Instabilities of extremal black holes in higher dimensions

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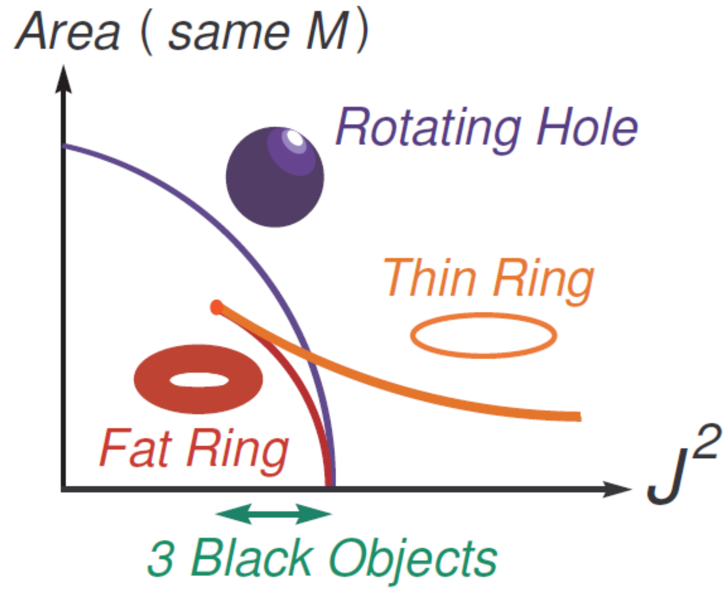
Talk based on arXiv:1408.0801 w/ S. Hollands

BH Classification problem in Higher Dimensions

- *No uniqueness* in D>4 GR

- Classification of them is yet under way



To classify ⇨ need to study their stability

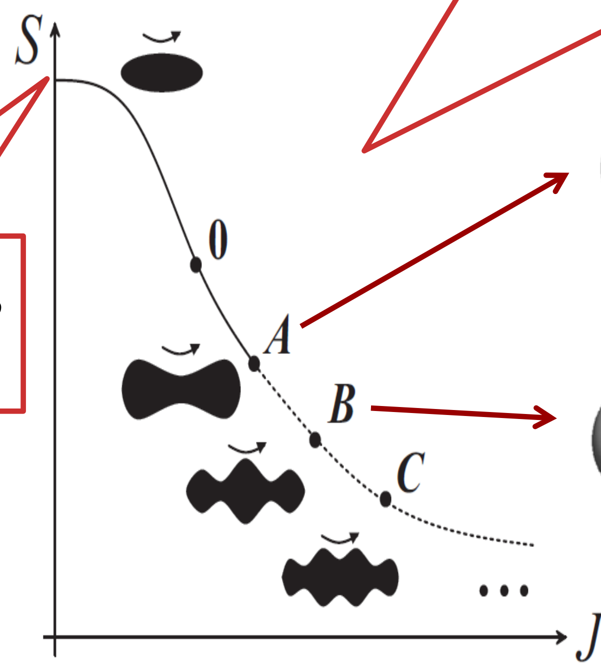
Instabilities are signals of bifurcation to something different, implying more variety of solutions.

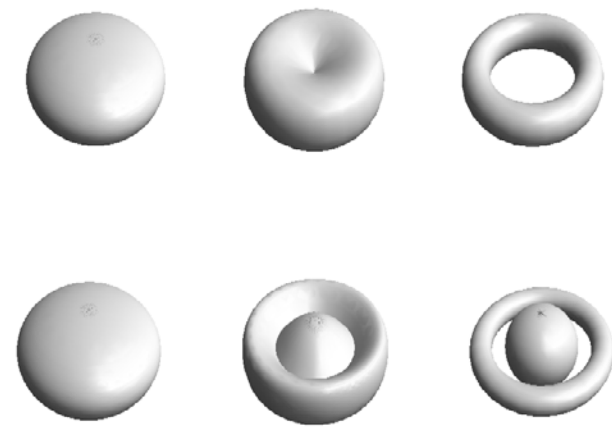
Phase space of Higher Dimensional BHs

S ： Horizon area

We need criteria for the onset of instabilities of *rotating* – BHs

Non-rotating BHs
Stable





Dias et al 09

J ： Angular momentum

Instability ⇒ Bifurcation: New branch of solutions

However, the phase space is so large…

Start with classifying *Extremal* black holes

- Limit of zero Hawking temperature

$$T_H \to 0$$

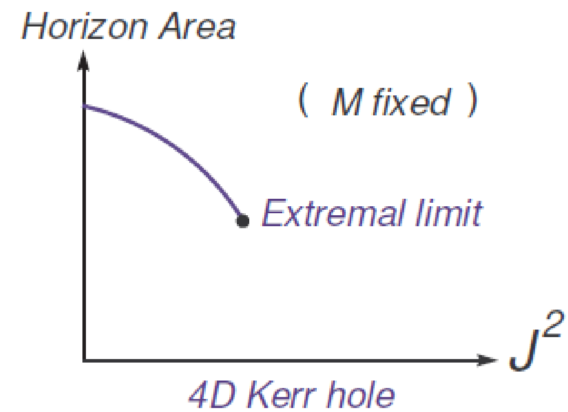
- Play an important role in various contexts

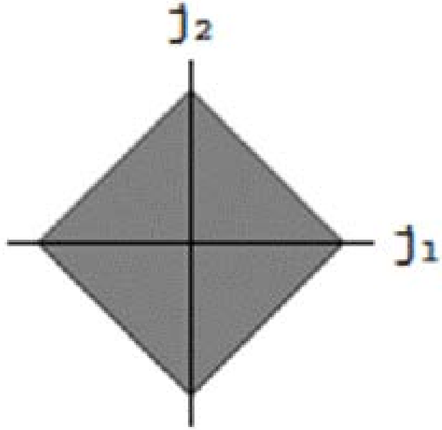
  e.g. Supergravity

  Entropy counting

  Kerr/CFT

  Holographic Superconductors





5D Myers-Perry hole

Classify "boundaries of the solution space" from the stability view point

Stability analysis

- Linear perturbation analyses

  Tractable when master equations available:

  e.g., Teukolsky equations in 4D

- Unfortunately there is *no* Teukolsky type master equation for higher dimensional (extremal/non-extremal) black holes

To classify extremal black holes …

- Helpful to study *"near-horizon geometries"* which
  * arise as a scaling limit of extremal black hole
  * satisfy the same dynamics
  * possess more symmetries
  * admit Teukolsky type master equations

Near Horizon Geometry (NHG)

$$ds^2 = 2du d\rho - \rho^2 \alpha du^2 - 2\rho du \beta_A dx^A + \mu_{AB} dx^A dx^B$$

- Diffeomorphism

$$(u, \rho, x^A) \mapsto \left(\frac{u}{\epsilon}, \epsilon\rho, x^A\right)$$

- Scalimg limit $\epsilon \to 0$

$\alpha,\ \beta_A,\ \mu_{AB}$ become functions of x^A

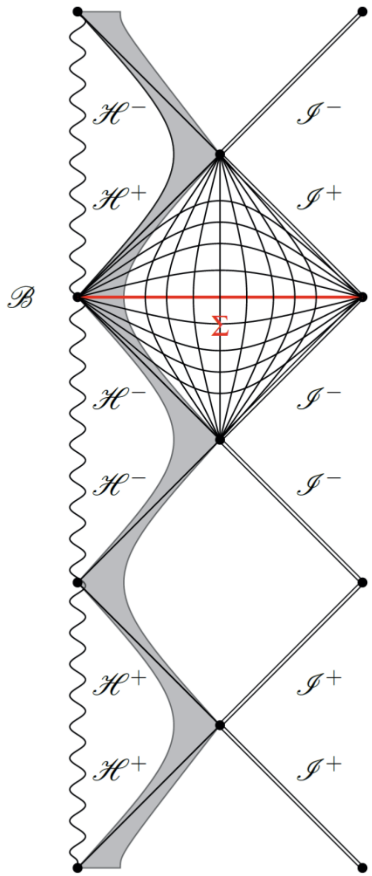
$$\mathrm{d}s^2 = L^2\, \mathrm{d}\hat{s}^2 + g_{IJ}(\mathrm{d}\phi^I + k^I\, \hat{A})(\mathrm{d}\phi^J + k^J\, \hat{A}) + \mathrm{d}\sigma^2_{d-n-2}$$

AdS_2 $\boxed{\mathrm{d}\hat{s}^2 = -R^2 \mathrm{d}T^2 + \frac{\mathrm{d}R^2}{R^2}}$

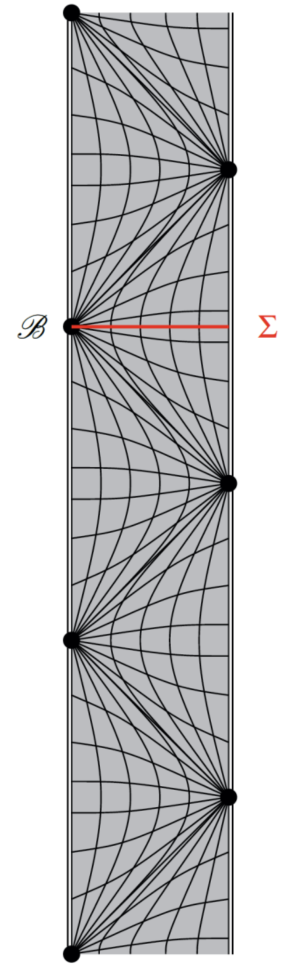
$\boxed{\hat{A} = -R\mathrm{d}T}$

Near-Horizon scaling

Carter 72



Scaling limit



AdS_2

A horizon neighborhood
of Extreme black hole

Near-Horizon Geometry

Durkee & Reall conjectured that

When axi-symmetric perturbations on the NHG violate AdS_2 -BF-bound on the NHG, then the original extremal BH is unstable $e^{im_I\phi^I}$ $m_I N^I(x) = 0.$

… supportd by numerical results:

Durkee–Reall 11

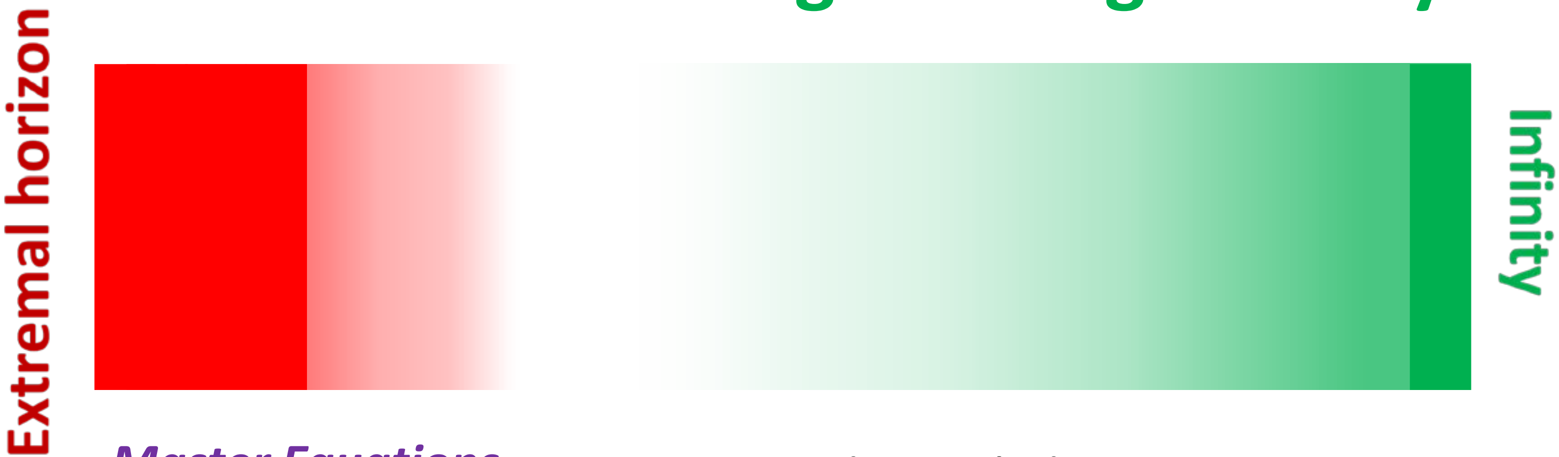
Purpose

We show this conjecture by using

- Hertz-potential
- Canonical energy method
- Initial data correction

Strategy for proving DR Conjecture

NHG **Original BH geometry**



Master Equations available

No decoupled Master Equtions
Apply ***Canonical-Energy Method for Initial Data***

Matching initial data by scaling

Attempt to construct ***negative*** energy initial data **on NHG**

Construct ***negative*** canonical Energy for **the BH initial data**

(1) Teukolsky type equations for Hertz potential on NHG

$$\left(\nabla^2_{AdS_2} - q^2 - \mathscr{A}\right) U = 0$$

$\mathscr{A}$: Laplace operator on horizon cross-section

(2) Metric perturbations γ_{ab} can be re-constructed as

$$\gamma_{ab} = -l_a l_b (C_{cedf} l^e n^f U^{cd}) + 2l_{(a} \text{þ} \eth^c U_{b)c} + 2l_{(a} (\tau^c + \tau'^c) \text{þ} U_{b)c} - \text{þ}^2 U_{ab}$$

(3)Construct the canonical energy on NHG $E_{\mathrm{NHG}}(\gamma)$

(4)Show when BF-bound is vaioated $E_{\mathrm{NHG}}(\gamma)$ becomes negative

$$\lambda < -1/4 \quad \Rightarrow \quad E_{NHG} < 0$$

(5) Show $E_{NHG} < 0 \quad \Rightarrow \quad E_{\mathrm{BH}} < 0$

Decoupled Master equations on NHG

Thanks to high symmetry of NHG, metric perturbations γ_{ab} can be written in terms of Hertz potential

$$U^{AB} = \psi \cdot Y^{AB} \cdot \exp(i\underline{m} \cdot \underline{\phi})$$

$$\gamma_{ab} = -l_a l_b (C_{cedf} l^e n^f U^{cd}) + 2 l_{(a} \text{þ} \eth^c U_{b)c} + 2 l_{(a} (\tau^c + \tau'^c) \text{þ} U_{b)c} - \text{þ}^2 U_{ab}$$

Hertz potential obeys AdS_2 Klein-Gordon equation

$$-\frac{1}{R^2}\frac{\partial^2 \psi}{\partial T^2} + \frac{\partial}{\partial R}\left(R^2 \frac{\partial \psi}{\partial R}\right) - \frac{2iq}{R}\frac{\partial \psi}{\partial T} - \lambda \psi = 0$$

$$\underline{\mathscr{A} Y = \lambda Y}\ , \qquad \underline{\pounds_{\partial/\partial \phi^I} Y = 0}$$

where $\mathscr{A}$ is 2nd-order operator on the horizon section

We show the following theorem:

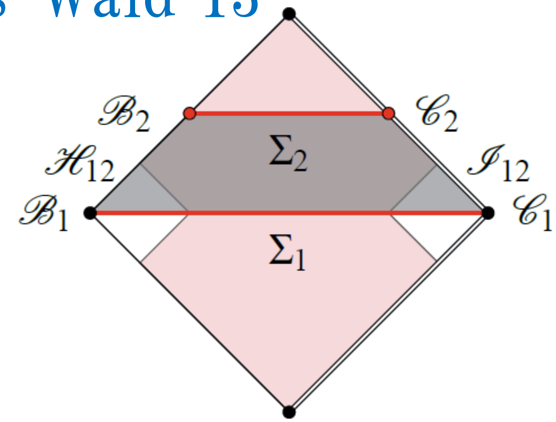
If the minimal eigenvalue of $\mathscr{A}$ *violates the effective BF-bound*

$$\lambda < -\frac{1}{4}$$

then the original extremal black hole is unstable

Canonical energy for initial data

Hollands–Wald 13



Symplectic current

$$w^a = \frac{1}{16\pi} P^{abcdef}(\gamma_{2\,bc}\nabla_d\gamma_{1\,ef} - \gamma_{1\,bc}\nabla_d\gamma_{2\,ef})$$

Symplectic form $\quad W(\Sigma;\gamma_1,\gamma_2) \equiv \int_\Sigma \star w(g;\gamma_1,\gamma_2)$

Canonical energy $\quad \mathscr{E}(\Sigma,\gamma) \equiv W(\Sigma;\gamma,\pounds_K\gamma) - B(\mathscr{B},\gamma) - C(\mathscr{C},\gamma)$

1) $\mathscr{E}$ is gauge invariant

$$B(\mathscr{B},\gamma) = \frac{1}{32\pi}\int_{\mathscr{B}} \gamma^{ab}\delta\sigma_{ab}$$

2) $\mathscr{E}$ is monotonically decreasing for any axi-symmetric perturbation

$$C(\mathscr{C},\gamma) = -\frac{1}{32\pi}\int_{\mathscr{C}} \tilde{\gamma}^{ab}\delta\tilde{N}_{ab}$$

Construction of a perturbation with negative canonical energy

- For initial data: $(f_0, f_1) \equiv \left(\psi \Big|_{T=0}, \frac{\partial}{\partial T} \psi \Big|_{T=0} \right)$

$$f_0(R) = \frac{R^N}{(R+\varepsilon)^{N+1/2}(1+R^N e^{1/(1-R)})}, \qquad f_1(R) = 0$$

for $0 < R < 1$ and $f_0(R) = 0$ for $R \geq 1$.

$$\mathscr{E} = \frac{1}{8\pi}\underline{(\lambda + \tfrac{1}{4})}(\lambda^2 + 2a^2\lambda + a^4 - 9a^2 + \tfrac{7}{2}) \log \varepsilon^{-1} + O(1)$$

where $a = \underline{k} \cdot \underline{m}$

If $a = 0$ $\quad \mathscr{E} < 0 \quad$ for $\lambda < -\frac{1}{4}$ and sufficiently small $\varepsilon > 0$

This energy expression holds only on NHG,

not on the original BH geometry.

One can *correct it to hold on the original BH geometry* by using Corvino-Schoen's initial data gluing method.

Corvino–Schoen 03

Summary

- We have proven Durkee-Reall conjecture that *extremal black holes are unstable when the eigenvalue* λ *of the operator* $\mathscr{A}$ *is less than the effective BF bound* $-1/4$

  Our proof uses

  (i) *Canonical energy method*

  (ii) *Symmetry of the NHG and Hertz potential*

  (iii) *Structure of the constraint equations*

- The stability analysis is thus reduced to an *analysis on the horizon cross-section*, which is a *much simpler* problem than analyzing the perturbed Einstein equations.