Strong magnetic fields around black holes

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Black holes in magnetic universes

- Harrison transformation applied to the Kerr-Newman solution gives the MKN black hole

$$\boldsymbol{g} = |\Lambda|^2\, \Sigma \left[-\frac{\Delta}{\mathscr{A}}\, \mathbf{d}t^2 + \frac{\mathbf{d}r^2}{\Delta} + \mathbf{d}\vartheta^2 \right] + \frac{\mathscr{A}}{\Sigma\, |\Lambda|^2} \sin^2\vartheta \,(\mathbf{d}\varphi - \omega\, \mathbf{d}t)^2$$

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- Function Λ is constructed from Ernst potentials

$$\begin{aligned}\Lambda = 1 + \frac{1}{4}B^2 \left(\frac{\mathscr{A} + a^2 Q^2\left(1+\cos^2\vartheta\right)}{\Sigma}\sin^2\vartheta + Q^2\cos^2\vartheta\right) + \\ + \frac{BQ}{\Sigma}\left[ar\sin^2\vartheta - \mathrm{i}\left(r^2+a^2\right)\cos\vartheta\right] - \\ - \frac{\mathrm{i}}{2}B^2 a\cos\vartheta\left[M\left(3-\cos^2\vartheta\right) + \frac{Ma^2\sin^2\vartheta - Q^2 r}{\Sigma}\sin^2\vartheta\right]\end{aligned}$$

- Complicated form of dragging potential ω and of electromagnetic field

Black holes in magnetic universes II

- **Gibbons**, G. W., **Mujtaba**, A. H., **Pope**, C. N. *Ergoregions in magnetized black hole spacetimes*. Classical and Quantum Gravity, 30. 2013.
- Problem with interpretation of the azimuthal coordinate, axis regular if

$$\varphi \in \left(0, 2\pi\left[1 + \frac{3}{2}B^2Q^2 + 2B^3MQa + B^4\left(\frac{1}{16}Q^4 + M^2a^2\right)\right]\right)$$

- Non-flat asymptotics resembling Melvin magnetic universe, approximately flat region if

$$r_+ \ll r \ll \frac{1}{B}$$

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- Note: Generalised electrostatic potential $\phi = -A_t - \omega A_\varphi$

Recipe for near-horizon limit of the metric

- **Carter**, Brandon. *Black hole equilibrium states*. Les Houches Lectures, 1972.
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- One can factorise out a function Δ from some metric coefficients

$$\boldsymbol{g} = -\Delta \tilde{N}^2 \, \mathbf{d}t^2 + g_{\varphi\varphi} \, (\mathbf{d}\varphi - \omega \, \mathbf{d}t)^2 + \frac{\tilde{g}_{rr}}{\Delta} \, \mathbf{d}r^2 + g_{\vartheta\vartheta} \, \mathbf{d}\vartheta^2$$

- Let us assume that the black hole is extremal and coordinate r is chosen so that hypersurface $r = r_0$ is the degenerate horizon
- Then set

$$\Delta = (r - r_0)^2$$

Recipe for near-horizon limit of the metric II

- Coordinate transformation depending on a limiting parameter p

$$r = r_0 + p\chi$$
$$t = \frac{\tau}{p}$$

- Assume that $\tilde{N}, \tilde{g}_{rr}, g_{\varphi\varphi}, g_{\vartheta\vartheta}$ as functions of χ have a finite, nonzero limit for $p \to 0$, so that just this expression is left to resolve

$$\mathbf{d}\varphi - \omega\,\mathbf{d}t$$

- Expansion of the dragging potential

$$\omega \doteq \omega_{\mathrm{H}} + \left.\frac{\partial\omega}{\partial r}\right|_{r_0} (r - r_0) = \omega_{\mathrm{H}} + \left.\frac{\partial\omega}{\partial r}\right|_{r_0} p\chi$$

Recipe for near-horizon limit of the metric III

- Plugging in the expansion we get

$$\mathbf{d}\varphi - \omega\,\mathbf{d}t \doteq \mathbf{d}\varphi - \left(\omega_{\mathrm{H}} + \left.\frac{\partial\omega}{\partial r}\right|_{r_0} p\chi\right)\frac{\mathbf{d}\tau}{p} = \mathbf{d}\varphi - \frac{\omega_{\mathrm{H}}}{p}\,\mathbf{d}\tau - \left.\frac{\partial\omega}{\partial r}\right|_{r_0}\chi\,\mathbf{d}\tau$$

- It is necessary to add „rewinding“ of the azimuthal angle

$$\varphi = \psi + \frac{\omega_{\mathrm{H}}}{p}\tau$$

Recipe for near-horizon limit of the metric III

- Plugging in the expansion we get

$$\mathbf{d}\varphi - \omega\,\mathbf{d}t \doteq \mathbf{d}\varphi - \left(\omega_{\mathrm{H}} + \left.\frac{\partial\omega}{\partial r}\right|_{r_0} p\chi\right)\frac{\mathbf{d}\tau}{p} = \mathbf{d}\varphi - \frac{\omega_{\mathrm{H}}}{p}\,\mathbf{d}\tau - \left.\frac{\partial\omega}{\partial r}\right|_{r_0}\chi\,\mathbf{d}\tau$$

- It is necessary to add „rewinding“ of the azimuthal angle

$$\varphi = \psi + \frac{\omega_{\mathrm{H}}}{p}\tau$$

- For the Kerr-Newman solution we get the following limiting metric

$$\boldsymbol{g} = \left[Q^2 + a^2\left(1 + \cos^2\vartheta\right)\right]\left(-\frac{\chi^2}{\left(Q^2 + 2a^2\right)^2}\,\mathbf{d}\tau^2 + \mathbf{d}\vartheta^2 + \frac{\mathbf{d}\chi^2}{\chi^2}\right) + \\ + \frac{\left(Q^2 + 2a^2\right)^2}{Q^2 + a^2\left(1 + \cos^2\vartheta\right)}\sin^2\vartheta\left(\mathbf{d}\psi + \frac{2a\sqrt{Q^2 + a^2}\chi}{\left(Q^2 + 2a^2\right)^2}\,\mathbf{d}\tau\right)^2$$

Near-horizon limit of the electromagnetic field

- Again, we should expand to the first order and rearrange the terms

$$\boldsymbol{A} = A_t\,\mathbf{d}t + A_\varphi\,\mathbf{d}\varphi \doteq$$

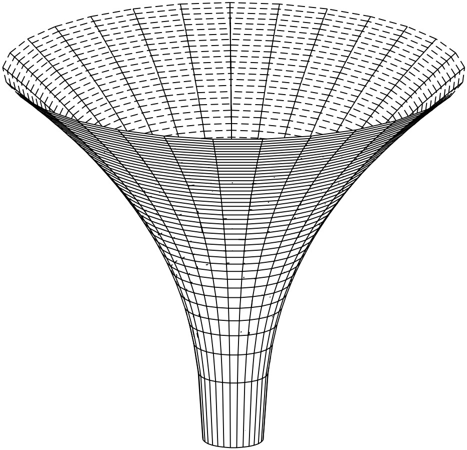
$$\doteq \left(A_t|_{r_0} + \left.\frac{\partial A_t}{\partial r}\right|_{r_0} p\chi \right) \frac{\mathbf{d}\tau}{p} + \left(A_\varphi|_{r_0} + \left.\frac{\partial A_\varphi}{\partial r}\right|_{r_0} p\chi \right) \left(\mathbf{d}\psi + \frac{\omega_{\mathrm{H}}}{p}\,\mathbf{d}\tau \right) \doteq$$

$$\doteq \left(A_t|_{r_0} + \omega_{\mathrm{H}}\, A_\varphi|_{r_0} \right) \frac{\mathbf{d}\tau}{p} + \left(\left.\frac{\partial A_t}{\partial r}\right|_{r_0} + \omega_{\mathrm{H}} \left.\frac{\partial A_\varphi}{\partial r}\right|_{r_0} \right) \chi\,\mathbf{d}\tau + A_\varphi|_{r_0}\,\mathbf{d}\psi =$$

$$= -\frac{\phi_{\mathrm{H}}}{p}\,\mathbf{d}\tau + \left(\left.\frac{\partial A_t}{\partial r}\right|_{r_0} + \omega_{\mathrm{H}} \left.\frac{\partial A_\varphi}{\partial r}\right|_{r_0} \right) \chi\,\mathbf{d}\tau + A_\varphi|_{r_0}\,\mathbf{d}\psi$$

- We assume the generalised electrostatic potential of the horizon ϕ_{H} to be constant, so we can get rid of the singular term by changing gauge
- It is possible to consider a different gauge, in which the singular term does not emerge at all, but this gauge does not respect the symmetry of the metric

Infinite throat



(Reissner-Nordström, $Q = M, t = \text{const.}$)

Stationary Ernst solution ($a = 0$)

$$\boldsymbol{g} = \left[\left(1+\frac{1}{4}B^2Q^2\right)^2 + B^2Q^2\cos^2\vartheta\right]\left(-\frac{\chi^2}{Q^2}\,\mathbf{d}\tau^2 + \frac{Q^2}{\chi^2}\,\mathbf{d}\chi^2 + Q^2\,\mathbf{d}\vartheta^2\right) + \\ + \frac{Q^2\sin^2\vartheta}{\left(1+\frac{1}{4}B^2Q^2\right)^2 + B^2Q^2\cos^2\vartheta}\left[\mathbf{d}\psi - \frac{2B}{Q}\left(1+\frac{1}{4}B^2Q^2\right)\chi\,\mathbf{d}\tau\right]^2$$

$$A_\tau = \frac{\chi}{Q}\frac{\left(1+\frac{1}{4}B^2Q^2\right)^2 - B^2Q^2\cos^2\vartheta}{\left(1+\frac{1}{4}B^2Q^2\right)^2 + B^2Q^2\cos^2\vartheta}\left(1-\frac{1}{4}B^2Q^2\right)$$

$$A_\psi = \frac{-BQ^2}{1+\frac{3}{2}B^2Q^2+\frac{1}{16}B^4Q^4}\,\frac{\left(1-\frac{1}{16}B^4Q^4\right)\sin^2\vartheta}{\left(1+\frac{1}{4}B^2Q^2\right)^2 + B^2Q^2\cos^2\vartheta}$$

- Interesting case: What if $|BQ| = 2$?

Stationary Ernst solution ($a = 0$)

$$\boldsymbol{g} = \left[\left(1 + \frac{1}{4}B^2Q^2\right)^2 + B^2Q^2\cos^2\vartheta\right]\left(-\frac{\chi^2}{Q^2}\,\mathbf{d}\tau^2 + \frac{Q^2}{\chi^2}\,\mathbf{d}\chi^2 + Q^2\,\mathbf{d}\vartheta^2\right) + \\ + \frac{Q^2\sin^2\vartheta}{\left(1+\frac{1}{4}B^2Q^2\right)^2 + B^2Q^2\cos^2\vartheta}\left[\mathbf{d}\psi - \frac{2B}{Q}\left(1 + \frac{1}{4}B^2Q^2\right)\chi\,\mathbf{d}\tau\right]^2$$

$$A_\tau = \frac{\chi}{Q}\frac{\left(1+\frac{1}{4}B^2Q^2\right)^2 - B^2Q^2\cos^2\vartheta}{\left(1+\frac{1}{4}B^2Q^2\right)^2 + B^2Q^2\cos^2\vartheta}\left(1 - \frac{1}{4}B^2Q^2\right)$$

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- Interesting case: What if $|BQ| = 2$?
- It actually becomes near-horizon Kerr with $|\hat{a}| = 2\,|Q|$!

Ernst-Wild solution ($Q = 0$)

$$\boldsymbol{g} = \left[\left(1 + B^4 a^4\right)\left(1 + \cos^2\vartheta\right) + 2B^2 a^2 \sin^2\vartheta\right]\left(-\frac{\chi^2}{4a^2}\,\mathbf{d}\tau^2 + \frac{a^2}{\chi^2}\,\mathbf{d}\chi^2 + a^2\,\mathbf{d}\vartheta^2\right) + \\ + \frac{4a^2\sin^2\vartheta}{\left(1 + B^4 a^4\right)\left(1 + \cos^2\vartheta\right) + 2B^2 a^2 \sin^2\vartheta}\left[\mathbf{d}\psi + \frac{\chi}{2a^2}\left(1 - B^4 a^4\right)\mathbf{d}\tau\right]^2$$

$$A_\tau = -B\chi\left(1 - \frac{2\left(1 + B^2 a^2\right)^2}{\left(1 + B^4 a^4\right)\left(1 + \cos^2\vartheta\right) + 2B^2 a^2 \sin^2\vartheta}\right)$$

$$A_\psi = \frac{1 - B^4 a^4}{1 + B^4 a^4}\,\frac{2Ba^2\sin^2\vartheta}{\left(1 + B^4 a^4\right)\left(1 + \cos^2\vartheta\right) + 2B^2 a^2 \sin^2\vartheta}$$

- Interesting case: What if $|Ba| = 1$?

Ernst-Wild solution ($Q = 0$)

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- Interesting case: What if $|Ba| = 1$?
- It actually becomes near-horizon Reissner-Nordström, i.e. Robinson-Bertotti with $\left|\hat{Q}\right| = 2a$!

General extremal MKN black hole, i.e. $aQB \neq 0$ (raw)

Out[12]=

$$\frac{1}{64\left(a^2+Q^2+a^2\cos[\theta]^2\right)}\left(3a^2+2Q^2+a^2\cos[2\theta]\right)\Big(48a^6B^4+96a^3B^3Q\sqrt{a^2+Q^2}+8aBQ\sqrt{a^2+Q^2}\left(4+5B^2Q^2\right)+8a^4B^2\left(4+9B^2Q^2\right)+2Q^2\left(16+16B^2Q^2+B^4Q^4\right)+a^2\left(48+104B^2Q^2+27B^4Q^4\right)+\left(16a^6B^4+16B^2Q^4+32a^3B^3Q\sqrt{a^2+Q^2}+8a^4B^2\left(-4+3B^2Q^2\right)+8aBQ\sqrt{a^2+Q^2}\left(-4+3B^2Q^2\right)+a^2\left(16-8B^2Q^2+9B^4Q^4\right)\right)\cos[2\theta]\Big)$$

Out[13]=

$$\frac{32a^4B^3Q+32a^2B^3Q^3+16a^5B^4\sqrt{a^2+Q^2}+16a^3B^4Q^2\sqrt{a^2+Q^2}+4BQ^3\left(4+B^2Q^2\right)+a\sqrt{a^2+Q^2}\left(-16+24B^2Q^2+3B^4Q^4\right)}{8\left(2a^2+Q^2\right)^2}$$

Out[14]=

$$\Big(128a^7B^5\sqrt{a^2+Q^2}+192a^5B^3\sqrt{a^2+Q^2}\left(4+B^2Q^2\right)-2Q^3\left(-2+BQ\right)\left(2+BQ\right)\left(16+B^4Q^4\right)-24a^4B^2Q\left(-48-8B^2Q^2+B^4Q^4\right)+8aBQ^2\sqrt{a^2+Q^2}\left(80-16B^2Q^2+3B^4Q^4\right)+8a^3B\sqrt{a^2+Q^2}\left(16+136B^2Q^2+5B^4Q^4\right)+a^2Q\left(64+1232B^2Q^2-148B^4Q^4-9B^6Q^6\right)-\Big(64B^2Q^5-16B^4Q^7+128a^7B^5\sqrt{a^2+Q^2}+64a^5B^3\sqrt{a^2+Q^2}\left(-4+3B^2Q^2\right)-24a^4B^2Q\left(16-24B^2Q^2+B^4Q^4\right)-8aBQ^2\sqrt{a^2+Q^2}\left(16-32B^2Q^2+3B^4Q^4\right)+8a^3B\sqrt{a^2+Q^2}\left(16+8B^2Q^2+5B^4Q^4\right)+a^2Q\left(64-304B^2Q^2+236B^4Q^4-9B^6Q^6\right)\Big)\cos[2\theta]-32a^6B^4Q\left(-20+B^2Q^2\right)\sin[\theta]^2\Big)\Big/\Big(4\left(2a^2+Q^2\right)\Big(48a^6B^4+96a^3B^3Q\sqrt{a^2+Q^2}+8aBQ\sqrt{a^2+Q^2}\left(4+5B^2Q^2\right)+8a^4B^2\left(4+9B^2Q^2\right)+2Q^2\left(16+16B^2Q^2+B^4Q^4\right)+a^2\left(48+104B^2Q^2+27B^4Q^4\right)+\left(16a^6B^4+16B^2Q^4+32a^3B^3Q\sqrt{a^2+Q^2}+8a^4B^2\left(-4+3B^2Q^2\right)+8aBQ\sqrt{a^2+Q^2}\left(-4+3B^2Q^2\right)+a^2\left(16-8B^2Q^2+9B^4Q^4\right)\right)\cos[2\theta]\Big)\Big)$$

Out[15]=

$$-\frac{2B\left(12Q^2+24aBQ\sqrt{a^2+Q^2}+B^2\left(16a^4+16a^2Q^2+Q^4\right)\right)}{16+B^2\left(24Q^2+32aBQ\sqrt{a^2+Q^2}+B^2\left(16a^4+16a^2Q^2+Q^4\right)\right)}+\Big(2\Big(48a^6B^3+72a^3B^2Q\sqrt{a^2+Q^2}+2BQ^4\left(8+B^2Q^2\right)+8a^4B\left(2+9B^2Q^2\right)+2aQ\sqrt{a^2+Q^2}\left(4+15B^2Q^2\right)+a^2BQ^2\left(52+27B^2Q^2\right)+\left(16a^6B^3+8BQ^4+24a^3B^2Q\sqrt{a^2+Q^2}+8a^4B\left(-2+3B^2Q^2\right)+a^2BQ^2\left(-4+9B^2Q^2\right)+2aQ\sqrt{a^2+Q^2}\left(-4+9B^2Q^2\right)\right)\cos[2\theta]\Big)\Big)\Big/\Big(48a^6B^4+96a^3B^3Q\sqrt{a^2+Q^2}+8aBQ\sqrt{a^2+Q^2}\left(4+5B^2Q^2\right)+8a^4B^2\left(4+9B^2Q^2\right)+2Q^2\left(16+16B^2Q^2+B^4Q^4\right)+a^2\left(48+104B^2Q^2+27B^4Q^4\right)+\left(16a^6B^4+16B^2Q^4+32a^3B^3Q\sqrt{a^2+Q^2}+8a^4B^2\left(-4+3B^2Q^2\right)+8aBQ\sqrt{a^2+Q^2}\left(-4+3B^2Q^2\right)+a^2\left(16-8B^2Q^2+9B^4Q^4\right)\right)\cos[2\theta]\Big)$$

General extremal MKN black hole, i.e. $aQB \neq 0$ (2013)

$$\boldsymbol{g} = \underset{\sim}{f}(\vartheta)\left(-\frac{\chi^2}{(Q^2+2a^2)^2}\,\mathbf{d}\tau^2 + \frac{\mathbf{d}\chi^2}{\chi^2} + \mathbf{d}\vartheta^2\right) + \frac{\left(Q^2+2a^2\right)^2\sin^2\vartheta}{\underset{\sim}{f}(\vartheta)}\left(\mathbf{d}\psi - \tilde{\omega}\chi\,\mathbf{d}\tau\right)^2$$

$$\underset{\sim}{f}(\vartheta) = \left(1+\frac{1}{4}B^2Q^2\right)^2\left[Q^2+a^2\left(1+\cos^2\vartheta\right)\right] + 2\left(1+\frac{1}{4}B^2Q^2\right)\left[B^2a^2\left(Q^2+a^2\right)+BQa\sqrt{Q^2+a^2}\right]\sin^2\vartheta + $$
$$+ B^2\left(Ba\sqrt{Q^2+a^2}+Q\right)^2\left[a^2+\left(Q^2+a^2\right)\cos^2\vartheta\right]$$

$$\tilde{\omega} = \frac{1}{(Q^2+2a^2)^2}\left\{2BQ^3\left(1+\frac{1}{4}B^2Q^2+2B^2a^2\right)+4B^3Qa^4 + \right.$$
$$\left. + a\sqrt{Q^2+a^2}\left[-2\left(1-B^4a^4\right)+2B^4Q^2a^2+3B^2Q^2+\frac{3}{8}B^4Q^4\right]\right\}$$

$$A_\tau = \frac{1}{\underset{\sim}{f}(\vartheta)}\frac{\chi}{Q^2+2a^2}\left[Q\left(1-\frac{1}{4}B^2Q^2\right)+2Ba\sqrt{Q^2+a^2}\right]\left\{\left[Q\left(1-\frac{1}{4}B^2Q^2\right)+\right.\right.$$
$$\left. + 2Ba\sqrt{Q^2+a^2}\right]^2 + \left[\left(1+\frac{1}{4}B^2Q^2\right)^2a^2 + B^2\left(Ba\sqrt{Q^2+a^2}+Q\right)^2\left(Q^2+a^2\right) - \right.$$
$$\left.\left. - 2\left(1+\frac{1}{4}B^2Q^2\right)\left[B^2a^2\left(Q^2+a^2\right)+BQa\sqrt{Q^2+a^2}\right]\right]\sin^2\vartheta\right\}$$

$$A_\psi = -\frac{\tilde{\omega}}{2\underset{\sim}{f}(\vartheta)}\,\frac{(Q^2+2a^2)^2\left[Q\left(1-\frac{1}{4}B^2Q^2\right)+2Ba\sqrt{Q^2+a^2}\right]\sin^2\vartheta}{1+\frac{3}{2}B^2Q^2+2B^3Qa\sqrt{Q^2+a^2}+B^4\left(\frac{1}{16}Q^4+Q^2a^2+a^4\right)}$$

General extremal MKN black hole, i.e. $aQB \neq 0$ (2014)

$$g = \underset{\sim}{f}(\vartheta)\left(-\frac{\chi^2}{(Q^2+2a^2)^2}\,\mathbf{d}\tau^2 + \frac{\mathbf{d}\chi^2}{\chi^2} + \mathbf{d}\vartheta^2\right) + \frac{\left(Q^2+2a^2\right)^2 \sin^2\vartheta}{\underset{\sim}{f}(\vartheta)}\left(\mathbf{d}\psi - \tilde{\omega}\chi\,\mathbf{d}\tau\right)^2$$

$$\underset{\sim}{f}(\vartheta) = \left[\sqrt{Q^2+a^2}\left(1+\frac{1}{4}B^2Q^2+B^2a^2\right)+BQa\right]^2 + \\ + \left[a\left(1-\frac{3}{4}B^2Q^2-B^2a^2\right)-BQ\sqrt{Q^2+a^2}\right]^2\cos^2\vartheta$$

$$\tilde{\omega} = -\frac{2}{(Q^2+2a^2)^2}\left[\sqrt{Q^2+a^2}\left(1+\frac{1}{4}B^2Q^2+B^2a^2\right)+ \\ + BQa\right]\left[a\left(1-\frac{3}{4}B^2Q^2-B^2a^2\right)-BQ\sqrt{Q^2+a^2}\right]$$

$$A_\tau = \frac{1}{\underset{\sim}{f}(\vartheta)}\frac{\chi}{Q^2+2a^2}\left[Q\left(1-\frac{1}{4}B^2Q^2\right)+2Ba\sqrt{Q^2+a^2}\right]\left\{\left[Q\left(1-\frac{1}{4}B^2Q^2\right)+ \right.\right. \\ \left.\left. + 2Ba\sqrt{Q^2+a^2}\right]^2 + \left[a\left(1-\frac{3}{4}B^2Q^2-B^2a^2\right)-BQ\sqrt{Q^2+a^2}\right]^2\sin^2\vartheta\right\}$$

$$A_\psi = -\frac{\tilde{\omega}}{2\underset{\sim}{f}(\vartheta)}\frac{(Q^2+2a^2)^2\left[Q\left(1-\frac{1}{4}B^2Q^2\right)+2Ba\sqrt{Q^2+a^2}\right]\sin^2\vartheta}{1+\frac{3}{2}B^2Q^2+2B^3Qa\sqrt{Q^2+a^2}+B^4\left(\frac{1}{16}Q^4+Q^2a^2+a^4\right)}$$

The equivalence

- The near-horizon limiting description of a MKN black hole can be obtained using effective parameters from the one of the Kerr-Newman solution (they are mathematically equivalent)

$$\hat{M} = \sqrt{Q^2 + a^2}\left(1 + \frac{1}{4}B^2Q^2 + B^2a^2\right) + BQa$$

$$\hat{a} = a\left(1 - \frac{3}{4}B^2Q^2 - B^2a^2\right) - BQ\sqrt{Q^2 + a^2}$$

$$\hat{Q} = Q\left(1 - \frac{1}{4}B^2Q^2\right) + 2Ba\sqrt{Q^2 + a^2}$$

The equivalence

- The near-horizon limiting description of a MKN black hole can be obtained using effective parameters from the one of the Kerr-Newman solution (they are mathematically equivalent)

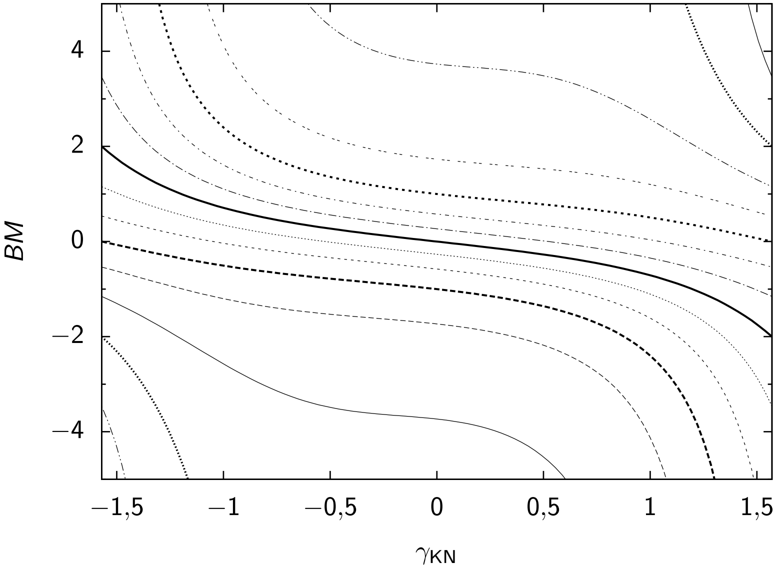
$$\hat{M} = \sqrt{Q^2 + a^2}\left(1 + \frac{1}{4}B^2Q^2 + B^2a^2\right) + BQa$$

$$\hat{a} = a\left(1 - \frac{3}{4}B^2Q^2 - B^2a^2\right) - BQ\sqrt{Q^2 + a^2}$$

$$\hat{Q} = Q\left(1 - \frac{1}{4}B^2Q^2\right) + 2Ba\sqrt{Q^2 + a^2}$$

- Magnetic field in the vicinity of the degenerate horizon has *internal* character, which is an indication of so called Meissner effect
- **Karas**, V., **Budínová**, Z. *Magnetic Fluxes Across Black Holes in a Strong Magnetic Field Regime*. Physica Scripta, 61, 253-256. 2000.

Parameter space of a near-horizon MKN: $a = M\cos\gamma_{KN}$, $Q = M\sin\gamma_{KN}$



Thank you for your attention