Temperature dependence of the absorption cross section for charged black holes

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String frame

- A **string frame lagrangian** naturally results from string perturbative calculations, given in terms of the string coupling constant e^{ϕ}:

$$\mathrm{e}^{-2\phi}\,(\cdots) + 1\,(\cdots) + \mathrm{e}^{2\phi}\,(\cdots) + \ldots$$

- At tree level, in d dimensions, with arbitrary terms $I_i(\mathcal{R}, \mathcal{M})$:

$$\frac{1}{16\pi G}\sqrt{-g}\;\mathrm{e}^{-2\phi}\left(\mathcal{R} + 4\,(\partial^\mu\phi)\,\partial_\mu\phi + \sum_i I_i(\mathcal{R}, \mathcal{M})\right).$$

- Each $I_i(\mathcal{R}, \mathcal{M})$ is a function, with conformal weight w_i, of any given order in α', of the Riemann tensor $\mathcal{R}_{\mu\nu\rho\sigma}$ and any other fields generically designated by $\mathcal{M}$.

From string to Einstein frame

- Metric redefined through a conformal transformation involving the dilaton:

$$g_{\mu\nu} \to \exp\left(\frac{4}{d-2}\phi\right) g_{\mu\nu},$$

$$\mathcal{R}_{\mu\nu}{}^{\rho\sigma} \to \exp\left(-\frac{4}{d-2}\phi\right) \widetilde{\mathcal{R}}_{\mu\nu}{}^{\rho\sigma},$$

$$\widetilde{\mathcal{R}} = \mathcal{R} + 4\frac{d-1}{d-2}\nabla^2\phi - 4\frac{d-1}{d-2}\left(\partial^\mu \phi\right)\partial_\mu \phi.$$

- The same term in the conventional **Einstein frame**:

$$\frac{1}{16\pi G}\sqrt{-g}\left(\mathcal{R} - \frac{4}{d-2}\left(\partial^\mu \phi\right)\partial_\mu \phi + \sum_i \mathrm{e}^{\frac{4}{d-2}(1+w_i)\phi} I_i(\widetilde{\mathcal{R}}, \mathcal{M})\right).$$

Gravitational α' corrections

- Effective action in the Einstein frame

$$\frac{1}{16\pi G}\int \sqrt{-g}\left[\mathcal{R} - \frac{4}{d-2}\left(\partial^{\mu}\phi\right)\partial_{\mu}\phi + \lambda\, \mathrm{e}^{\frac{4}{d-2}(1+w)\phi} Y(\mathcal{R})\right] \mathrm{d}^d x,$$

  $Y(\mathcal{R})$: scalar polynomial in the Riemann tensor with conformal weight w.
  λ : suitable power of α', up to a numerical factor.

- Field equations

$$\begin{aligned}
\mathcal{R}_{\mu\nu} \quad &+ \quad \lambda\, \mathrm{e}^{\frac{4}{d-2}(1+w)\phi}\left(\frac{\delta Y(\mathcal{R})}{\delta g^{\mu\nu}} + \frac{1}{d-2} Y(\mathcal{R}) g_{\mu\nu} - \frac{1}{d-2} g_{\mu\nu} g^{\rho\sigma}\frac{\delta Y(\mathcal{R})}{\delta g^{\rho\sigma}}\right) = 0;\\
\nabla^2\phi \quad &- \quad \frac{\lambda}{2}\, \mathrm{e}^{\frac{4}{d-2}(1+w)\phi}\, Y(\mathcal{R}) = 0.
\end{aligned}$$

Background nonextremal black hole

- Asymptotically flat, spherically symmetric metric in the Einstein frame of the type

$$d\,s^2 = -f(r)\,d\,t^2 + g^{-1}(r)\,d\,r^2 + r^2\,d\,\Omega^2_{d-2};$$

- General assumption for the α' corrected solution:

$$f(r) = f_0(r)\,(1 + \lambda f_c(r))\,,\;\; g(r) = f_0(r)\,(1 + \lambda g_c(r))\,.$$

- Tangherlini solution: $f_0(r) =: f_0^T(r) = 1 - \left(\frac{R_H}{r}\right)^{d-3};$
- Encoding charges and string effects:

$$f_0(r) = c(r)\left(1 - \left(\frac{R_H}{r}\right)^{d-3}\right).$$

- Horizon radius – string frame: R_H^S; Einstein frame: R_H^E.

The absorption cross–section

- In classical EH gravity, for *any* spherically symmetric black hole in arbitrary d, the absorption cross–section of minimally–coupled massless scalars in the low–frequency limit is (Das, Gibbons, Mathur, 1997)

$$\sigma = A_H = 4GS.$$

A_H : horizon area with respect to the induced metric.

- In the presence of leading α' corrections, for the generic metrics considered (Moura, 2013),

$$\sigma = A_H \left(1 - \lambda \, \frac{f_c(R_H) + g_c(R_H)}{2}\right).$$

- We should obtain a **covariant, frame–independent** formula for σ, in terms of **physical quantities**.

Black hole temperature

- Wick–rotate to Euclidean time $t = i\tau$; the resulting manifold has no conical singularities as long as τ is a periodic variable, with a period $\beta = \frac{1}{T}$.
- Smoothness condition:

  $2\pi = \lim_{r\to R_H} \frac{\beta}{g^{-\frac{1}{2}}(r)} \frac{df^{\frac{1}{2}}(r)}{dr}$,

  or

$$T = \lim_{r\to R_H} \frac{\sqrt{g}}{2\pi} \frac{d\sqrt{f}}{d\,r}.$$

- In our case,

$$T = \frac{f_0'(R_H)}{4\pi} \left(1 + \lambda\, \frac{f_c(R_H) + g_c(R_H)}{2}\right)$$

  or concretely

$$T = \frac{d-3}{4\pi R_H} \left(1 + \lambda \delta T\right), \; \delta T = -\delta\sigma.$$

Black hole mass

- The mass can be written with a perturbative multiplicative λ–correction to the classical Tangherlini mass. In the Einstein frame:

$$M = (1 + \lambda\, \delta M_E)\, \frac{(d-2)\, \Omega_{d-2}}{16 \pi G}\, \left(R_H^E\right)^{d-3} .$$

- Both temperature and mass do not depend either on systems of coordinates or on field redefinitions (frames/schemes). In the string frame:

$$M = (1 + \lambda\, \delta M_S)\, \frac{(d-2)\, \Omega_{d-2}}{16 \pi G}\, \left(R_H^S\right)^{d-3} .$$

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Relation between string/Einstein frame

- Equating expressions for the mass, one obtains a relation between the horizon locations in the two different frames:

$$R_H^E = R_H^S \left(1 + \lambda \frac{\delta M_S - \delta M_E}{d-3}\right).$$

- Proceeding analogously with the temperature:

$$R_H^E = R_H^S \left(1 + \lambda \left(\delta T_E - \delta T_S\right)\right).$$

- There must be a relation between the mass and temperature λ corrections such that the two expressions above are the same.

Black hole entropy

- Wald entropy: $S=-2\pi G\int_\Sigma \frac{\partial\mathcal{L}}{\partial\mathcal{R}_{\mu\nu\rho\sigma}}\varepsilon_{\mu\nu}\varepsilon_{\rho\sigma}\sqrt{h}\,d\Omega_{d-2};$
- For spherically symmetric metrics, the only nonzero component of the binormal is $\varepsilon_{tr}=\sqrt{\frac{f}{g}};$
- $S=\frac{1}{4G}\int_H\left(1-2\lambda'\frac{\partial Y(\mathcal{R})}{\partial\mathcal{R}^{trtr}}\right)\sqrt{h}\,d\,\Omega_{d-2}=$
  $\frac{A_H}{4G}-\frac{\lambda'}{2G}\int_H\frac{\partial Y(\mathcal{R})}{\partial\mathcal{R}^{trtr}}\sqrt{h}\,d\,\Omega_{d-2}.$
- In general $S=\frac{A_H}{4G}\left(1+\lambda\,\delta S\right).$

The Callan-Myers-Perry black hole I

- For $Y(\mathcal{R}) = \mathcal{R}^{\mu\nu\rho\sigma}\mathcal{R}_{\mu\nu\rho\sigma}$;
- The only free parameter is the horizon radius R_H (secondary hair), which is not changed;
- $f_0(r) =: f_0^T(r) = 1 - \left(\frac{R_H}{r}\right)^{d-3}$;
- Einstein frame (CMP,1989):

$$f_c(r) = g_c(r) = f_c^{CMP}(r) := -\frac{(d-3)(d-4)}{2}\left(\frac{R_H}{r}\right)^{d-3}\frac{1-\left(\frac{R_H}{r}\right)^{d-1}}{1-\left(\frac{R_H}{r}\right)^{d-3}}.$$

- Temperature:

$$T = \frac{d-3}{4\pi R_H^E}\left(1 + \delta T_E^{CMP}\frac{\alpha'}{4(R_H^E)^2}\right),\ \delta T_E^{CMP} = -\frac{(d-1)(d-4)}{2}.$$

- Mass:

$$M = \left(1 + \delta M_E^{CMP}\frac{\alpha'}{4(R_H^E)^2}\right)\frac{(d-2)\Omega_{d-2}}{16\pi G}\left(R_H^E\right)^{d-3},\ \delta M_E^{CMP} = \frac{(d-3)(d-4)}{2}.$$

Leading α'-corrected dilaton

$$\begin{aligned}\varphi(r) &= \frac{\phi(r)}{\lambda} = \frac{(d-2)^2}{4R_H^2} \ln\left(1-\left(\frac{R_H}{r}\right)^{d-3}\right) - \frac{(d-3)(d-2)^2}{8(d-1)r^2}\left[(d-1)\right.\\ &+ \left. +2\left(\frac{R_H}{r}\right)^{d-3} - 2\frac{d-1}{d-3}\left(\frac{r}{R_H}\right)^2 B\left(\left(\frac{R_H}{r}\right)^{d-3}; \frac{2}{d-3}, 0\right)\right] < 0,\\ \varphi'(r) &= \frac{(d-3)(d-2)^2}{4}\frac{R_H^{d-3}}{r^{d-2}}\frac{1-\left(\frac{R_H}{r}\right)^{d-1}}{1-\left(\frac{R_H}{r}\right)^{d-3}} > 0\end{aligned}$$

with $B(x;\, a, b) = \int_0^x t^{a-1}\,(1-t)^{b-1}\,dt$ (Moura, 2010).
At the horizon,

$$\phi\left(R_H\right) = -\frac{\lambda}{R_H^2}\frac{(d-2)^2}{8(d-1)}\left(d^2-2d+2(d-1)\left(\psi^{(0)}\left(\frac{2}{d-3}\right)+\gamma\right)-3\right),$$

with

$$\psi(z) = \frac{\Gamma'(z)}{\Gamma(z)},\ \psi^{(n)}(z) = \frac{d^n\,\psi(z)}{d\,z^n},\ \gamma = \lim_{n\to\infty}\left(\sum_{k=1}^n \frac{1}{k} - \ln n\right).$$

The Callan-Myers-Perry black hole II

String frame:

$$f_S^{CMP}(r) = f_0^T \left(1 + \frac{\alpha'}{2(R_H^S)^2}\mu(r)\right), \; g_S^{CMP}(r) = f_0^T \left(1 - \frac{\alpha'}{2(R_H^S)^2}\epsilon(r)\right)$$

$$\begin{aligned}
\epsilon(r) &= \frac{(d-3)R_H^{d-5}}{4\left(r^{d-3} - R_H^{d-3}\right)} \left[\frac{(d-2)(d-3)}{2} - \frac{2(2d-3)}{d-1}\right. \\
&+ \left.(d-2)\left(\psi^{(0)}\left(\frac{2}{d-3}\right) + \gamma\right) + d\left(\frac{R_H}{r}\right)^{d-1} + \frac{4R_H^2}{d-2}\varphi(r)\right] \\
\mu(r) &= -\epsilon(r) + \frac{2}{d-2}\left(\varphi(r) - r\,\varphi'(r)\right).
\end{aligned}$$

The Callan-Myers-Perry black hole III

- Temperature (string frame):

$$
\begin{aligned}
T &= \frac{d-3}{4\pi R_H^S}\left(1+\delta T_S^{CMP}\,\frac{\alpha'}{4\left(R_H^S\right)^2}\right),\\
\delta T_S^{CMP} &= -\frac{3d(d-3)\left(d-\frac{5}{3}\right)-2(d-1)^2+2(d-2)(d-1)\left(\psi^{(0)}\left(\frac{2}{d-3}\right)+\gamma\right)}{4(d-1)}.
\end{aligned}
$$

- Mass (string frame):

$$
\begin{aligned}
M &= \left(1+\delta M_S^{CMP}\,\frac{\alpha'}{4\left(R_H^S\right)^2}\right)\frac{(d-2)\,\Omega_{d-2}}{16\pi G}\left(R_H^S\right)^{d-3},\\
\delta M_S^{CMP} &= (d-3)\left(-\delta T_S^{CMP}-\frac{(d-2)(d-4)}{2}\right).
\end{aligned}
$$

The Callan-Myers-Perry black hole IV

- For $Y(\mathcal{R}) = \mathcal{R}^{\mu\nu\rho\sigma}\mathcal{R}_{\mu\nu\rho\sigma}$, $\frac{\partial Y(\mathcal{R})}{\partial \mathcal{R}^{trtr}} = \mathcal{R}_{trtr}$, $\mathcal{R}_{trtr} = \frac{1}{2}f''$;
- $8\pi G \frac{\partial \mathcal{L}}{\partial \mathcal{R}^{\mu\nu\rho\sigma}} \varepsilon^{\mu\nu}\varepsilon^{\rho\sigma} = \left(-\frac{f}{g} + \mathrm{e}^{\frac{4}{d-2}\phi}\frac{\alpha'}{4}f''\right)\frac{g}{f}$;
- One gets for the black hole entropy $\delta S_E \neq \delta S_S$ (effect of the dilaton!):

$$\begin{aligned} S &= \frac{A_H^E}{4G}\left(1 + (d-3)(d-2)\frac{\alpha'}{4\left(R_H^E\right)^2}\right) \\ &= \frac{A_H^S}{4G}\left(1 + (d-2)\left(\delta T_S^{CMP} - \frac{(d-2)(d-5)}{2}\right)\frac{\alpha'}{4\left(R_H^S\right)^2}\right). \end{aligned}$$

- In both frames $\delta S \neq -\delta T$ and therefore $\sigma \neq 4GS$.

Frame-independent entropy formulae

- One can invert mass/temperature relations in order to obtain $R_H(M)$, $R_H(T)$:

$$
\begin{aligned}
R_H(M) &= \frac{1}{\sqrt{\pi}} \left(\frac{8G\, M \Gamma\left(\frac{d-1}{2}\right)}{d-2} \right)^{\frac{1}{d-3}} \left[1 - \frac{\lambda}{(d-3)} \delta M \right], \\
R_H(T) &= \frac{d-3}{4\pi T} \left(1 + \lambda \delta T \right).
\end{aligned}
$$

- Replacing these results in the entropy, we get

$$
\begin{aligned}
S(M) &= 2^{\frac{2d-3}{d-3}} \sqrt{\pi} \left(G \Gamma \left(\frac{d-1}{2} \right) \right)^{\frac{1}{d-3}} \left(\frac{M}{d-2} \right)^{\frac{d-2}{d-3}} \left[1 + \lambda \left(\delta S - \frac{d-2}{d-3} \delta M \right) \right], \\
S(T) &= \frac{\Omega_{d-2}}{4G} \left(\frac{d-3}{4\pi T} \right)^{d-2} \left(1 + \lambda \left(\delta S + (d-2) \delta T \right) \right).
\end{aligned}
$$

The CMP black hole entropy

- One can check that indeed

$$\delta S_E^{CMP} - \frac{d-2}{d-3}\delta M_E^{CMP} = \delta S_S^{CMP} - \frac{d-2}{d-3}\delta M_S^{CMP}$$

,

$$\delta S_E^{CMP} + (d-2)\delta T_E^{CMP} = \delta S_S^{CMP} + (d-2)\delta T_S^{CMP}.$$

- This allows us to obtain

$$\begin{aligned} S(M) &= 2^{\frac{2d-3}{d-3}}\sqrt{\pi}\left(G\Gamma\left(\frac{d-1}{2}\right)\right)^{\frac{1}{d-3}}\left(\frac{M}{d-2}\right)^{\frac{d-2}{d-3}} \\ &\quad \left[1+\alpha'\frac{(d-2)^2}{8}\pi\left(\frac{d-2}{8G\,M\Gamma\left(\frac{d-1}{2}\right)}\right)^{\frac{2}{d-3}}\right], \\ S(T) &= \frac{\Omega_{d-2}}{4G}\left(\frac{d-3}{4\pi T}\right)^{d-2}\left(1-\alpha'\frac{(d-2)^2(d-5)}{8}\left(\frac{4\pi T}{d-3}\right)^2\right) \end{aligned}$$

- These are **frame-independent formulas**!

The CMP absorption cross section

- Proceeding analogously

$$\begin{aligned}\sigma(M) &= 2^{\frac{4d-9}{d-3}}\sqrt{\pi}\left(\Gamma\left(\frac{d-1}{2}\right)\right)^{\frac{1}{d-3}}\left(\frac{GM}{d-2}\right)^{\frac{d-2}{d-3}}\left(1-\lambda\left(\delta T+\frac{d-2}{d-3}\delta M\right)\right),\\ \sigma(T) &= \left(\frac{d-3}{4\pi T}\right)^{d-2}\Omega_{d-2}\left(1+(d-3)\lambda\delta T\right).\end{aligned}$$

- One can check that for the CMP solution

$$\delta T_E^{CMP}+\frac{d-2}{d-3}\delta M_E^{CMP}\neq\delta T_S^{CMP}+\frac{d-2}{d-3}\delta M_S^{CMP}$$

,

$$\delta T_E^{CMP}\neq\delta T_S^{CMP}.$$

- The string-corrected absorption cross section cannot be expressed exclusively as a function of the black hole mass (or temperature) in a way which is independent of metric redefinitions!

Covariant frame-indep cross section

- Only possibility:

$$\sigma = \frac{d-3}{4\pi T} \Omega_{d-2}^{\frac{1}{d-2}} A_H^{\frac{d-3}{d-2}}.$$

- This formula is valid for Tangherlini–like d-dimensional solutions with leading α' corrections.
- Could it be valid for non-spherically symmetric solutions?
- Could it be valid to all orders in α'?
- How about charged solutions?

$d-$dimensional Reissner-Nordström

- $$f(r) = \left(1 - \left(\frac{R_Q}{r}\right)^{d-3}\right)\left(1 - \left(\frac{R_H}{r}\right)^{d-3}\right);$$
  $$R_H^{d-3} = \mu + \sqrt{\mu^2 - q^2}, \qquad R_Q^{d-3} = \mu - \sqrt{\mu^2 - q^2},$$
  $$\mu = \frac{8\pi}{\Omega_{d-2}(d-2)} M, \qquad q^2 = \frac{2}{(d-2)(d-3)} Q^2;$$

- Temperature $T_H = \frac{d-3}{4\pi R_H}\left(1 - \left(\frac{R_Q}{R_H}\right)^{d-3}\right)$, from which $R_H(T_H) = R_Q + \frac{4\pi R_Q^2}{(d-3)^2} T_H + \mathcal{O}\left(T_H^2\right)$ (well defined extremal limit);

- $c(R_H) = 1 - \left(\frac{R_Q}{R_H}\right)^{d-3} = \frac{2\sqrt{\mu^2 - q^2}}{\mu + \sqrt{\mu^2 - q^2}}$ **(frame-independent)**.

Generalization for charged black holes

- Charged solutions: $f(r) = c(r)\left(1 - \left(\frac{R_H}{r}\right)^{d-3}\right);$
- Temperature: $T = \frac{(d-3)c(R_H)}{4\pi R_H};$
- Extremal limit: $c(R_H) \equiv 0;$
- Cross section: $\sigma = \frac{(d-3)c(R_H)}{4\pi T}\Omega_{d-2}^{\frac{1}{d-2}} A_H^{\frac{d-3}{d-2}};$
- Well defined extremal limit (in principle!);
- By defining $\tilde{t} = c(R_H)t, \widetilde{T_H} = \frac{T_H}{c(R_H)}$, σ reduces to the noncharged case.

D-1 D-5 system

- Classical action compactified on $S^1 \otimes T^4$:
  $\frac{1}{16\pi G_{10}} \int \sqrt{-g} e^{-2\phi} \left[\mathcal{R} + 4(\nabla\phi)^2 - \frac{1}{2} |H_3|^2 \right] d^{10}x$
- x^5 with period $2\pi R$, x^i, $i = 6, ..., 9$, with period $2\pi V^{1/4}$.
- $f(r) = 1 - \frac{r_0^2}{r^2}$, $h_{1,5}(r) = 1 + \frac{r_{1,5}^2}{r^2}$, $\begin{pmatrix} t' \\ x'_5 \end{pmatrix} = \begin{pmatrix} \cosh\varsigma & -\sinh\varsigma \\ -\sinh\varsigma & -\cosh\varsigma \end{pmatrix} \begin{pmatrix} t \\ x_5 \end{pmatrix}$
- R-R 2-form field strength: $H_{(3)} = 2r_5^2 \epsilon_3 + 2e^{-2\phi} r_1^2 \star_6 \epsilon_3$,
  $\epsilon_3 = \frac{1}{8} d\theta \wedge \sin\theta d\phi \wedge d\psi$, $\star_6$ is the Hodge dual in $x^0, .., x^5$.
- $$\begin{aligned} ds_{10}^2 &= \frac{1}{\sqrt{h_1(r)h_5(r)}} \left(-f(r)dt'^2 + dx_5'^2 \right) + \sqrt{\frac{h_1(r)}{h_5(r)}} dx_i dx^i \\ &+ \sqrt{h_1(r)h_5(r)} \left(\frac{dr^2}{f(r)} + r^2 d\Omega_{S^3}^2 \right) \end{aligned}$$
- $e^{2\phi} = \frac{h_5(r)}{h_1(r)}$ (Horowitz, Maldacena, Strominger 1996)

Three charges

- Define r_n, α, γ:

$$r_n \equiv r_0 \sinh \varsigma,$$

$$r_1 \equiv r_0 \sinh \alpha, \;\; r_5 \equiv r_0 \sinh \gamma.$$

- Four independent parameters: $r_0,\ r_1,\ r_5,\ r_n,$ or $r_0,\ \alpha,\ \gamma,\ \varsigma$, in terms of which one may write the black hole mass and three $U(1)$ charges.

$$Q_1 \equiv \frac{1}{(2\pi)^2\, g_s} \int_{S^3} e^{2\phi} \star_6 H_{(3)} = \frac{V}{2g_s} r_0^2 \sinh 2\alpha,$$

$$Q_5 \equiv \frac{1}{(2\pi)^2\, g_s} \int_{S^3} H_{(3)} = \frac{1}{2g_s} r_0^2 \sinh 2\gamma,$$

$$n \equiv RP = \frac{R^2 V}{2g_s^2} r_0^2 \sinh 2\varsigma.$$

D-brane description

- **Bound state:** Q_1 fundamental strings wrapping S^1 and Q_5 NS5-branes wrapping $T^4 \otimes S^1$.
- **Excitations:** transverse oscillations, within the NS5-branes, of a single effective string wrapped $Q_1 Q_5$ times around the S^1. These oscillations carry the momentum n and are described by a gas of left and right movers on the string .
- Solution in the **dilute gas limit**: interactions between left and right moving oscillations can be neglected.
- Reduction to five dimensions:

$$ds_5^2 = -h^{-2/3}(r) f(r) dt^2 + h^{1/3}(r) \left(\frac{dr^2}{f(r)} + r^2 d\Omega_3^2 \right),$$

$$h(r) = h_1(r) h_5(r) h_n(r),\ h_n(r) = 1 + \frac{r_n^2}{r^2}.$$

Two large charges

- $r_0, r_n \ll r_1, r_5$: large values of α, γ; Q_1, Q_5 are large compared to n.
- Left and right moving temperatures

$$T_L = \frac{1}{2\pi}\frac{r_0 e^{\varsigma}}{r_1 r_5},\ T_R = \frac{1}{2\pi}\frac{r_0 e^{-\varsigma}}{r_1 r_5}$$

related to Hawking temperature by $T_H^{-1} = \frac{1}{2}(T_L^{-1} + T_R^{-1})$:

$$T_H = \frac{1}{2\pi}\frac{r_0}{r_1 r_5 \cosh\varsigma} \Leftrightarrow e^{\varsigma} = \frac{r_0 + \sqrt{r_0^2 - 4\pi^2 r_1^2 r_5^2 T_H^2}}{2\pi r_1 r_5 T_H}.$$

- Replacing in n and solving for r_0:

$$r_0^2 = 2\frac{g_s \pi r_1 r_5}{R}\sqrt{\frac{n}{V}}T_H + \pi^2 r_1^2 r_5^2 T_H^2 + \frac{3\pi^3 r_1^3 r_5^3 R}{4 g_s}\sqrt{\frac{V}{n}}T_H^3 + \frac{\pi^4 r_1^4 r_5^4 R^2 V}{2 g_s^2 n}T_H^4 + \mathcal{O}\left(T_H^5\right).$$

- r_0^2 has a well defined extremal limit, and that limit is 0.

D-brane spectroscopy

- Absorption cross section can be fully computed in the dilute gas limit (Maldacena, Strominger, 1997):

$$\sigma_{abs}(\omega) = 2\pi^2 r_1^2 r_5^2 \frac{\pi\omega}{2} \frac{e^{\frac{\omega}{T_H}} - 1}{\left(e^{\frac{\omega}{2T_L}} - 1\right)\left(e^{\frac{\omega}{2T_R}} - 1\right)}.$$

- Low frequency limit:

$$\sigma_{abs} \approx \frac{\pi r_0^2}{T_H}$$

($1/T_H$ dependence, finite extremal limit).

One large charge

- $r_0, r_1, r_n \ll r_5$: large value of γ; Q_5 is large compared to Q_1, n (Klebanov, Mathur, 1997).

- $$T_L = \frac{1}{2\pi r_5 \cosh(\alpha - \varsigma)},\ T_R = \frac{1}{2\pi r_5 \cosh(\alpha + \varsigma)},\ T_H = \frac{1}{2\pi r_5 \cosh\alpha \cosh\varsigma}.$$

- Expressing e^α, e^ς in terms of Q_1, n, we obtain
$$T_H = \sqrt{\frac{R}{nQ_1}}\frac{V}{4\pi g_s r_5} r_0^2 \left(1 - \frac{V}{2g_s}\left(\frac{1}{Q_1} + \frac{R}{n}\right) r_0^2\right) + \mathcal{O}\left(r_0^5\right).$$

- $$\sigma_{abs}(\omega) = \pi^3 r_5^2 r_0^2 (1 + \sinh^2\alpha + \sinh^2\varsigma)\omega \frac{e^{\frac{\omega}{T_H}} - 1}{\left(e^{\frac{\omega}{2T_L}} - 1\right)\left(e^{\frac{\omega}{2T_R}} - 1\right)} \approx \frac{\pi r_0^2}{T_H}.$$

- Low frequency limit: again, σ_{abs} exhibits $1/T_H$ dependence, finite extremal limit.

Four dimensional dyonic solution

- $$\begin{aligned} ds_4^2 &= -h^{-1/2}(r)f(r)dt^2 + h^{1/2}(r)\left(\frac{dr^2}{f(r)} + r^2 d\Omega_2^2\right), \\ f(r) &= 1 - \frac{r_0}{r} \\ h(r) &= h_1(r)h_2(r)h_3(r)h_n(r),\ h_n(r) = 1 + \frac{r_n}{r},\ h_i(r) = 1 + \frac{r_i}{r},\ i = 1, 2, 3. \end{aligned}$$

  (Cvetič, Tseytlin, Youm, 1997).
- Define $r_1 \equiv r_0 \sinh^2 \alpha$, $r_n \equiv r_0 \sinh^2 \varsigma$, with

  $$n_w = 4r_2r_3\frac{\pi r_0}{\kappa_4^2}\sinh(2\alpha),\ n_p = \frac{\pi r_0}{\kappa_4^2}\sinh(2\varsigma).$$
- We work in the range $r_0, r_1, r_n \ll r_2, r_3$, which physically means the charges associated to r_2, r_3 are much larger than those associated to r_1, r_n.

D-brane spectroscopy in $d = 4$

- $$T_L = \frac{1}{4\pi\sqrt{r_2 r_3}\cosh(\alpha - \varsigma)},\ T_R = \frac{1}{4\pi\sqrt{r_2 r_3}\cosh(\alpha + \varsigma)},\ T_H = \frac{1}{4\pi\sqrt{r_2 r_3}\cosh\alpha\cosh\varsigma}.$$

- Expressing $e^{\alpha}, e^{\varsigma}$ in terms of n_w, n_p, r_0, r_2, r_3, we obtain
$$T_H = \frac{r_0}{2\kappa^4\sqrt{n_p n_w}}\left(1 - \frac{(n_w + 4 r_2 r_3 n_p)}{\kappa^4 n_p n_w}\pi r_0\right) + \mathcal{O}\left(r_0^3\right).$$

- $$\sigma_{abs}(\omega) = 2\pi\sqrt{r_2 r_3}\, r_0(\cosh 2\alpha + \cosh 2\varsigma)\omega \frac{e^{\frac{\omega}{T_H}} - 1}{\left(e^{\frac{\omega}{2T_L}} - 1\right)\left(e^{\frac{\omega}{2T_R}} - 1\right)} \approx \frac{r_0}{T_H}.$$

- Low frequency limit: again, σ_{abs} exhibits $1/T_H$ dependence, finite extremal limit. This is a characteristic of all the cases we have considered.

Summary

- α' corrections naturally induce (and require) an explicit dependence of the low frequency absorption cross section on the black hole temperature of the form $1/T_H$.
- Assuming fundamental string states to be in 1 to 1 correspondence with black hole states, classically one obtains a low frequency cross section with the same explicit dependence of the form $1/T_H$. We verified this property for different regimes of the D1-D5 system in $d = 5$ and a $d = 4$ dyonic four-charged black hole.
- What seemed just a way to encode α' corrections to the cross section may actually be the most natural way to write it in the context of string theory, since it is valid classically and with leading α' corrections.
- This cross section is well defined in the extremal limit, but this result cannot be valid for extremal black holes.