### Gravity as gauge theory squared: from amplitudes to black holes

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Based on work with D. O'Connell and C. D. White

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- massless vector: 2 helicities  $\pm$
- massless 2-tensor: 2 gravitons  $(\pm \otimes \pm)$ dilaton + axion  $(\pm \otimes \mp)$

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#### Interactions

- *n*-particle gluon amplitude:  $\mathcal{A}_n(\{k_i^{\mu}, \epsilon_i^{\mu}, a_i\})$
- *n*-particle graviton amplitude:  $\mathcal{M}_n(\{k_i^{\mu}, \epsilon_i^{\mu\nu}\})$

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- relations between scattering amplitudes: double copy

Gauge theory amplitude depends on kinematics  $(k_i, \epsilon_i)$  and colour  $(a_i)$ :

$$\mathcal{A}_n = \sum_{\alpha \in \text{cubic}} \frac{n_\alpha \mathbf{C}_\alpha}{D_\alpha}$$

• colour factors - 
$$c_{\alpha} = f^{\cdots}f^{\cdots}\cdots f^{\cdots}$$
  $f^{abc} = tr([T^a, T^b]T^c)$ 

• propagators -  $D_{\alpha}(k_i)$ 

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**Statement**: it is possible to write gauge theory amplitudes such that numerators  $n_{\alpha}(k_i, \epsilon_i)$  have symmetries of colour factors  $c_{\alpha}(a_i)$ 

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Kinematic algebra for numerators?

## Double Copy to Gravity

Statement: gravity amplitudes are obtained from gauge theory as

$$\mathcal{M}_n = \sum_{\alpha \in \text{cubic}} \frac{n_\alpha \tilde{n}_\alpha}{D_\alpha}$$

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Tree level: Similar relations connect many theories of massless particles. [Cachazo, He, Yuan '141 [Kawai, Lewellen, Tve '86]

Loop level conjecture: Allows study of supergravity divergences. [Bern et al '08-'14]

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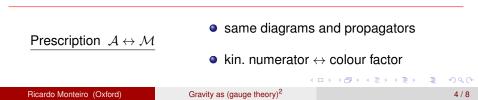
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Use light-cone coords: ds^2 = -du dv + dw d\bar{w}
u = t - z, v = t + z, w = x + iy, \bar{w} = x - iy
define \hat{k}_{\mu} = (0, \partial_w, 0, \partial_u)
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#### Self-dual gauge theory

[Bruschi, Levi, Ragnisco '82]

$$F_{\mu\nu} = \frac{i}{2} \, \varepsilon_{\mu\nu\alpha\beta} \, F^{\alpha\beta}$$

$$m{A}_{\mu}=\hat{m{k}}_{\mu}\Phi, \quad \Phi=\phi^{a}T^{a}$$

 $\Box \Phi + [\partial_w \Phi, \partial_u \Phi] = 0$ 

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where  $\{f, g\} = \partial_w f \partial_u g - \partial_u f \partial_w g$ 

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Colour-kinematics duality and double copy manifest.[RM, O'Connell '11] $[\cdot, \cdot]\{\cdot, \cdot\} \rightarrow \{\cdot, \cdot\}^2$  $(\neg, \cdot)^2$ Ricardo Monteiro (Oxford)Gravity as (gauge theory)^25/8

### **Kerr-Schild Spacetimes**

Consider Kerr-Schild metric

[Kerr, Schild '65]

 ${m g}_{\mu
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u}+\phi\,{m k}_{\mu}{m k}_{
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where  $k_{\mu}$  is null and geodesic wrt  $\eta_{\mu\nu}$  ( $\eta^{\mu\nu}k_{\mu}k_{\nu} = 0$ ,  $\eta^{\mu\nu}k_{\mu}\partial_{\nu}k_{\lambda} = 0$ ), therefore also wrt  $g_{\mu\nu}$ .

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Leads to linearisation of the Einstein equations:

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$$g^{\mu\nu} = \eta^{\mu\nu} - \phi k^{\mu}k^{\nu}$$
  
•  $R^{\mu}{}_{\nu} = \frac{1}{2}\partial_{\alpha} \left[\partial^{\mu} \left(\phi k^{\alpha}k_{\nu}\right) + \partial_{\nu} \left(\phi k^{\alpha}k^{\mu}\right) - \partial^{\alpha} \left(\phi k^{\mu}k_{\nu}\right)\right] \qquad \qquad \partial^{\mu} \equiv \eta^{\mu\nu}\partial_{\nu}$ 

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Stationary Einstein equations give  $(k_0 = 1)$ :

$$R^{0}_{0} = \frac{1}{2}\nabla^{2}\phi = 0 \qquad \qquad R^{i}_{0} = \frac{1}{2}\partial_{j}\left[\partial^{i}\left(\phi k^{j}\right) - \partial^{j}\left(\phi k^{i}\right)\right] = 0$$

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[RM, O'Connell, White '14]

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$$g_{\mu\nu} = \eta_{\mu\nu} + \phi \, k_{\mu} k_{\nu} \quad \longrightarrow \quad \boxed{A^a_{\mu} = \phi \, k_{\mu} \, c^a} \qquad (c^a \, \mathrm{const})$$

- keep spatial propagator  $\phi$
- keep one copy of  $k_{\mu}$
- colour? Linearisation in gravity  $\rightarrow$  Abelianisation in gauge theory

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$$\mathbf{0} = \partial_{\mu} F^{a\mu\nu} = c^{a} \begin{cases} -\nabla^{2}\phi & \nu = \mathbf{0} \\ -\partial_{j} \left[ \partial^{j} \left( \phi k^{j} \right) - \partial^{j} \left( \phi k^{j} \right) \right] & \nu = i \end{cases}$$

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<u>Schwarzschild ~ (Coulomb)^2</u>:  $\phi(r) = \frac{2GM}{r}, \quad k_{\mu} = \left(1, \frac{\vec{x}}{r}\right)$ •  $A_{\mu} = \phi k_{\mu}, \quad \phi(r) = \frac{Q}{r}$ • after gauge transformation,  $A'_{\mu} = \frac{Q}{r} (1, \vec{0})$ 

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Also for Kerr, black branes. Similar for plane waves.

Ricardo Monteiro (Oxford)

Gravity as (gauge theory)<sup>2</sup>

### Conclusions

 New structures connecting perturbative gauge theory and gravity. Related to unexpected UV cancellations in supergravity.

• Simple manifestation in self-dual theories.

• Perturbative relations extend to exact solutions in examples.

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