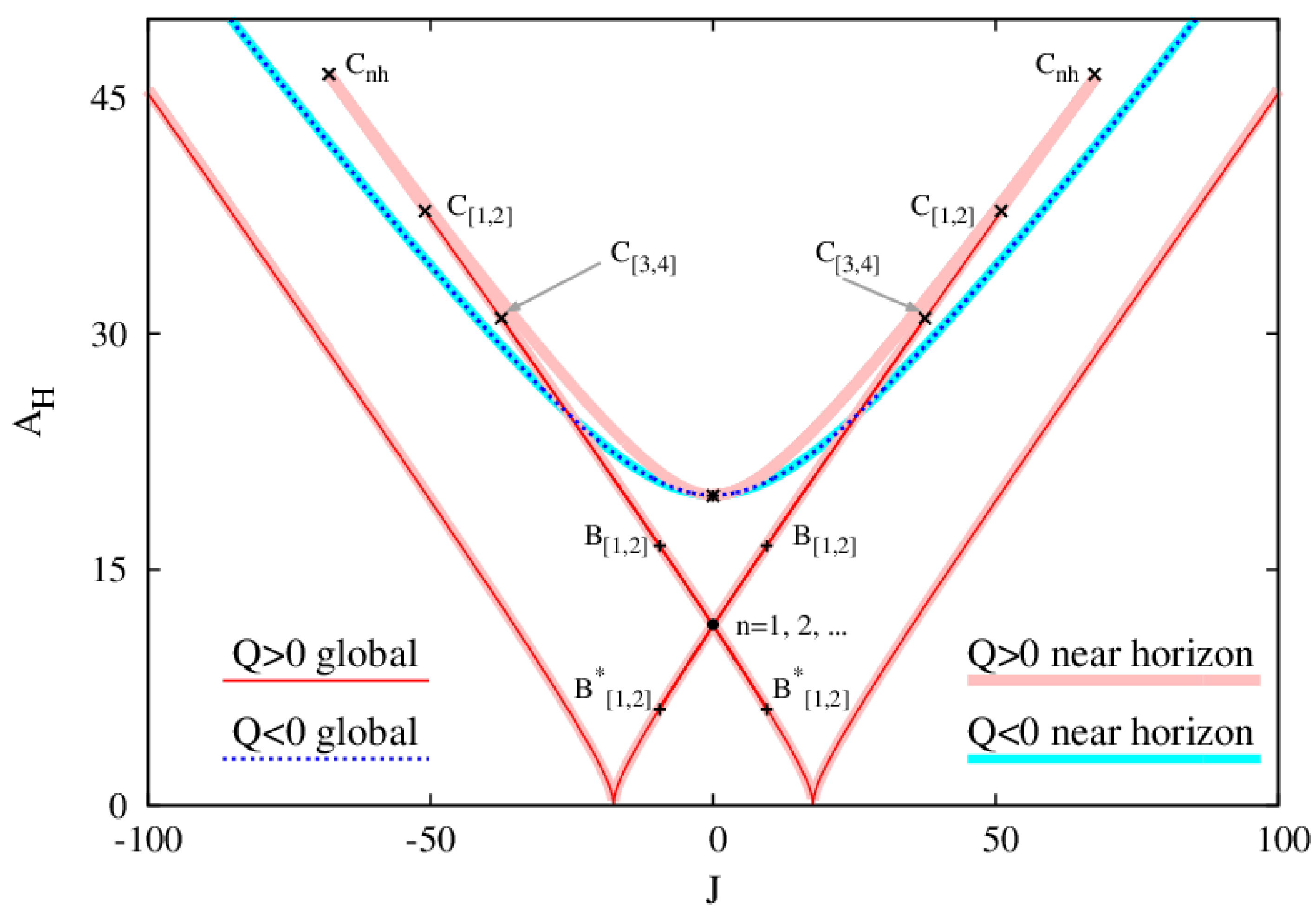
3. Numerical results

Global solutions and branch structure

$\lambda > 2$

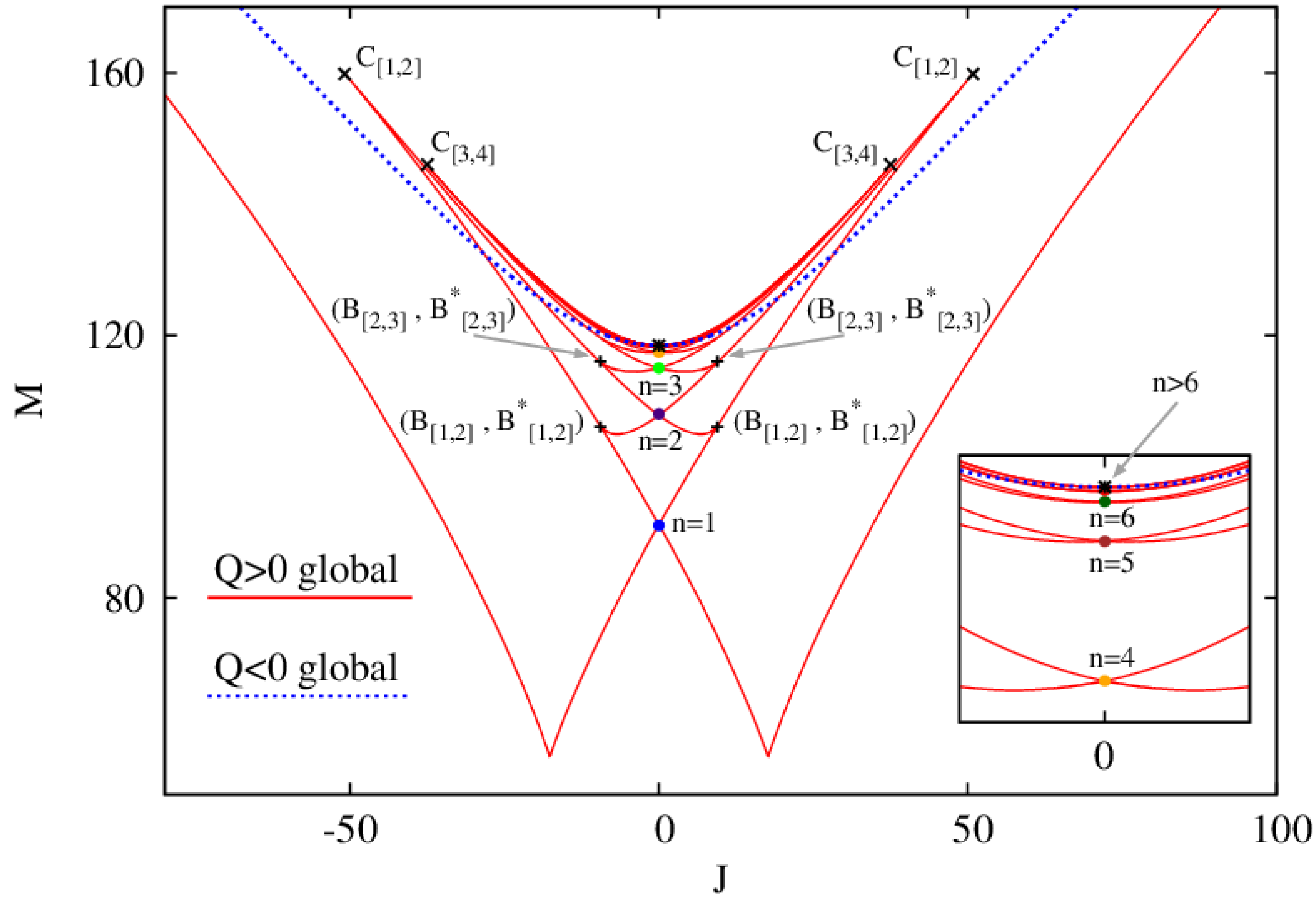
|| 3. Numerical Results ||

Area vs angular momentum (near-horizon and global solutions)



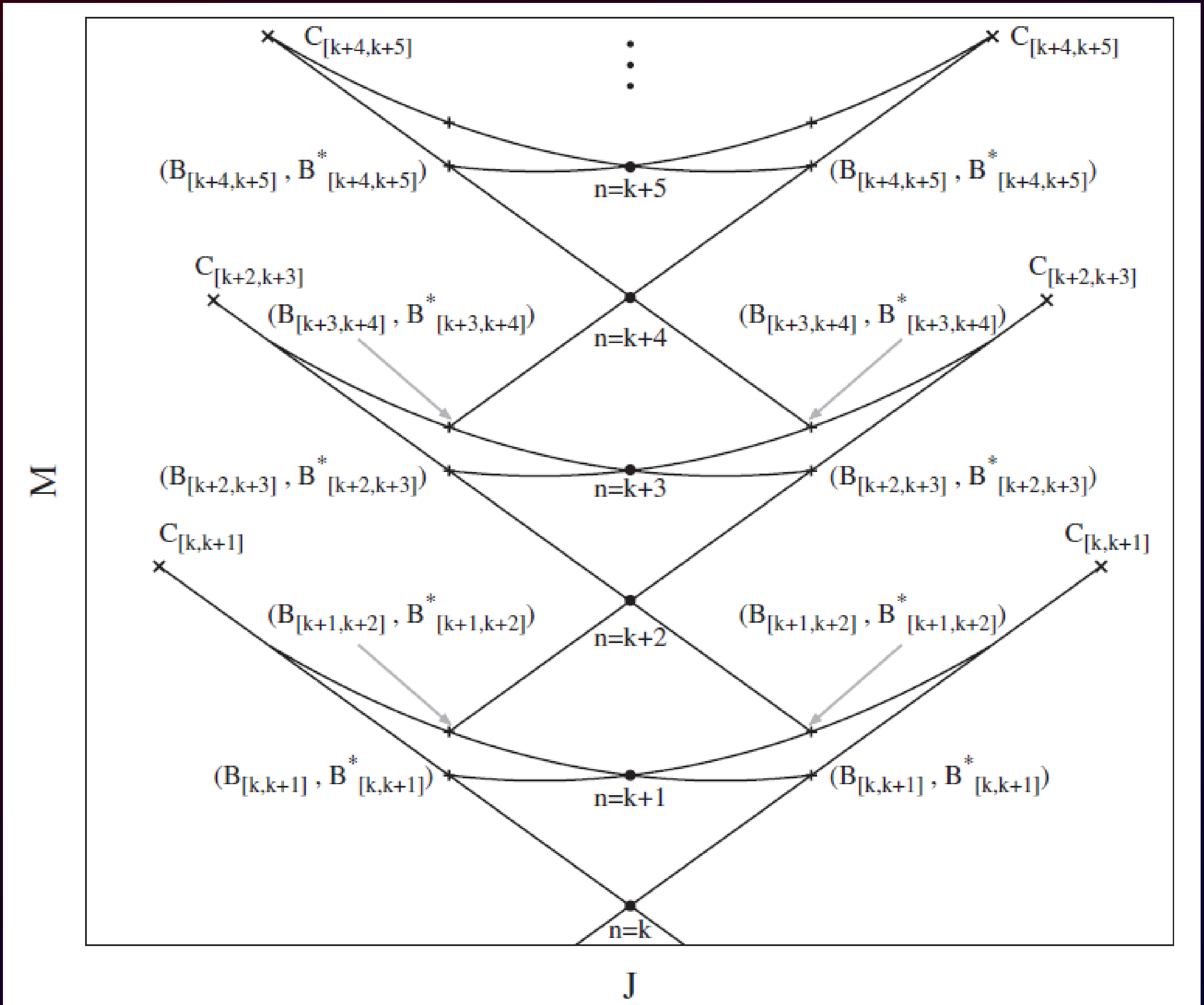
|| 3. Numerical Results ||

Branch structure in the mass vs angular momentum plot:



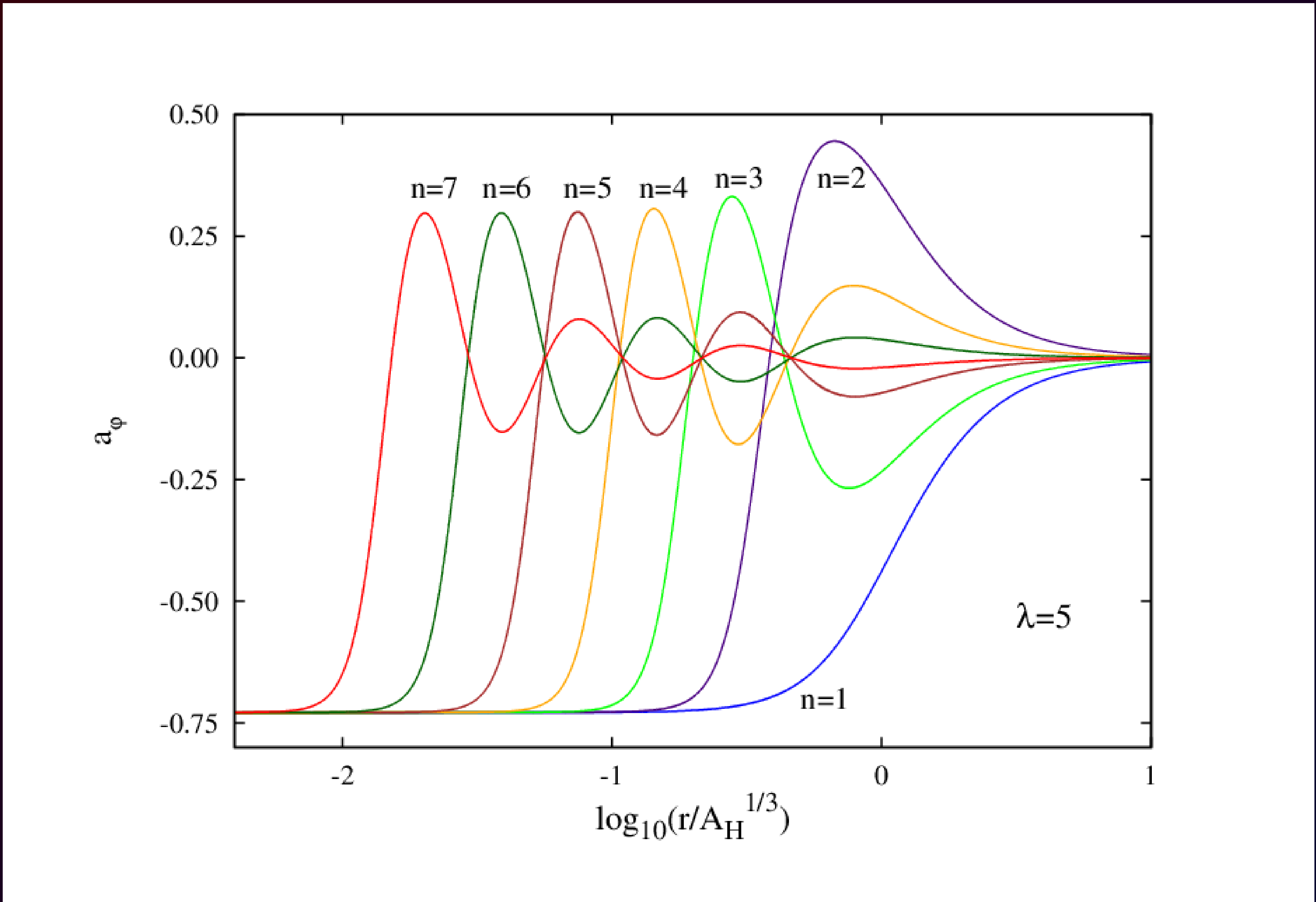
|| 3. Numerical Results ||

Branch structure scheme:



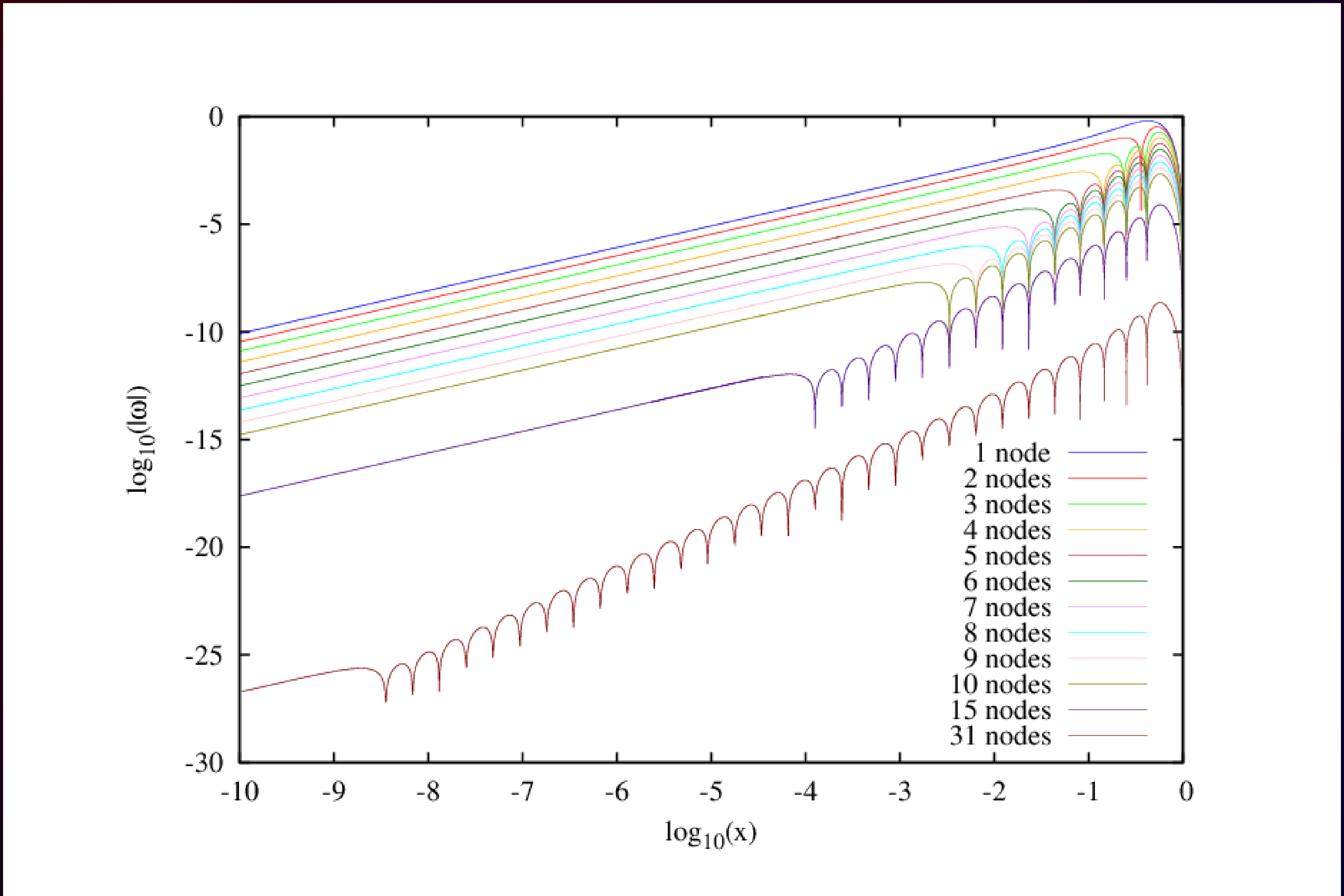
|| 3. Numerical Results ||

J=0 non-static solutions



|| 3. Numerical Results ||

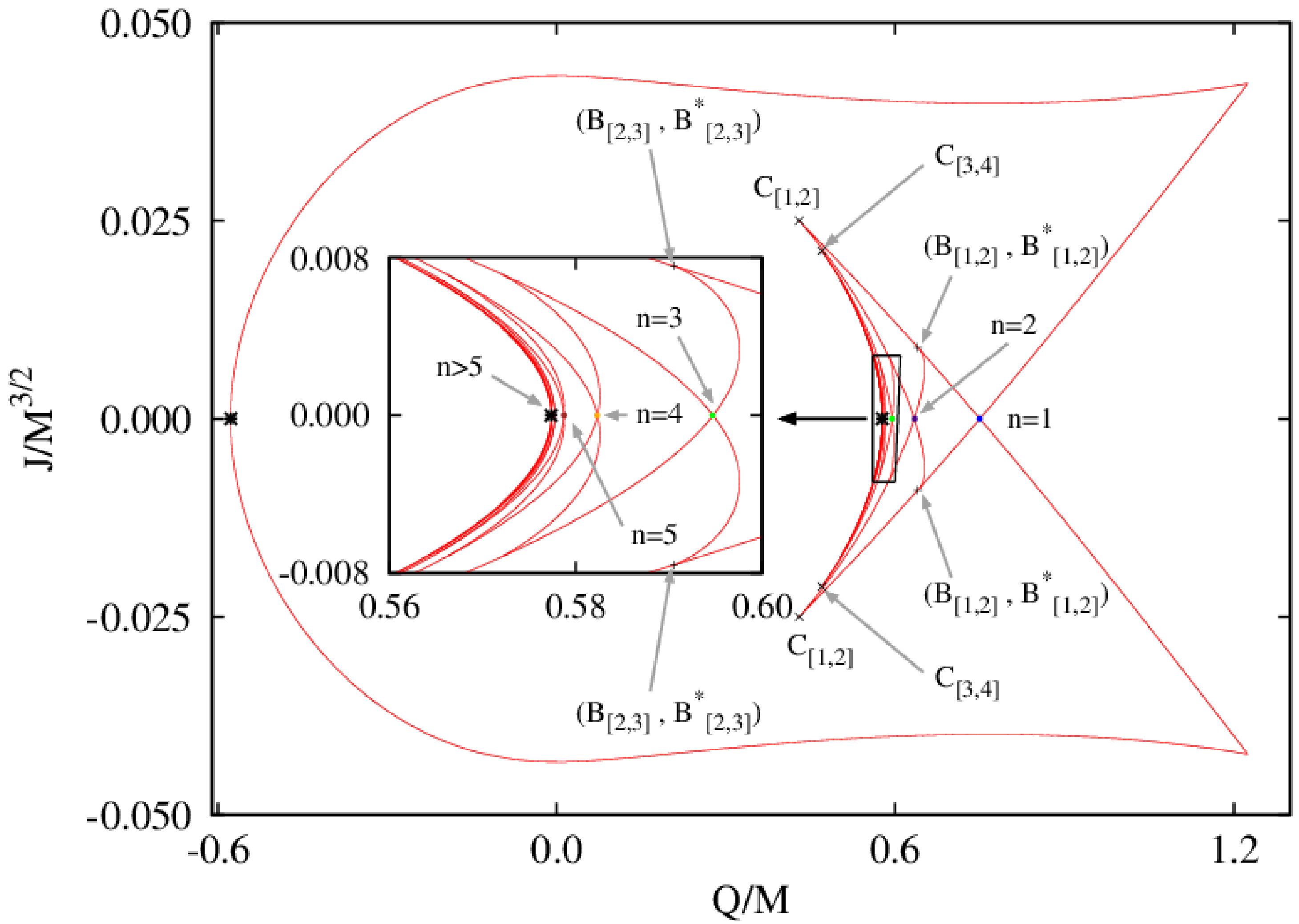
J=0 non-static solutions



3. Numerical results

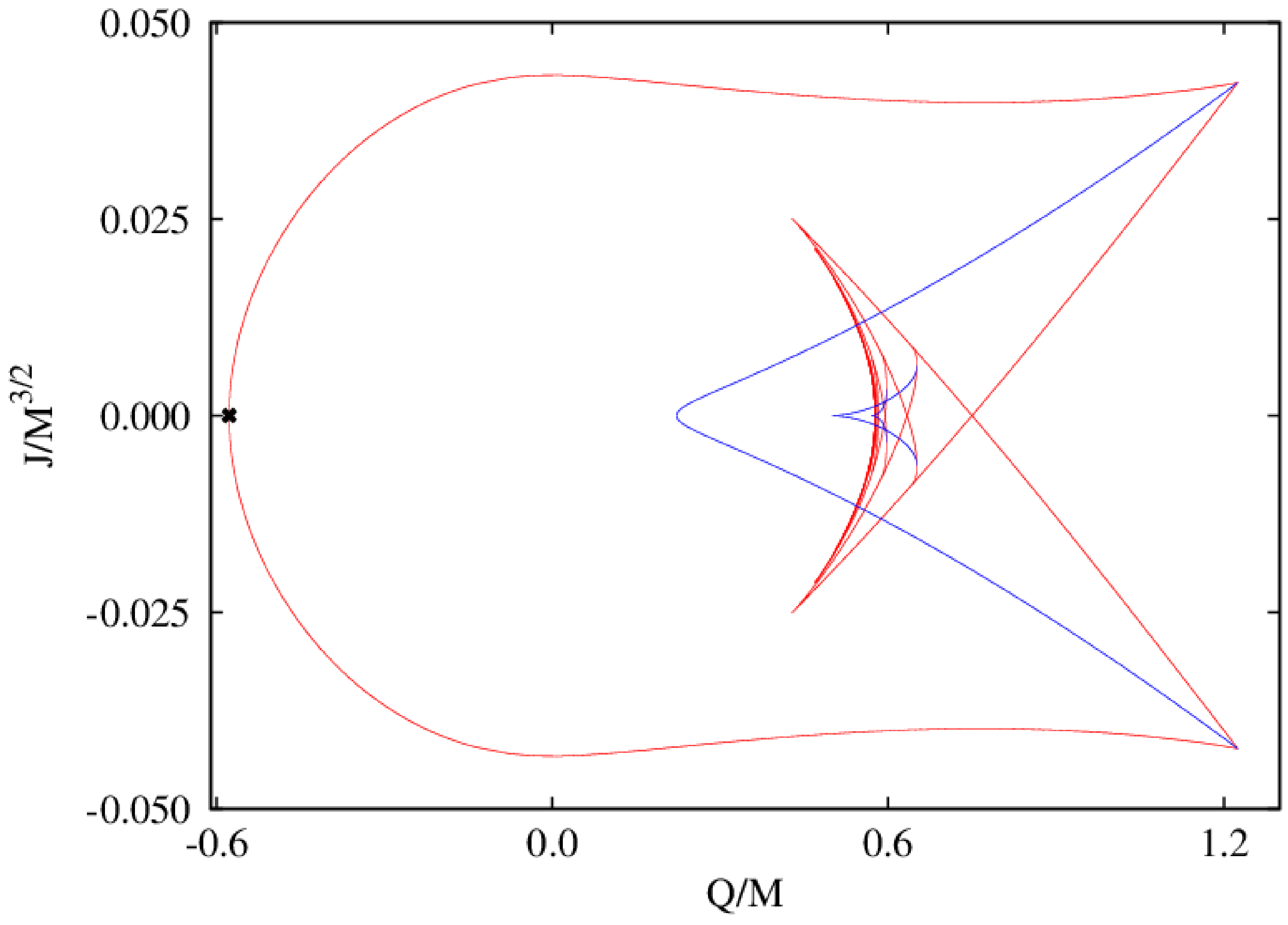
Domain of existence

Domain of existence: Extremal solutions



|| 3. Numerical Results ||

Extremal vs $\Omega_H=0$



4. Future work:
Including negative cosmological constant

|| 4. Future work ||

Black holes in Einstein-Maxwell-Chern-Simons with negative cosmological constant.

- Black holes no longer asymptotically flat: Anti-de-Sitter
- Same subset of solutions: equal-magnitude angular momenta, spherical topology...
- Non-extremal black holes in EM where previously studied in

Jutta Kunz, Francisco Navarro-Lérida, Eugen Radu, Higher dimensional rotating black holes in Einstein-Maxwell theory with negative cosmological constant, Physics Letters B 649 (2007) 463-471

- Extremal black holes also have non-integer exponents near the horizon
- The near-horizon formalism gives us some interesting analytical results: richer branch structure
- Branch structure in global charges similar to the EMCS flat case for a certain range of the parameters of the theory

Thank you for your attention!

Jose Luis Blazquez-Salcedo, Jutta Kunz,
Francisco Navarro Lerida, Eugen Radu,
Sequences of Extremal Radially Excited
Rotating Black Holes, Physical Review
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