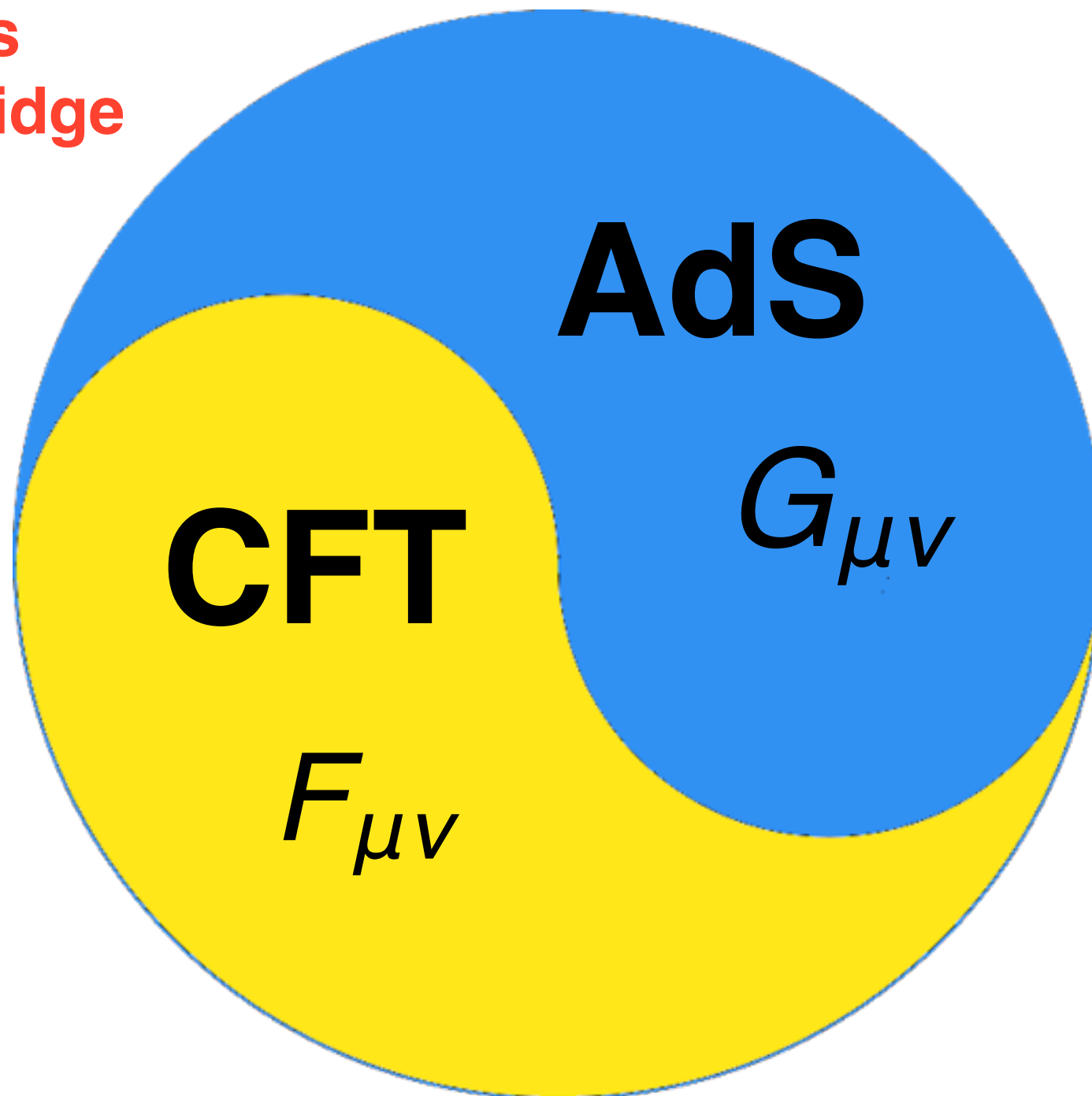


Quantum Gravity corrections in Holography

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CFT computes Quantum Gravity corrections

AdS5xS⁵

$$G_N \sim 1/N^2$$

$$L^2 \sim \sqrt{\lambda}$$

SU(N) N=4 SYM

$$\langle \mathcal{O} \dots \rangle = \sum_k F_k(\lambda) N^{2-2k}$$

$$Ng_{YM}^2 \sim \lambda$$

Classical SUGRA

$$N, \lambda \rightarrow \infty$$

Quantum SUGRA

$$\infty > N, \lambda \gg 1$$

How to compute $1/N$ in String Theory?

1) Perturbative

$$G_{\mu\nu} = G_{\mu\nu}^0 + h_{\mu\nu}$$

Too HARD!!

2) Supersymmetric Localization

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2) Supersymmetric Localization

**Certain Path Integrals Localize
on a finite dimensional locus!**

1-loop EXACT!

Compute all N corrections!!

What is Localisation?

$$\int_{\mathcal{M}} a \quad Q^2 = \partial_V$$

$$Qa = 0 \text{ but } a \neq Q\beta$$

$$\int_{\mathcal{M}} Q\gamma = \sum_{p \in |\partial_V|=0} \gamma(p)$$

$$a \simeq Q\gamma \quad \text{Poincaré Lemma}$$

What is Localisation?

$$\int e^{-S} = \sum_{\sigma \in \delta\psi=0} e^{-S(\sigma)} \times \text{sdet}(\delta^2)$$

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Semiclassical approximation is exact!

Two problems

A) Entropy of Supersymmetric Black Holes

$$\ln d(Q) = \ln Z_{AdS_2}$$

B) Free energy of N M2-branes

$$F_{CFT_3} = \ln Z_{AdS_4}$$

Two problems

A) Entropy of Supersymmetric Black Holes

$$\ln d(Q) = \ln Z_{AdS_2}$$

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$$F_{CFT_3} = \ln Z_{AdS_4}$$

Known exactly

A) Entropy of SUSY black holes

Exact Hardy-Ramanujan-Rademacher expansion (1/8 BPS D1-D5)

$$d(Q) = \int_{-i\infty}^{i\infty} \frac{dt}{t^{9/2}} e^{t + \frac{Q}{t}} + \sum_{c \geq 1} Kl(Q, c) \int_{-i\infty}^{i\infty} \frac{dt}{t^{9/2}} e^{t + \frac{Q}{c^2 t}}$$

Kloosterman Sums

$$Kl(Q, c) = \sum_{\substack{d \in \mathbb{Z}/c\mathbb{Z} \\ ad=1 \bmod(c)}} e^{2\pi i Q \frac{d}{c}} e^{-2\pi i \frac{a}{c}} M(a, d, c)$$

Bekenstein-Hawking entropy

$$d(Q) \simeq e^{\sqrt{Q} - 2 \ln(Q) + \dots} \quad \sqrt{Q} = \frac{A}{4}$$

A) Entropy of SUSY black holes

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$$d(Q) = \int_{-i\infty}^{i\infty} \frac{dt}{t^{9/2}} e^{t + \frac{Q}{t}} + \sum_{c \geq 1} KI(Q, c) \int_{-i\infty}^{i\infty} \frac{dt}{t^{9/2}} e^{t + \frac{Q}{c^2 t}}$$

Localizes to a
finite dimensional integral

$$\int_{AdS_2 \times S^2} e^{-S_{SUGRA}} \longrightarrow \int_{-i\infty}^{i\infty} \frac{dt}{t^{9/2}} e^{t + \frac{Q}{t}} + \dots$$

[Dabholkar, JG, Sameer Murthy]

Global Contributions
CS action

$$\int \text{Tr} A \wedge dA + 2/3 A^3 \longrightarrow \sum_{c \geq 1} KI(Q, c) \int_{-i\infty}^{i\infty} \frac{dt}{t^{9/2}} e^{t + \frac{Q}{c^2 t}}$$

[Dabholkar, JG, Sameer Murthy]

B) Free Energy of N M2-branes

Low energy of N M2-branes

ABJM=3D CS-matter theory

$$Z_{ABJM/S^3} = Ai(k^{1/3} N) + Z_{Npt}$$

[Kapustin, Yaakov, Willet]

[Fuji, Hirano, Moriyama;
Drukker, Marino, Putrov]

$$Ai(z) = \int_{e^{-\frac{i\pi}{3}} \infty}^{e^{\frac{i\pi}{3}} \infty} e^{\frac{1}{3}x^3 - zx} dx$$

B) Free Energy of N M2-branes

Asymptotics for large N

$$-\ln Z_{ABJM} = \frac{\sqrt{2}\pi}{3} k^{1/2} N^{3/2} + \frac{1}{4} \ln N + \dots, \quad N \gg 1$$

Localization in the Bulk!

M-th on $AdS_4 \times S^7$

Localizes to a
finite dimensional integral

$$\int e^{-S_{SUGRA}} \longrightarrow \int e^{x^3/3 - k^{1/3} N x} dx$$
$$S_{SUGRA} = \frac{1}{G_4} \int_{AdS_4} R - 2\Lambda + \dots$$

Conclusions

