Stationary Bianchi black brane solutions and Holographic Lattices

VII Black Holes Workshop@Aveiro

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1 Introduction

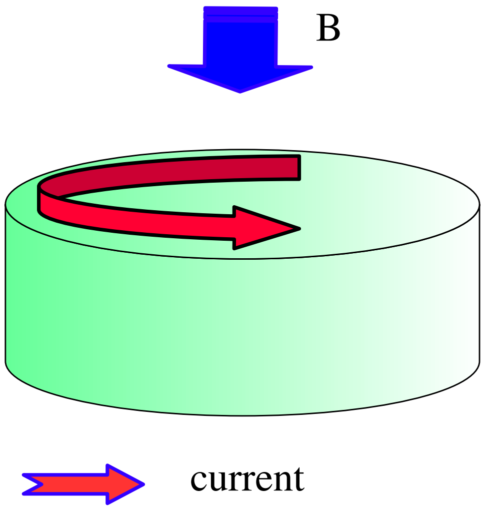
AdS/CMP correspondence:

d-dim. strongly coupled condensed matter physics (CMP) is described by a $d+1$-dim. gravity model in asymptotically AdS spacetime

Ex). Holographic superconductor model $\Longrightarrow$ a complex scalar field model coupled with $U(1)$ gauge field in asymptotically AdS spacetime

Properties of superconductors

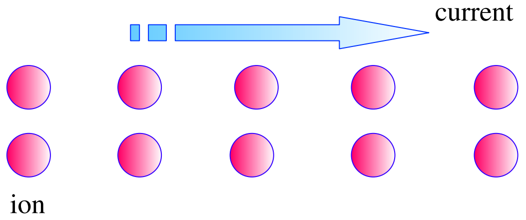
- (DC)-Conductivity becomes infinite
- persistent current exists

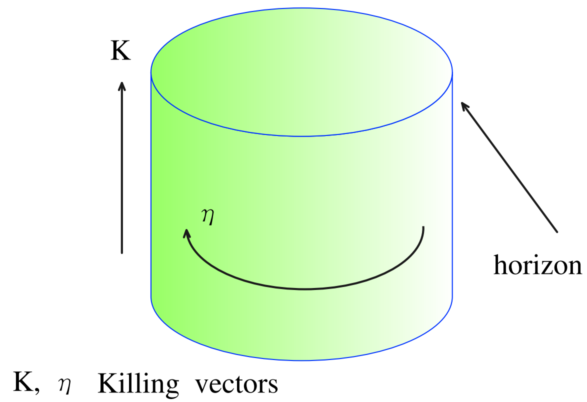


Remark: Translational symmetry should be violated along the current, due to the lattice structure or impurities of the usual condensed matter systems.

persistent current $\Longleftrightarrow$ **stationary black branes with momentum or rotation along the latticed direction**

This seems to be **incompatible** with **Black hole Rigidity theorem**: Stationary rotating black hole must be *axisymmetric*





In this talk, we clarify this apparent discrepancy by constructing **stationary 5-dim**. Bianchi black brane solutions.

- **Helical lattices from Bianchi type VII$_0$ -** Three Killing vectors:

$$\xi_1 = \partial_{x^2}\,,\;\; \xi_2 = \partial_{x^3}\,,\;\; \xi_3 = \partial_{x^1} - x^3\partial_{x^2} + x^2\partial_{x^3}\,,$$

with $\left[\xi_i, \xi_j\right] = C^k_{ij}\,\xi_k$ with $C^1_{23} = -C^1_{32} = -1$, $C^2_{13} = -C^2_{31} = 1$ and with the invariant one-form

$$\omega^1 = \cos(x^1)dx^2 + \sin(x^1)dx^3\,,$$
$$\omega^2 = -\sin(x^1)dx^2 + \cos(x^1)dx^3\,, \quad \omega^3 = dx^1$$

Metric ansatz:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + e^{2v_3(r)}(\omega^3 - \Omega(r)dt)^2 + e^{2v_1(r)}(\omega^1)^2 + e^{2v_2(r)}(\omega^2)^2$$

$$\partial_{x^1} : \mathbf{Non-killingvectors}$$

Holographic model:

$$\mathcal{L} = R + \frac{12}{L^2} - \frac{1}{4}F^2 - \frac{1}{4}W^2 - |D\Phi|^2 - m^2|\Phi|^2$$
$$F = dA, \quad W = dB$$

Ansatz:

$$\Phi = \phi(r)\,, \quad A_\mu dx^\mu = A_{x^1}(r)\,\omega^3 + A_t(r)dt$$
$$B_\mu dx^\mu = b(r)\,\omega^1$$

Equation of motion for $\xi := v_1 - v_2$ (lattice structure disappears when $v_1 = v_2$)

$$\begin{aligned} &f\xi'' + \{f' + f(v_1' + v_2' + v_3')\}\xi' \\ &\quad - 2e^{-2v_3}\left(1 - e^{2v_3}f^{-1}\Omega^2\right)\sinh 2\xi \\ &= \frac{1}{2}e^{-2(v_2+v_3)}\left(1 - e^{2v_3}f^{-1}\Omega^2\right)b^2 - \frac{1}{2}e^{-2v_1}fb'^2 \end{aligned}$$

$\Longrightarrow$

The gauge field b induces *lattice structure*.

General Properties of Horizon:

Null generator on the horizon H:

$$l = \partial_t + \Omega_h \partial_{x^1} \quad , \qquad \Omega_h \equiv \Omega|_{r=r_h}$$

and on H

$$R_{\mu\nu} l^\mu l^\nu = -2\Omega_h^2 (\sinh(v_1 - v_2))^2 \leq 0$$

$$\bigtriangledown$$

To satisfy the null energy condition, two possibilities exist:

- $\Omega_h \neq 0$ and $v_1 = v_2$ just on the horizon $\quad \times \Longleftarrow \quad$ Field Eq. for b.
- $\Omega_h = 0$ on the horizon $\quad \bigcirc$

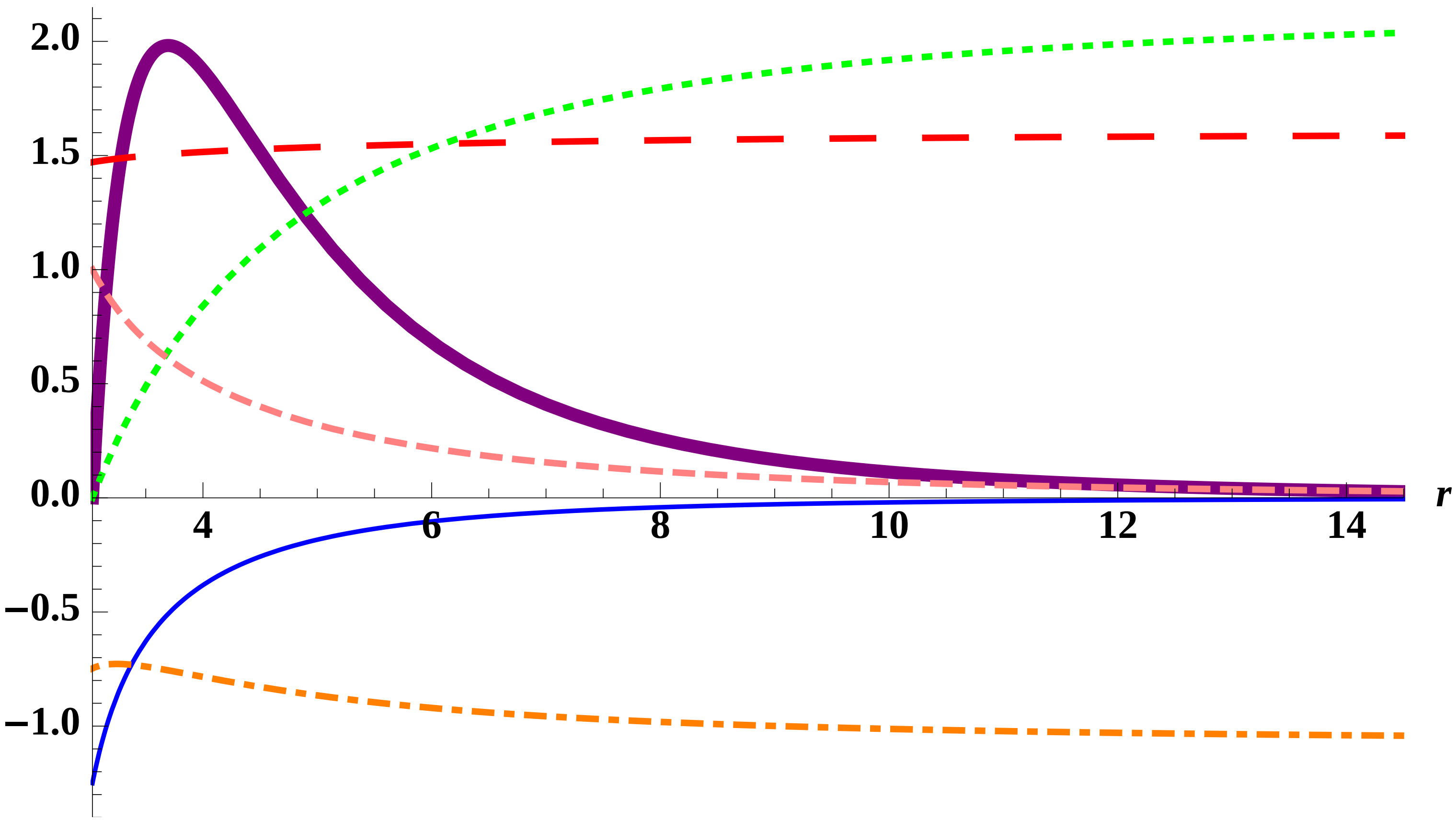
The asymptotic boundary conditions:

$$\lim_{r\to\infty} \Omega(r) = 0, \qquad \lim_{r\to\infty} \xi(r) \equiv v_1(r) - v_2(r) = 0$$

Asymtotically AdS condition

Numerical solutions for $m^2 = -15/8$, $L^2 = 2$:

$$\phi \simeq C_+ r^{-\frac{5}{2}} + C_- r^{-\frac{3}{2}} \text{ Asymptotic b. c.} \Longrightarrow C_- = 0$$



100Ω: thick solid purple, 10ξ: thin solid blue, $0.5A_t$: dotted green, A_{x^1}: dot-dashed orange, ϕ small dashed pink, $0.2b$: large dashed

♠ Superfluid hydrodynamics

The asymptotic behaviors of A_{x^1} and Ω:

$$A_{x^1} \simeq a_{x0} + \frac{a_{xN}}{r^2}, \qquad \Omega \simeq \Omega_0\ (=0) + \frac{\Omega_N}{r^4}$$

AdS/CFT dictionary $\Longrightarrow$

$$\sqrt{2} a_{xN} \Longleftrightarrow \langle\, j_{x^1} \,\rangle, \qquad -2\sqrt{2}\Omega_N \Longleftrightarrow \langle\, T_{tx^1} \,\rangle$$

We numerically find that the relation

$$\frac{\langle\, T_{tx^1} \,\rangle}{\mu \langle\, j_{x^1} \,\rangle} = -1.000 \pm O(10^{-4})$$

holds in all of the solutions!

$$\triangle$$

Landau and Tisza's two fluid model

Landau and Tisza's two fluid model:

$$T_{\mu\nu} = (\epsilon + P)u_\mu u_\nu + P\eta_{\mu\nu} + \mu\rho_s v_\mu v_\nu \,,$$

$$j_\mu = \rho_n u_\mu + \rho_s v_\mu$$

u^μ : velocity of normal component, $\quad v^\mu$: velocity of superfluid component

Josephson Eq. $\Longrightarrow$

$$v_\mu u^\mu = -1$$

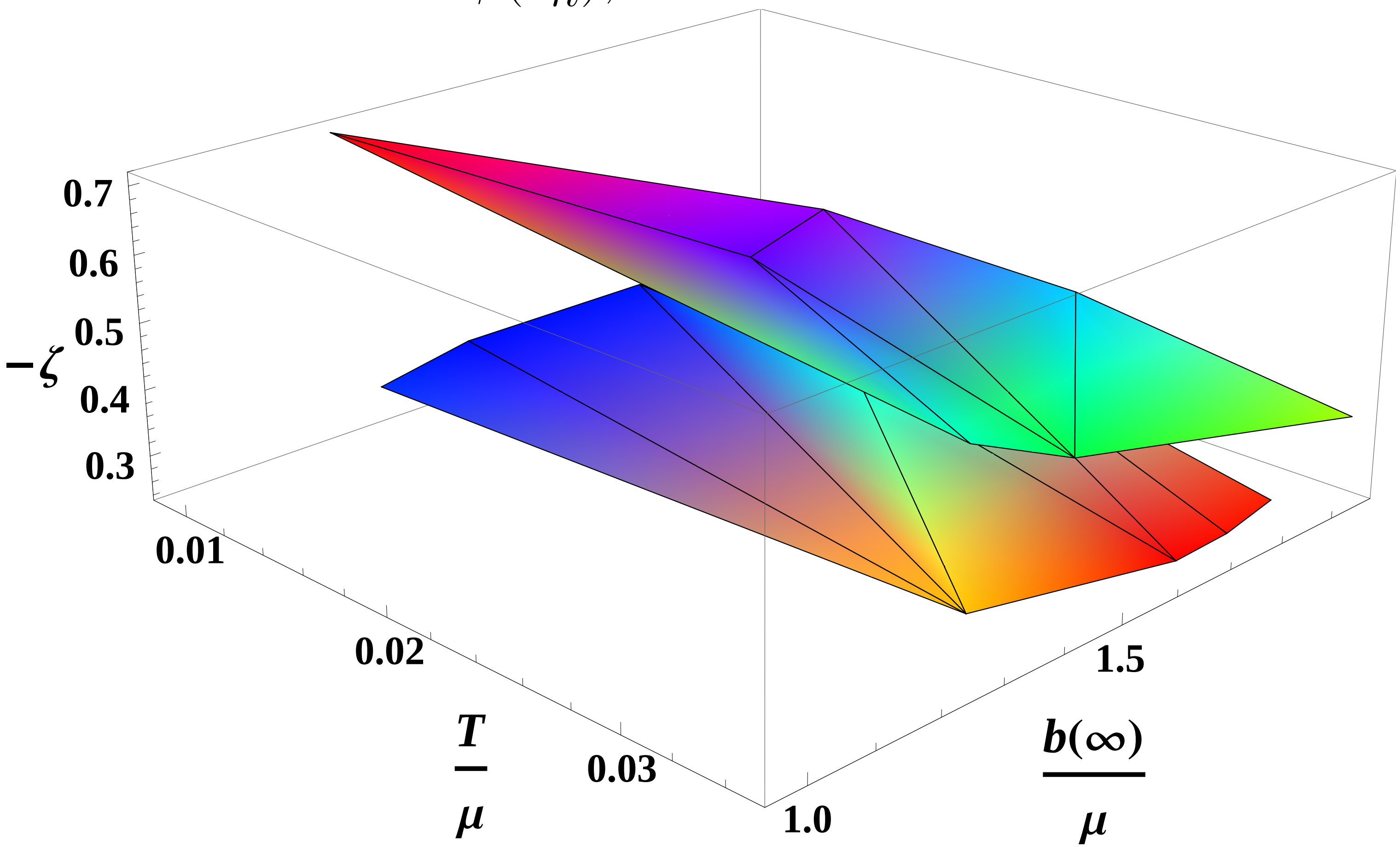
When $u_{x^1} = 0$, we find that

$$\frac{T_{tx^1}}{\mu\, j_{x^1}} = v_t = -(u^t)^{-1} = -1,$$

which agrees with our numerical calculations!

Properties of the solutions:

For a fixed value of $\phi(r_h)$, we find



$\zeta := A_{x^1}(\infty)/A_t(\infty)$: superfluid fraction

General properties of rotating Bianchi black brane solutions

We consider general stationary 5-dim. Bianchi black brane as

$$ds_5^2 = -h\,dt^2 + \frac{dr^2}{f} + 2\tilde{N}_I dt\,\omega^I + \tilde{g}_{IJ}\omega^I\omega^J, \quad I,\ J = 1,2,3$$

$$[\xi_I,\, \xi_J] = C^K{}_{IJ}\xi_K$$

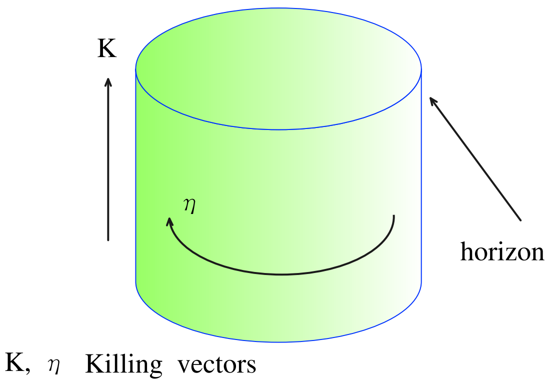
In general Bianchi type, the spatial cross section of the event horizon Σ is non-compact:

$$\bigtriangledown$$

The rigidity theorem does not exclude the possibility that *the horizon rotates along the direction with no translational symmetry*

Theorem: Consider a 5-dimensional stationary black brane with the stationary Killing vector k, and choose a horizon cross-section Σ. Then k is uniquely decomposed into a null vector field n normal to H and a spacelike vector η as $k = n - \eta$

Then, for the Bianchi symmetry of the type II, VI_0, or VII_0, either $\eta = 0$, or η must be a Killing vector of Σ, under the null energy condition irrespective of Σ being compact or non-compact.



proof) In Type II, VI_0, VII_0, $\theta = \nabla_\mu l^\mu = 0$.

Then, from the Raychaudhuri Eq.

$$\frac{d\theta}{d\lambda} = -\sigma_{ij}\sigma^{ij} - R_{\mu\nu}l^\mu l^\nu - \frac{1}{3}\theta^2,$$

we find

$$R_{\mu\nu}l^\mu l^\nu = -\sigma_{ij}\sigma^{ij} \geq 0 \quad \Longleftrightarrow \quad \sigma_{ij} = 0, \quad l^\mu : \text{ null geodesic}$$

Question) Can we find a stationary black brane solution with a rotating horizon along the direction with no translational symmetry?

$\bigtriangledown$

Yes!

We find a stationary Bianchi VII_0 black brane solution with a rotating horizon along the direction with no translational symmetry with action

$$S = \int d^5x\sqrt{-g}\left(R + \frac{6}{L^2} - \frac{1}{4}F^2 - \frac{1}{4}W^2 - V(C)\right),$$

$A,\ C$: 1-form potentials,

$$F = dA\,, \quad W = dC$$

The potential $V(C)$ for 1-form C:

$$V(C) = a_0(C - C_0)^2 + a_1(C - C_0)^4, \quad C_0 \equiv c_0\,\omega_1$$

Contradicts with the Rigidity Theorem?

- The null energy condition ○
- Compactness of the horizon cross section ○
- Analyticity on the horizon ×
  (η is killing just on H, but it's not outside of H)

This is a first example for stationary black brane solution with a *rotating horizon along the direction with no translational symmetry*

Thank you!