# Composite localized field solutions in the Einstein-Yang-Mills theory in AdS spacetime

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# Outline

# Bartnik-McKinnon Solutions

**Bartnik-McKinnon Solutions** - particle like numerical solutions of EYM equation.
**Spherically symmetric SU(2) gauge ansatz:**

$$A = \omega(\mathbf{T_2} d\theta - \mathbf{T_1} \sin\theta d\varphi) + \mathbf{T_3} \cos\theta d\varphi$$

**Spacetime metric:**

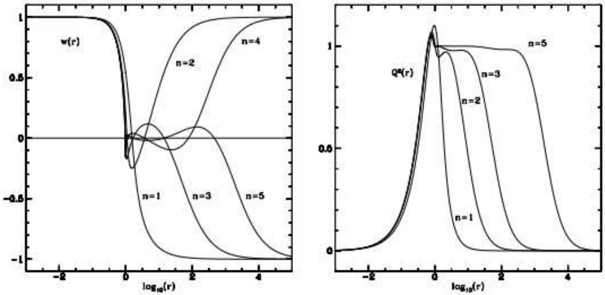
$$ds^2 = \frac{dr^2}{N} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) - \sigma N dt^2, \text{ with } N = 1 - \frac{2m}{r}$$

**Vacuum states:** $\omega(\infty) = (-1)^k$

R. Bartnik and J. McKinnon, "Particle - Like Solutions of the Einstein Yang-Mills Equations", Phys. Rev. Lett. **61** (1988) 141.

# Bartnik-McKinnon Solutions

**Effective charge density:** $N(r) = 1 - \frac{2M}{r} + \frac{Q^2(r)}{r}$



# The Model

**Action of $SU(2)$ Einstein-Yang-Mills theory**

$$I_{bulk} = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}(R-2\Lambda) - \mathrm{Tr}\{\frac{1}{2}F_{\mu\nu}F^{\mu\nu}\}\right]$$

with

- $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ie[A_\mu, A_\nu]$ - $SU(2)$ field strength tensor
- $R$ - Ricci scalar
- $\Lambda$ - cosmological constant

# Ansatz

Axially symmetric ansatz invariant under the action of Killing vectors $\partial/\partial\varphi$ and $\partial/\partial t$:

$$A_\mu dx^\mu = \left(\frac{H_1}{r}dr + (1-H_2)d\theta\right)\frac{u_\varphi^{(n)}}{2e}$$
$$-n\sin\theta\left(H_3\frac{u_r^{(n)}}{2e} + (1-H_4)\frac{u_\theta^{(n)}}{2e}\right)d\varphi,$$

where $u_a^{(n)}$ - $SU(2)$-matrices.

**Gauge transformation:**

$$U = \exp\{i\Gamma(r,\theta)u_\varphi^{(n)}/2\}$$

**Gauge condition:**

$$r\partial_r H_1 - \partial_\theta H_2 = 0$$

# Boundary Conditions at $\infty$

## Odd $m$

$$H_1 = 0, \quad H_2 = 1 - m + w_0$$
$$H_3 = \frac{\cos\theta}{\sin\theta}\Big(\cos((m-1)\theta) - 1\Big) + w_0 \sin((m-1)\theta)$$
$$H_4 = -\frac{\cos\theta}{\sin\theta}\sin((m-1)\theta) + w_0\cos((m-1)\theta)$$

**Magnetic charge:** $Q_M = n|1 - w_0^2|$

## Even $m$

$$H_1 = 0, \quad H_2 = 1 - mw_0$$
$$H_3 = w_0 \frac{\cos((m-1)\theta) - \cos\theta}{\sin\theta}$$
$$H_4 = 1 - w_0 - w_0 \frac{\sin((m-1)\theta}{\sin\theta}$$

**Magnetic charge:** $Q_M = \frac{mn}{2}|(1 - w_0)w_0|$

# Einstein-deTurck Equation

- M. Headrick, S. Kitchen and T. Wiseman, Class. Quant. Grav. **27** (2010) 035002 [arXiv:0905.1822 [gr-qc]].
- A. Adam, S. Kitchen and T. Wiseman, Class. Quant. Grav. **29** (2012) 165002 [arXiv:1105.6347 [gr-qc]].
- T. Wiseman, arXiv:1107.5513 [gr-qc].

**Harmonic Einstein Equation:**

$$R_{\mu\nu} - \nabla_{(\mu}\xi_{\nu)} - \Lambda g_{\mu\nu} = 0$$

**DeTurck choice of $\xi$:**

$$\xi^\mu = g^{\nu\rho}(\Gamma^\mu_{\nu\rho} - \bar{\Gamma}^\mu_{\nu\rho})$$

**Advantage:** better quality of the numerical results

# Reference Metric

The **AdS₄** spacetime metric in global coordinates

$$ds^2 = \frac{dr^2}{1+\frac{r^2}{\ell^2}} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) - (1+\frac{r^2}{\ell^2})dt^2, \Lambda = -\frac{3}{\ell^2}.$$

# Reference Metric

The **AdS$_4$** spacetime metric in global coordinates

$$ds^2 = \frac{dr^2}{1+\frac{r^2}{\ell^2}} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) - (1+\frac{r^2}{\ell^2})dt^2, \Lambda = -\frac{3}{\ell^2}.$$

**Schwarzschild-AdS** (SAdS) black hole background

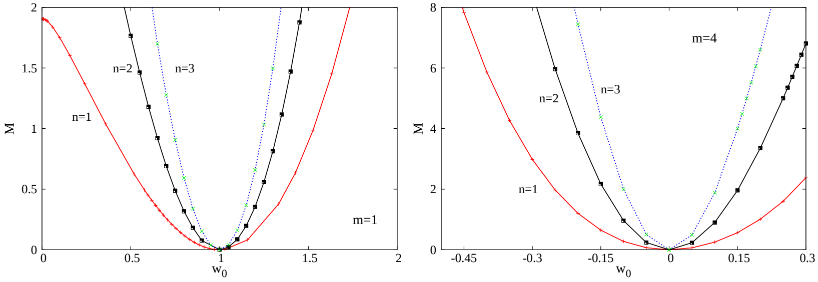
$$ds^2 = \frac{dr^2}{N(r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) - N(r)dt^2, \text{ with } N(r) = 1 - \frac{2M}{r} + \frac{r^2}{\ell^2},$$

In terms of $r_h$

$$N(r) = \left(1 - \frac{r_h}{r}\right)\left(1 + \frac{1}{\ell^2}(r^2 + rr_h + r_h^2)\right).$$

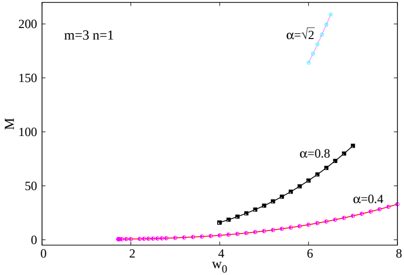
**Hawking temperature**: $T_H = \frac{1}{r_h} + \frac{3r_h}{\ell^2}$

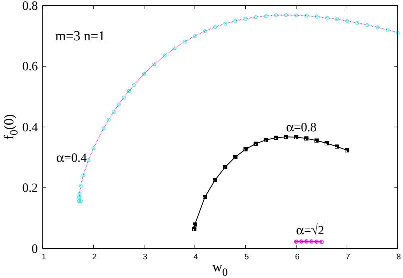
# Results




Solutions always exist for a single $w_0$-interval only, with $w_0^{min} < w_0 < w_0^{max}$ ($w_0$ defines $Q_M$)

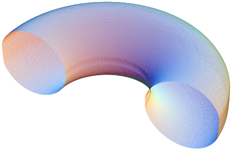
# Results




- $w_0$-interval decreases with increasing $\alpha$
- the decrease of the size of the $w_0$-interval results in the disappearance of the solutions with a zero net magnetic charge, $Q_M = 0$

# Composite Configurations

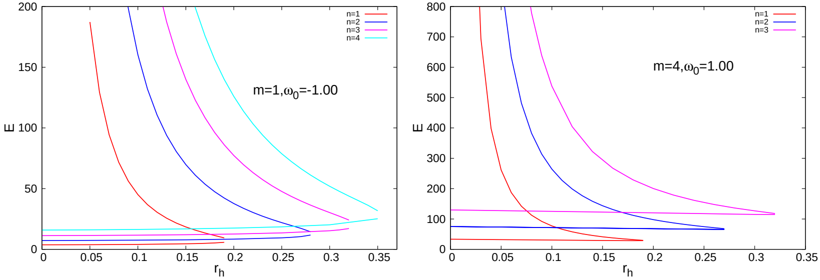


- For $m = 1, 3$ the energy is located in a small region around the origin. The energy density exhibits a torus-like structure, with a maximum being located in the equatorial plane. A double-tori structure appears for large enough values of $n$.
- For $m > 3$ solutions are composite configurations with a dumbbell-like structure. Such configurations have two distinct components, located symmetrically with respect to the equatorial plane, and kept in balance due to the repulsive stresses of the YM fields.

# Including Black Hole Horizon

**Schwarzschild-AdS** (SAdS) black hole background

$$ds^2 = \frac{dr^2}{N(r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) - N(r)dt^2, \text{ with } N(r) = 1 - \frac{2M}{r} + \frac{r^2}{\ell^2}$$



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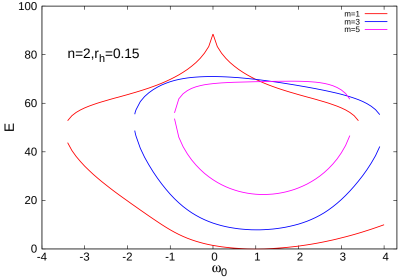


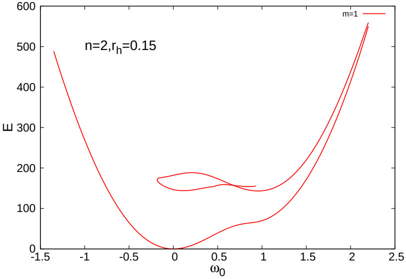

# Conclusions

- In case of back reaction there are no magnetically neutral configurations
- There are balanced, regular composite configurations, with several distinct components.

**Further work:**

- Include an electric potential.
- Constract a black hole solution using deTurck approach

# Thank You for Your Attention!