Black hole hair in Horndeski scalar-tensor gravity

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Beyond scalar-tensor theory

We already know that more general couplings lead to "hairy" solutions! Such solutions have been found for couplings of the type

$$e^{\phi}(R^2 - 4R^{\mu\nu}R_{\mu\nu} + R^{\mu\nu\kappa\lambda}R_{\mu\nu\kappa\lambda})$$

P. Kanti et al., Phys. Rev. D 54, 5049 (1996).

No real hope for a no-hair theorem that would cover every case!

There has been a no-hair proof claim for a rather general class though: shift-symmetric generalised galileons!

L. Hui, A. Nicolis, Phys. Rev. Lett. 110, 241104 (2013).

Generalised Galileons

…or Horndeski's theory: the most general scalar-tensor action that leads to second order field equations

G. W. Horndeski, Int. J. Theor. Phys. 10, 363 (1974)
C. Deffayet et al., Phys. Rev. D 80, 064015 (2009)

Shift-symmetric restriction:

$$
\begin{aligned}
L =& K(X) - G_3(X)\Box\phi + G_4(X)R \\
& + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] + G_5(X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \\
& - \frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]
\end{aligned}
$$

The proof

The equation for the scalar is a current conservation

$$\nabla_\mu J^\mu = 0$$

Assumptions:

- Staticity and spherical symmetry
- Asymptotic flatness

<u>Step 1</u>: Show that $J^\mu = 0$

<u>Step 2</u>: Argue that this implies $\partial_\mu \phi = 0$

However, there is actually a loophole!

T.P.S. and S.-Y. Zhou, Phys. Rev. Lett. 112, 251102 (2014);
arXiv:1408.1698 [gr-qc]

A simple exception

Consider the action

$$S = \frac{m_P^2}{8\pi} \int d^4x \sqrt{-g} \left(\frac{R}{2} - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \alpha \phi \mathcal{G} \right)$$

The corresponding scalar equation is

$$\Box \phi + \alpha \mathcal{G} = 0$$

The Gauss-Bonnet term vanishes only in flat space!

This theory corresponds to

$$K = X, \qquad G_3 = G_4 = 0, \qquad G_5 = -4\alpha \ln |X|$$

T. Kobayashi et al., Prog. Theor. Phys. 126, 511 (2011).

Uniqueness

We need a term in the action that

- leads to an ϕ-independent contribution to the field equations: $\phi A[g]$

- is shift-symmetric: $A[g]$ should be a total divergence

- it should lead to no more than second order derivatives in the field equations

There is only one such choice:

$$A[g] = \mathcal{G} \equiv R^2 - 4R^{\mu\nu}R_{\mu\nu} + R^{\mu\nu\kappa\lambda}R_{\mu\nu\kappa\lambda}$$

Avoiding the offensive term

- There is no symmetry protecting the action from the linear coupling

- It is not technically natural to exclude it!

- The generality of the no-hair theorem is compromised

One could impose symmetry under $\phi \to -\phi$

Most general action:

$$L = K(X) + G_4(X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$

Perturbative solution

To first order in α

- metric is Schwarzschild
- non-trivial scalar profile: $$\phi' = \alpha \frac{16M^2 - Cr^3}{r^4(r-2M)}$$

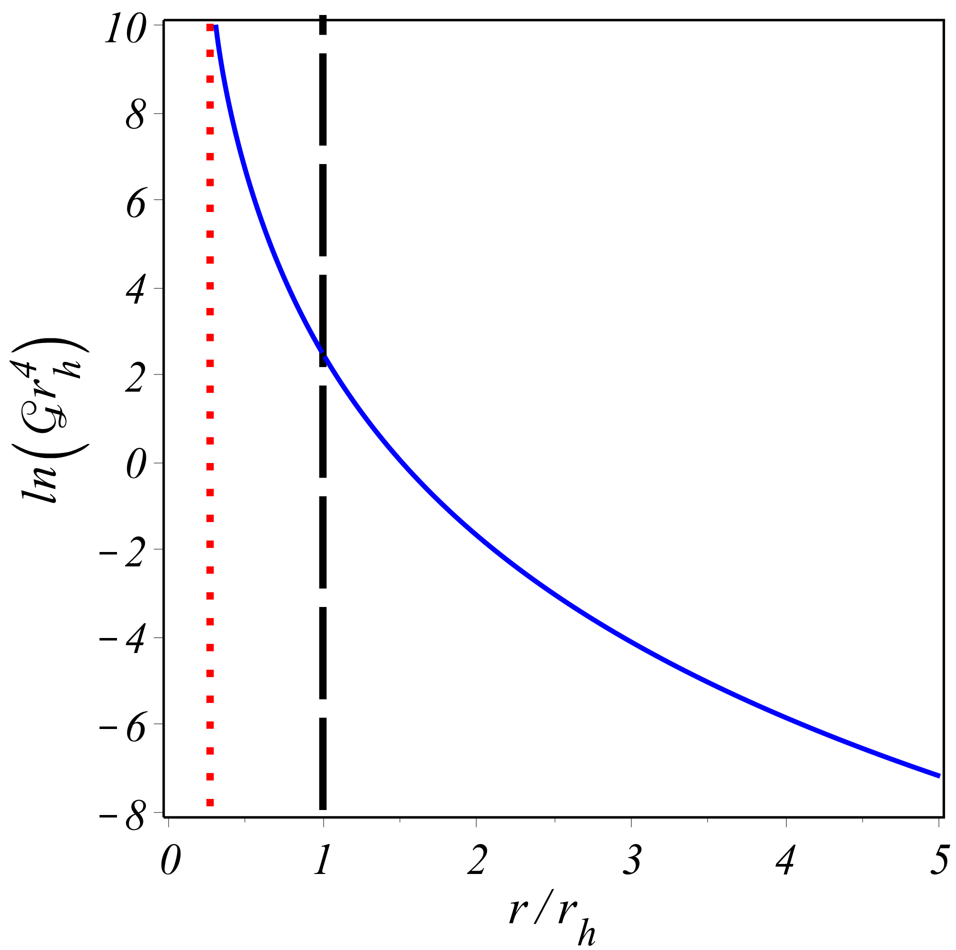
Regularity on the horizon implies $C = 2/M$

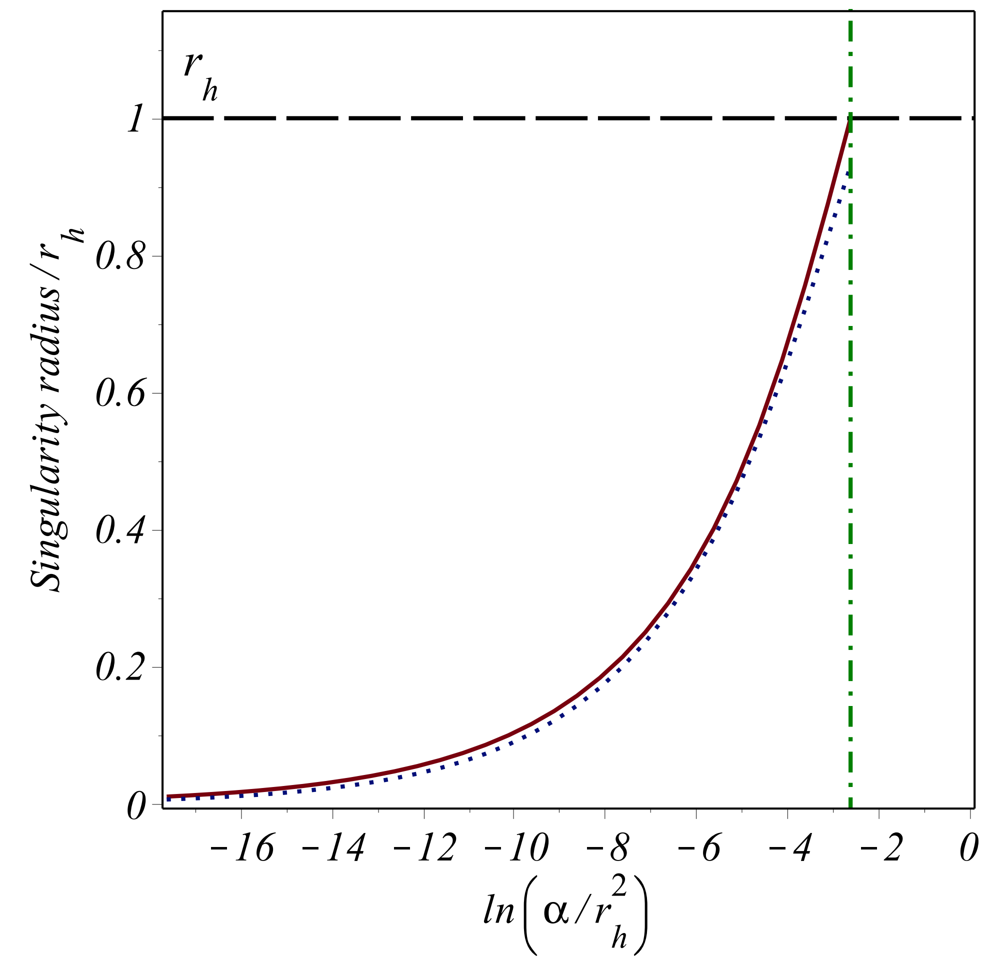
The scalar charge is fixed to be $$P = \frac{2\alpha}{M}$$

To second order in α

- metric no longer Schwarzschild $$M = m\left(1 + \frac{49\alpha^2}{40m^4}\right)$$
- Scalar profile unchanged

Numerical solution





Main characteristics

- Fixed scalar charge, finite area singularity!
- Black holes have a minimum size!
- Perturbative treatments breaks down near singularity