AdS and Af Horndeski black hole solutions in four dimensions

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Introduction

Horndeski theory

Is the most general scalar-tensor theory in four dimensions, on a Lorentzian manifold and constructed out with a Levi-Civita connection, which gives second order equations of motion for both fields, the metric and the scalar degree of freedom.

A particular case

Kinetic terms with non-minimal couplings given by the Einstein tensor

$$H(\phi, \rho) G_{\mu\nu} \nabla^{\mu} \nabla^{\nu} \phi \to G_{\mu\nu} \nabla^{\mu} \phi \nabla^{\nu} \phi \quad (1)$$

with $H(\phi, \rho)$ an arbitrary function of the scalar field and its kinetic term $\rho = \nabla_{\mu} \phi \nabla^{\mu} \phi$.

- Massimiliano Rinaldi: Black holes with non minimal derivative coupling, arXiv:1208.0103 $/\Lambda = 0$
- Eugeny Babichev, Christos Charmousis: Dressing a black hole with a time-dependent Galileon, arXiv:1312.3204.
- Andres Anabalón, Adolfo Cisterna,Julio Oliva: Asymptotically locally AdS and flat black holes in Horndeski theory, arXiv:1312.3597.
- Masato Minamitsuji: Solutions in the scalar-tensor theory with non minimal derivative coupling, arXiv:1312.3759.
- Moises Bravo-Gaete,Mokhtar Hassaine: Lifshitz black holes with arbitrary dynamical exponent in Horndeski theory, arXiv:1312.7736.
- Adolfo Cisterna, Cristián Erices: Asymptotically locally AdS and flat black holes in the presence of an electric field in the Horndeski scenario, arXiv:1401.4479.
- Tsutomu Kobayashi (et al.), Sergey V. Sushkov (et al.), Thomas P. Sotiriou (et al.), Kaixi Feng (et al.).

The action

Consider the following action

$$I[g,\phi]=\int\sqrt{-g}\ \left[\kappa\left(R-2\Lambda\right)-\frac{1}{2}\left(\alpha g_{\mu\nu}-\eta G_{\mu\nu}\right)\nabla^{\mu}\phi\nabla^{\nu}\phi-\frac{1}{4}F^{2}\right]d^{4}x \qquad (2)$$

α and η are two parameters and $\kappa=\frac{1}{16\pi G}$.

From the scalar field equation we can see

$$\nabla_{\mu}\left[\left(\alpha g^{\mu\nu}-\eta G^{\mu\nu}\right)\nabla_{\nu}\phi\right]=0\ \rightarrow\sqrt{-g}\left(\alpha g^{\mu\nu}-\eta G^{\mu\nu}\right)\nabla_{\nu}\phi=C_{0} \qquad (3)$$

In order to look for solutions we use the ansatz

$$ds^{2}=-F(r)dt^{2}+G(r)dr^{2}+r^{2}d\Sigma_{K}^{2}\ , \qquad (4)$$

$d\Sigma_{K}^{2}\rightarrow$ 2-dimensional Euclidean space with $K=0,\pm1$ and $\phi=\phi\left(r\right)$.

Uncharged AdS solution

Setting $C_0 = 0$ we obtain the following spherically symmetric solution (K=1)

$$F(r) = \frac{r^2}{l^2} + \frac{1}{\alpha}\sqrt{\alpha\eta}\left(\frac{\alpha+\Lambda\eta}{\alpha-\Lambda\eta}\right)^2 \frac{\arctan\left(\frac{\sqrt{\alpha\eta}}{\eta}r\right)}{r} - \frac{\mu}{r} + \frac{3\alpha+\Lambda\eta}{\alpha-\Lambda\eta}\,, \quad (5)$$

$$G(r) = \frac{\alpha^2((\alpha-\eta\Lambda)\,r^2+2\eta)^2}{(\alpha-\eta\Lambda)^2(\alpha r^2+\eta)^2 F(r)} \quad (6)$$

$$\psi^2(r) = -\frac{2r^2\kappa\alpha^2(\alpha+\eta\Lambda)((\alpha-\eta\Lambda)\,r^2+2\eta)^2}{\eta(\alpha-\eta\Lambda)^2(\alpha r^2+\eta)^3 F(r)} \quad (7)$$

where $l^{-2} = \frac{\alpha}{3\eta}$ and μ is the only integration constant.

Features

- α and η must possess the same sign $\rightarrow$ the space-time is asymptotically AdS.
- A real salar field for $r > r_H$ implies $(\alpha + \eta\Lambda) < 0$.
- Under this conditions and for $\mu > 0$ the solution describe a black hole with a single non-degenerate horizon.
- It is not possible to switch off the scalar field.
- The scalar field vanishes at $r = r_H$ but it is not analytic there.
- $\mu = 0$ represents an AdS gravitational soliton. In fact

$$ds^2_{soliton} \underset{r\to 0}{\sim} -\left(1 - \frac{\Lambda}{3}r^2 + O(r^4)\right) dt^2 + \left(1 - \frac{3\alpha + 2\Lambda\eta}{3\eta}r^2 + O(r^4)\right) dr^2 + r^2 d\Omega^2 \qquad (8)$$

Thermodynamic

Using the Hawking and Page approach we need to compute

$$I_{reg} = I_E\left[g_{\mu\nu}, \phi\right] - I_E\left[g^{(0)}_{\mu\nu}, \phi^{(0)}\right] \tag{9}$$

To do so we use the redshift condition

$$\beta^2 F\left(r = r_c, \mu\right) = \beta_0^2 F\left(r = r_c, \mu = 0\right) \tag{10}$$

where the euclidean period is

$$\beta = 4\pi\sqrt{\frac{G'}{F'}}\bigg|_{r=r_+} = \frac{4\pi\eta\left(\alpha - \eta\Lambda\right) r_+}{\alpha\left(2\eta + \left(\alpha - \Lambda\eta\right) r_+^2\right)} = \left[\frac{\sqrt{3}x_+}{4\pi l} + \frac{\sqrt{3}l_0^2}{2\pi l\left(l_0^2 + l^2\right) x_+}\right]^{-1}. \tag{11}$$

Here $l_0 := \sqrt{-\frac{3}{\Lambda}}$ and $x_+ := \sqrt{\frac{\alpha}{\eta}} r_+$.

Thermodynamic

Then

$$I_{reg} = \frac{8\pi^2\kappa}{9}\frac{l^2x_+}{l_0^2\left(2l_0^2+\left(l^2+l_0^2\right)x_+^2\right)}\left[3\left(l^2-l_0^2\right)^2\arctan\left(x_+\right)\right.$$
$$\left.+\left(l^2-2l_0^2\right)\left(l^2+l_0^2\right)x_+^3+3\left(l_0^4-l^4+2l^2l_0^2\right)x_+\right]\ . \tag{12}$$

Using the thermodynamical relations coming from the canonical ensemble

$$I_{reg} = \beta F \tag{13}$$

$$M = \frac{\partial I_{reg}}{\partial\beta} \text{ and } S = \beta\frac{\partial I_{reg}}{\partial\beta} - I_{reg} \tag{14}$$

the entropy reads

$$S = \frac{8\pi^2l^2\kappa x_+^2}{3l_0^2}\left[\frac{\left(l^2+l_0^2\right)\left(l^2-2l_0^2\right)x_+^4+l_0^2\left(l^2-l_0^2\right)x_+^2+2l_0^4}{\left(1+x_+^2\right)\left(2l_0^2-\left(l_0^2+l^2\right)x_+^2\right)}\right]. \tag{15}$$

Entropy analysis

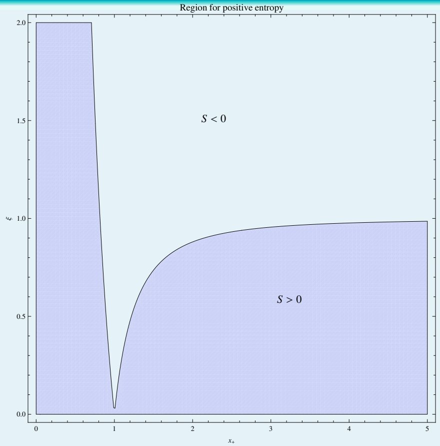

Figure: The grey region corresponds to the region with positive entropy in the plane $\xi := \frac{l^2}{l_0^2} - 1$ vs x_+, while the white regions stands for negative entropy. For $\xi > 1$, requiring $S > 0$ implies an upper bound on the black holes radius while for $0 < \xi < 1$ there is a gap on the possible radius of black holes with positive entropy.

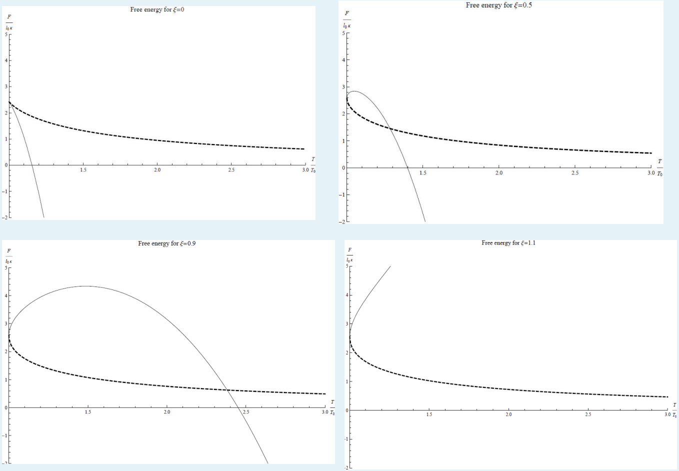

Figure: The free energy for large (continuous) and small (dashed) black holes in terms of $\frac{T}{T_0}$, for $\xi = 0$ (2.a), $\xi = 0.5$ (2.b), $\xi = 0.9$ (2.c) and $\xi = 1.1$ (2.d). The x-axis do not start at $T = 0$.

Charged Af solution

Considering α and Λ vanishing

$$I[g,\phi] = \int \sqrt{-g}\left[\kappa R + \frac{\eta}{2} G_{\mu\nu}\nabla^{\mu}\phi\nabla^{\nu}\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right] d^4x \tag{16}$$

The solution take the form

$$ds^2 = -F(r)dt^2 + \frac{3(8\kappa r^2 - q^2)^2}{r^4}\frac{dr^2}{F(r)} + r^2 d\Omega^2 \tag{17}$$

with

$$F(r) = 192\kappa^2 - \frac{\mu}{r} + 48\kappa\frac{q^2}{r^2} - \frac{q^4}{r^4}$$

$$\psi(r)^2 = -\frac{15}{2}\frac{(8\kappa r^2 - q^2)^2}{r^6\eta}\frac{q^2}{F(r)}$$

$$A_0(r) = \sqrt{15}\left(\frac{q^3}{3r^3} - 8\kappa\frac{q}{r}\right)$$

- The solution is asymptotically flat

$$ds^2 = -\left(1 - \frac{\mu}{r} + O(r^{-2})\right) dt^2 + \left(1 + \frac{\mu}{r} + O(r^{-2})\right) dr^2 + r^2 d\Omega^2$$

- For a non degenerated horizon $r = r_H$ the scalar field vanish at the horizon, as in the previous cases, is not analytic there.
- A real scalar field outside of the horizon implies

$$\eta < 0$$

- For any value of the integration constant μ we have the curvature singularities

$$r_0 = 0 \ ,$$

$$r_1 = \sqrt{\frac{1}{8\kappa}|q|} \ .$$

- The electric field goes to zero at infinity.
- Taking the limit when $q \to 0$ we obtain a trivial scalar field and then we recover the Schwarzschild solution.

Further remarks

- Asymptotically AdS and asymptotically flat solutions were described for a particular action contained in the Horndeski theory.
- The spherically symmetric uncharged AdS solution exhibits a phase transition between thermal AdS and large black holes.
- The entropy in this later case does not agree with Wald formula, $S = \frac{A}{4}$.
- For static configurations the scalar field is regular at the horizon but not analytic.

Muito obrigado