Gauss–Bonnet Boson Stars in AdS

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in collaboration with

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LABORES Scientific Research Lab
Models of Gravity

VII BLACK HOLES WORKSHOP
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Agenda

- Describe the **model** to construct **non-rotating Gauss-Bonnet boson stars in AdS** in $D = 5$ dimensions
- Investigate **effect of Gauss-Bonnet term** to boson star solutions
- Outlook

Why study Q-balls and boson stars?

Boson stars

- Simple **toy models for a wide range of objects** such as particles, compact stars, e.g. neutron stars and even centres of galaxies
- We are interested in the **effect of Gauss-Bonnet gravity** and will study these objects in **the minimal number of dimensions** in which the term does not become a **total derivative.**
- Toy models for **studying properties of AdS space-time**
- Toy models for **AdS/CFT correspondence.** Planar **boson stars** in AdS have been interpreted as Bose-Einstein condensates of glueballs

Model for Gauss–Bonnet Boson Stars

- **Action**

$$
\begin{aligned}
S &= \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\,(R-2\Lambda+\alpha\mathcal{L}_{GB}+16\pi G_5\mathcal{L}_{\mathrm{matter}}) \\
\mathcal{L}_{GB} &= \left(R^{MNKL}R_{MNKL}-4R^{MN}R_{MN}+R^2\right) \qquad (1)
\end{aligned}
$$

- **Matter Lagrangian** $\mathcal{L}_{matter} = -\left(\partial_\mu\Phi\right)^*\partial^\mu\Phi - U(\Phi)$
- **Gauge mediated potential**

$$U_{\mathrm{SUSY}}(|\Phi|) = m^2\eta_{\mathrm{susy}}^2\left(1-\exp\left(-\frac{|\Phi|^2}{\eta_{\mathrm{susy}}^2}\right)\right) \qquad (2)$$

$$U_{\mathrm{SUSY}}(|\Phi|) = m^2|\Phi|^2 - \frac{m^2|\Phi|^4}{2\eta_{\mathrm{susy}}^2} + \frac{m^2|\Phi|^6}{6\eta_{\mathrm{susy}}^4} + O\left(|\Phi|^8\right) \qquad (3)$$

A. Kusenko, Phys. Lett. B **404** (1997), 285; Phys. Lett. B **405** (1997), 108, L. Campanelli and M. Ruggieri, Phys. Rev. D **77** (2008), 043504

Model for Gauss–Bonnet Boson Stars

- **Einstein Equations are derived from the variation of the action with respect to the metric fields**

$$G_{MN} + \Lambda g_{MN} + \frac{\alpha}{2} H_{MN} = 8\pi G_5 T_{MN} \tag{4}$$

where H_{MN} is given by

$$\begin{aligned} H_{MN} &= 2\left(R_{MABC}R_N^{ABC} - 2R_{MANB}R^{AB} - 2R_{MA}R_N^A + RR_{MN}\right) \\ &- \frac{1}{2}g_{MN}\left(R^2 - 4R_{AB}R^{AB} + R_{ABCD}R^{ABCD}\right) \end{aligned} \tag{5}$$

- **Energy-momentum tensor**

$$\begin{aligned} T_{MN} &= -g_{MN}\left[\frac{1}{2}g^{KL}\left(\partial_K\Phi^*\partial_L\Phi + \partial_L\Phi^*\partial_K\Phi\right) + U(\Phi)\right] \\ &+ \partial_M\Phi^*\partial_N\Phi + \partial_N\Phi^*\partial_M\Phi \end{aligned} \tag{6}$$

Model continued

- The **Klein-Gordon equation** is given by:

$$\left(\Box - \frac{\partial U}{\partial|\Phi|^2}\right)\Phi = 0 \qquad (7)$$

- $\mathcal{L}_{matter}$ is invariant under the **global U(1) transformation**

$$\Phi \rightarrow \Phi e^{i\chi} \quad . \qquad (8)$$

- Locally conserved **Noether current** j^M

$$j^M = -\frac{i}{2}\left(\Phi^*\partial^M\Phi - \Phi\partial^M\Phi^*\right) ; j^M_{;M} = 0 \qquad (9)$$

- The globally conserved **Noether charge** Q reads

$$Q = -\int d^4x\sqrt{-g}j^0 \ . \qquad (10)$$

Ansatz non-rotating

- **Metric Ansatz**

$$ds^2 = -N(r)A^2(r)dt^2 + \frac{1}{N(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2 + \sin^2\theta \sin^2\varphi d\chi^2\right) \quad (11)$$

where

$$N(r) = 1 - \frac{2n(r)}{r^2} \quad (12)$$

- **Stationary Ansatz for complex scalar field**

$$\Phi(r,t) = \phi(r)e^{i\omega t} \quad (13)$$

Boundary Conditions for asymptotic AdS space time

- If $\Lambda < 0$ the scalar field function **falls of** with

$$\phi(r >> 1) = \frac{\phi_\Delta}{r^\Delta} \;\; , \;\; \Delta = 2 + \sqrt{4 + L_{eff}^2} \; . \tag{14}$$

- Where L_{eff} **is the effective AdS-radius**:

$$L_{eff}^2 = \frac{2\alpha}{1 - \sqrt{1 - \frac{4\alpha}{L^2}}} ; L^2 = \frac{-6}{\Lambda} \tag{15}$$

- The boundary field is **dual to an operator of dimension in the CFT**

$$\phi_\Delta \leftrightarrow \langle O \rangle ; r \to \infty \tag{16}$$

- **Mass for** $\kappa > 0$ we define the **gravitational mass** at AdS boundary

$$M_G \sim n(r \to \infty)/\kappa \tag{17}$$

Boundary Conditions for asymptotic AdS space time

- α is constrained simultaneously by the **positivity of the energy constraints** in conformal field theories and **causality in their dual gravity description.**

$$\frac{(D-3)(3D-1)}{4(D+1)^2} \leq \alpha \geq \frac{(D-3)(D-4)(D^2-3D+8)}{4(D^2-5D-10)^2} \tag{18}$$

- Case $\alpha > 0$: **Chern-Simons limit**

$$\alpha = \frac{L^2}{4}; L_{eff}^2 = \frac{2\alpha}{1-\sqrt{1-\frac{4\alpha}{L^2}}}; L^2 = \frac{-6}{\Lambda} \tag{19}$$

AdS Gauss–Bonnet Boson Stars with $\alpha \geq 0$

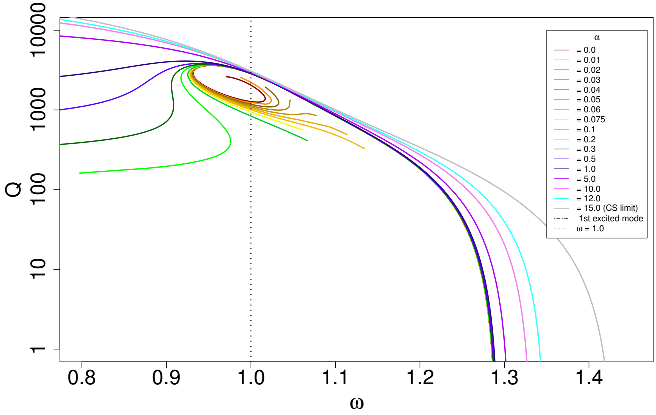


Figure : Charge Q in dependence on the frequency ω for $\Lambda < 0$, $\kappa > 0$ and different values of $\alpha \geq 0$. ω_{max} **shift**: $\omega_{max} = \frac{\Delta}{L_{eff}}$

AdS Gauss–Bonnet Boson Stars with $\alpha \leq 0$

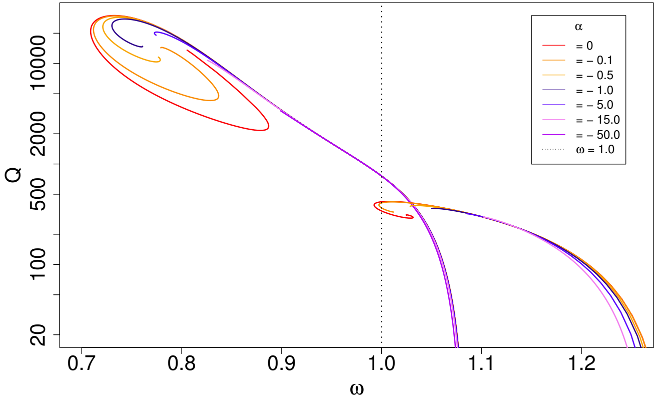


Figure : **Work in progress:** Charge Q in dependence on the frequency ω for $\Lambda < 0$, $\kappa > 0$ and different values of $\alpha \leq 0$. ω_{max} **shift**: $\omega_{max} = \frac{\Delta}{L_{eff}}$

AdS Gauss–Bonnet Boson Stars with $\alpha \leq 0$

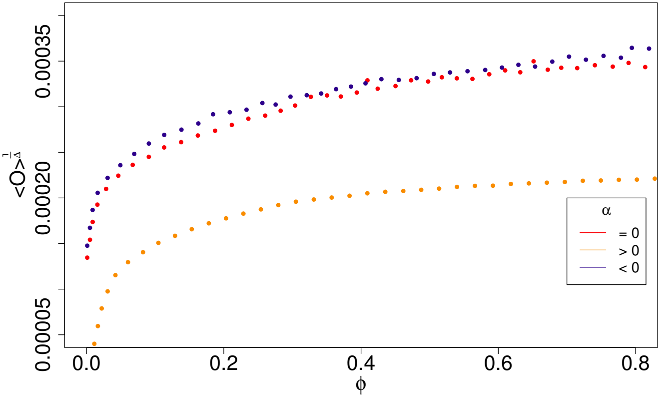


Figure : **Work in progress:** Expectation value of the dual operator on the AdS boundary $\langle O \rangle^{\frac{1}{\Delta}}$

Summary Effect of Gauss-Bonnet Correction

Boson stars

- **Small coupling to GB term, i.e. small** α, we find **similar spiral like characteristic** as for boson stars in **pure Einstein gravity**.
- When the **Gauss-Bonnet parameter** α **is positive and large** enough the **spiral 'unwinds'**.
- When $\alpha > 0$ **and the coupling to gravity (**κ**)** are of the **same magnitude**, only **one branch of solutions** survives.
- When $\alpha < 0$ and negative enough the **spiral 'shrinks' and pulls back** to larger frequencies to becomes **one branch**.
- For $\alpha > 0$ **condensation is harder** at the AdS boundary and for $\alpha < 0$ it **enhances the condensation.**

Outlook

- Analysis the **effect of the Gauss-Bonnet term** on **stability of boson stars.**
- To do a **stabiliy analysis** similar to the one done for non-rotating minimal boson stars by **Bucher et al 2013 ([arXiv:1304.4166 [gr-qc]])**
- See whether **our arguments** related to the **classical stability** of our solutions **agrees with a full perturbation analysis**
- Further study of Gauss-Bonnet Boson Stars and **AdS/CFT correspondance**
- Boson Stars in **general Lovelock theory**

Thank You

Thank You!

Finding solutions: fixing ω

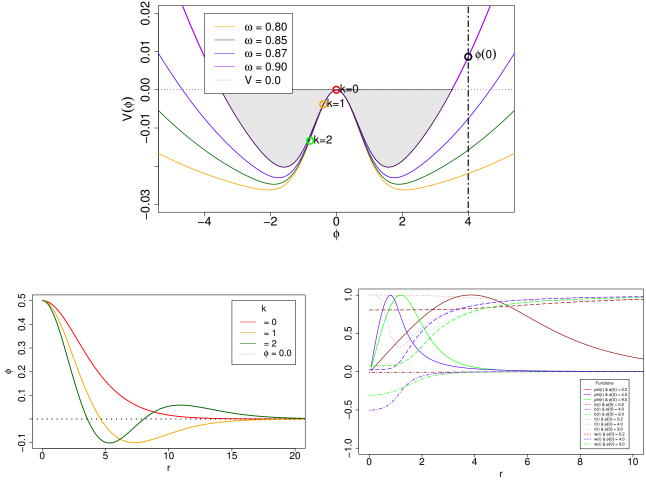


Figure : Effective potential $V(f) = \omega^2 f^2 - U(f)$.

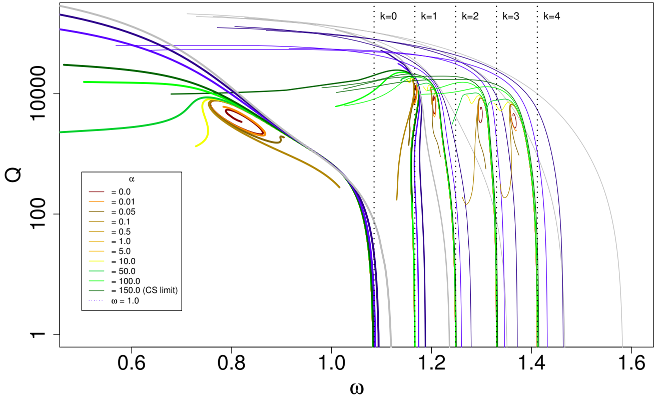
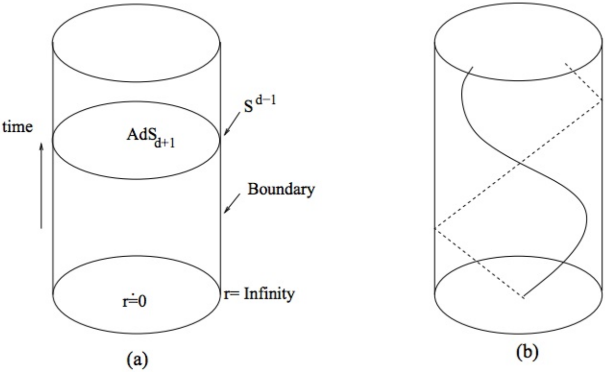


Figure : Charge Q in dependence on the frequency ω for $\Lambda = -0.01$, $\kappa = 0.02$, and different values of α. ω_{max} **shift**: $\omega_{max} = \frac{\Delta+2k}{L_{eff}}$

AdS Space-time



(4)

Figure : (a) Penrose diagram of AdS space-time, (b) massive (solid) and massless (dotted) geodesic.

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(4) J. Maldacena, The gauge/gravity duality, arXiv:1106.6073v1