Abstract

It has long been known that Lovelock gravity, being of Cauchy-Kowalevskaya type, admits a well defined initial value problem for analytic data. However, this does not address the physically important issues of continuous dependence of the solution on the data and the domain of dependence property. In this note we fill this gap in our understanding of the (local) dynamics of the theory. We show that, by a known mathematical trick, the fully nonlinear harmonic-gauge-reduced Lovelock field equations can be made equivalent to a quasilinear PDE system. Due to this equivalence, an analysis of the principal symbol, as has appeared in recent works by other authors, is sufficient to decide the issue of local well-posedness of perturbations about a given background.

Local well-posedness in Lovelock gravity

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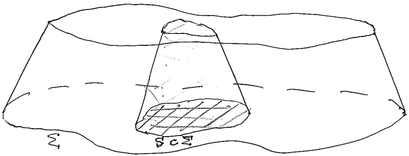
Background

Q: Why Lovelock? Most general second derivative theory in the metric. Can play a role in higher dimensional theories.

Q: What does well-posed mean? Existence, uniqueness of solutions on some domain, for an appropriately chosen initial surface. (diagram) Continuous dependence of solution on data; domain of dependence theorem.



Q: Why does it matter? Physics is predictable; approximate solutions have physical meaning; finite propagation speed.

The field equations

- Lovelock theory field equations ($d \geq 5$ dimensions):

$$\delta^{bd_1 d_2}_{ac_1 c_2} R^{d_1 d_2}{}_{c_1 c_2} + \lambda \delta^{bd_1 \ldots d_4}_{ac_1 \ldots c_4} R^{d_1 d_2}{}_{c_1 c_2} R^{d_3 d_4}{}_{c_3 c_4} + \cdots = T^b_a \,. \tag{1}$$

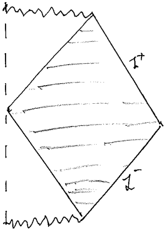
- Harmonic gauge reduced equations: [Choquet-Bruhat 1988]

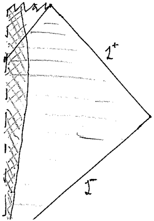
$$g^{cd} \partial_c \partial_d g_{ab} + \lambda F_{ab}(g, \partial g, \partial\partial g) = B_{ab}(g, \partial g) \tag{2}$$

  F is nonlinear in second derivatives.
- Cachy-Kowalevskaya form. F is linear in $g_{ab,00}$. Existence and uniqueness for analytic data.
- However, standard well-posedness results do not apply to the equations in this form (they are not quasilinear hyperbolic).

Characteristic surfaces and black holes in Lovelock theory

- Boulware-Deser black hole event horizon is null. It is also characteristic [Izumi 2014]. Also conformal infinity (asymptotically flat) is characteristic in an asymptotoc sense.
- Same result holds in higher order Lovelock theory [Reall et a 2014l]. Superluminal propagation can occur in the shaded region, question of global hyperbolicity of shaded region is open.





- Reall et a [2014] found that the hyperbolic nature of field equations can break down for B.D black hole in 5 and 6 dimensions. Therefore perturbations is ill posed problem.

Missing piece in the jigsaw

- In order to be able to make positive statements about well-posedness we need to reformulate the theory as a quasilinear system.
- Introduce new variable $v_{\mu\nu i}$, and imposing initial conditions $v_{\mu\nu i} = \partial_i g_{\mu\nu}$, $i = 1, \ldots, n-1$, and its first time derivative, one can arrive at an equivalent system of equations: [Willison 2014 to appear in Class. Quant. Grav.]

$$g^{\rho\sigma}\partial_\rho\partial_\sigma g_{\mu\nu} = \cdots \tag{3}$$

$$g^{\rho\sigma}\partial_\rho\partial_\sigma v_{\mu\nu i} - \lambda A^{jk\mu\nu}_{\alpha\beta}\partial_k\partial_i v_{\mu\nu j} - \lambda B^{j\mu\nu}_{\alpha\beta}\partial_j\partial_0 v_{\mu\nu i} = \cdots \tag{4}$$

- This is quasilinear. An analysis of the characteristics is therefore sufficient to decide the issue of well-posedness.
- In particular, we note that for small perturbations about Minkowski space, the system is (Leray) hyperbolic.