Wormholes and black holes in ghost-free bigravity

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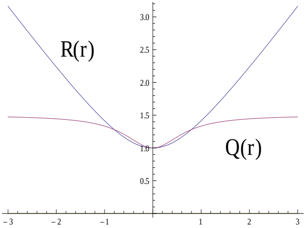
S.V.Sushkov and M.S.V., arXiv.1412.xxxx

M.S.V., Phys.Rev. D85 (2012) 124043

Wormholes – bridges between universes

$$ds^2 = -Q^2(r)dt^2 + dr^2 + R^2(r)(d\vartheta^2 + \sin^2\vartheta d\varphi^2),$$



$G_{\mu\nu} = 8\pi G T_{\mu\nu} \Rightarrow \rho + p < 0,\ p < 0 \Rightarrow$ violation of the energy conditions $\Rightarrow$ vacuum polarization, phantom fields, higher derivative gravity, Gauss-Bonnet, branworld gravity, non-minimal coulings, Horndeski, Galileon $\Rightarrow$ Massive gravity.

Massive gravity

Non-linear extension of the linear Pauli-Fierz theory of massive gravitons /1939/.

Non-unique, generaically propagates 6 DoF,

$$6 = 5 \text{ graviton polarizations} + 1 \text{ Boulware-Deser ghost}$$

The ghost has a negative kinetic energy $\Rightarrow$ instability /1972/.

There is a unique massive gravity theory which propagates only 5 DoF /de Rahm, Gabadadze, Tolley 2010/. The theory admits

- self-accelerating cosmologies
- black holes
- $G_{\mu\nu} = m^2 T_{\mu\nu}(g, f)$ where $T_{\mu\nu}(g, f)$ does not satisfy the local energy conditions. This suggests that wormholes could exist. This risks to invalidate the theory.

Ghost-free bigravity – two dynamical metrics $g_{\mu\nu}$ and $f_{\mu\nu}$.

$$S = \frac{m^2}{M_{\rm Pl}^2} \int \left(\frac{1}{2\kappa_1} R(g)\sqrt{-g} + \frac{1}{2\kappa_2} R(f)\sqrt{-f} - \mathcal{U}\sqrt{-g} \right) d^4x$$

$$\begin{aligned} \mathcal{U} &= b_0 + b_1 \sum_A \lambda_A + b_2 \sum_{A<B} \lambda_A \lambda_B \\ &+ b_3 \sum_{A<B<C} \lambda_A \lambda_B \lambda_C + b_4\, \lambda_0 \lambda_1 \lambda_2 \lambda_3 \end{aligned}$$

where λ_A are eigenvalues of $\gamma^\mu{}_\nu = \sqrt{g^{\mu\alpha} f_{\alpha\nu}}$.

$$\begin{aligned} G^\mu_\nu(g) &= \kappa_1\, T^\mu{}_\nu(g,f), \\ G^\mu_\nu(f) &= \kappa_2\, \mathcal{T}^\mu{}_\nu(g,f), \end{aligned}$$

A massive + a massless graviton = 7 DoF.

Reduction to the S-sector

$$
\begin{aligned}
ds_g^2 &= -Q^2 dt^2 + \frac{R'^2}{N^2}\, dr^2 + R^2 d\Omega^2 \\
ds_f^2 &= -q^2 dt^2 + \frac{U'^2}{Y^2}\, dr^2 + U^2 d\Omega^2
\end{aligned}
$$

Q, N, R, q, Y, U depend on r, one can impose 1 gauge condition.
5 independent equations

$$
\begin{aligned}
G^0_0(g) &= \kappa_1\, T^0_0, \\
G^r_r(g) &= \kappa_1\, T^r_r, \\
G^0_0(f) &= \kappa_2\, \mathcal{T}^0_0, \\
G^r_r(f) &= \kappa_2\, \mathcal{T}^r_r, \\
T^{r\prime}_r &+ \frac{Q'}{Q}(T^r_r - T^0_0) + \frac{2}{r}(T^\vartheta_\vartheta - T^r_r) = 0.
\end{aligned}
$$

Equations

Three coupled ODEs

$$\begin{aligned} N' &= \mathcal{D}_N(N, Y, U, R), \\ Y' &= \mathcal{D}_Y(N, Y, U, R), \\ U' &= \mathcal{D}_U(N, Y, U, R), \end{aligned} \tag{1}$$

where R is subject to gauge-fixing, plus one extra ODE,

$$Q' = \frac{1}{2} F(N, Y, U, R) Q,$$

plus an algebraic constraint

$$q = \Sigma(N, Y, R, U)\, Q.$$

Eqs.(1) were studied in the black hole context, their boundary conditions are the same for black holes and for wormholes.

Black holes versus wormholes in Schwarzschild gauge

$$ds^2 = -Q^2 dt^2 + \frac{dR^2}{N^2} + R^2 d\Omega^2$$

black holes: for $R = h > 0$ (horizon) both Q^2 and N^2 vanish,

$$Q^2 \propto N^2 \propto \varepsilon \quad \text{for} \quad R = h + \varepsilon$$

wormholes: for $R = h > 0$ (neck) only N^2 vanishes,

$$Q^2 = \mathcal{O}(1), \quad N^2 \propto \varepsilon \quad \text{for} \quad R = h + \varepsilon$$

Passing to $r = \int_h^R dR/N$ gives the standard wormhole form

$$ds^2 = -Q^2 dt^2 + dr^2 + R^2 d\Omega^2$$

with $R = h + \mathcal{O}(r^2)$ and $Q = Q(0) + \mathcal{O}(r^2)$.

Boundary conditions

$$\begin{aligned} N' &= \mathcal{D}_N(N, Y, U, R), \\ Y' &= \mathcal{D}_Y(N, Y, U, R), \\ U' &= \mathcal{D}_U(N, Y, U, R), \end{aligned} \tag{2}$$

where for $R = h > 0$ one has

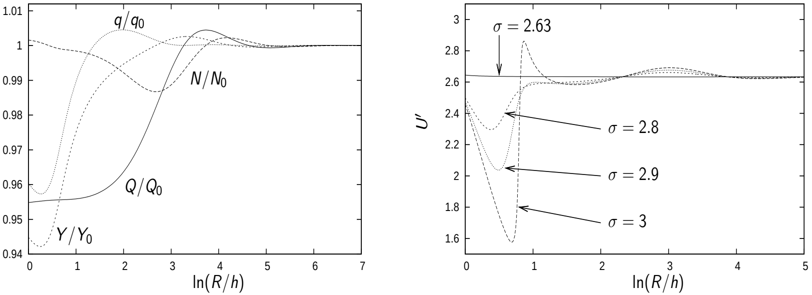
$$N = Y = 0, \quad U = \sigma \tag{3}$$

Solutions are labeled by h and by σ. They determine

$$(Q^2)' = F(N, Y, U, R)Q^2.$$

For generic h, σ solutions of (2,3) the function F has a pole at $R = h \Rightarrow Q^2$ vanishes $\Rightarrow$ black holes.
For special h, σ there are solutions of (2,3) for which F is finite at $R = h \Rightarrow Q^2$ is also finite $\Rightarrow$ wormholes.

Black holes



Exist for generic h, σ, approach AdS: N_0, Q_0, Y_0, q_0 correspond to the AdS solution. M.S.V., Phys.Rev. D85 (2012) 124043

For specially fine-tuned h, σ there are asymptotically flat black holes. R.Brito, V.Cardoso, P.Pani, Phys.Rev. D88 (2013) 064006

A different fine-tunung of h, σ gives wormholes.

Wormholes – local solution

$$ds_g^2 = -Q^2 dt^2 + dr^2 + R^2 d\Omega^2$$
$$ds_f^2 = -q^2 dt^2 + \frac{U'^2}{Y^2} dr^2 + U^2 d\Omega^2$$

$$Y = Y_1 r + Y_3 r^3 + \dots \quad Q = Q_0 + Q_2 r^2 + \dots \quad R = h + R_2 r^2 + \dots$$
$$q = q_0 + q_2 r^2 + \dots \qquad U = \sigma h + U_2 r^2 + \dots$$

Expanding the field equations gives in the leading order

$$\left(\kappa_1 \mathrm{P}_0 - \frac{1}{h^2}\right) Q_0 + \kappa_1 \mathrm{P}_1\, q_0 = 0,$$
$$\left(\kappa_2 \mathrm{P}_2 - \frac{1}{h^2}\right) q_0 + \kappa_2 \mathrm{P}_1\, Q_0 = 0,$$

with $\mathrm{P}_m = b_m + 2b_{m+1}\sigma + b_{m+2}\,\sigma^2$. To have non-zero Q_0, q_0, the determinant of this system must vanish. This gives

Master condition

$$\boxed{\left(\kappa_1 h^2 \mathrm{P}_0 - 1\right)\left(\kappa_2 h^2 \mathrm{P}_2 - 1\right) - \kappa_1\kappa_2 h^4 \mathrm{P}_1^2 = 0}$$

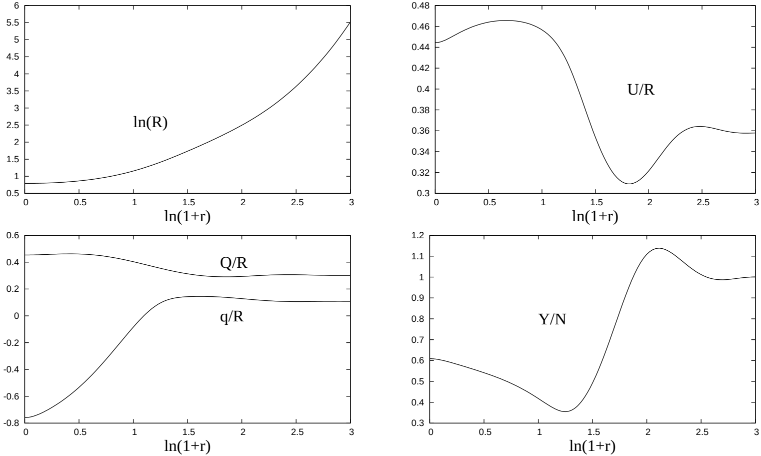
– an algebraic equation for σ. A real solution exists if $h > 1/\sqrt{3}$ (in $1/m$ units) so that the wormhole throat is cosmologically large. One then determines the lowest expansion coefficients

$$\begin{aligned}
R_2 &= -\frac{\kappa_1\sigma\alpha}{2\kappa_2}\, Y_1, \\
U_2 &= \frac{\alpha}{2}\left(1 + \frac{2\sigma Y_1}{\kappa_2 h \mathrm{P}_1}\right) Y_1, \\
Q_2 &= -\left(\frac{\kappa_1}{4}\left(\frac{2U_2}{Y_1} - \sigma\right)(d\mathrm{P}_0 + \alpha d\mathrm{P}_1) + \frac{R_2}{h} + \frac{1}{2h^2}\right) Q_0, \\
q_2 &= \frac{U_2}{Y_1}\left(2Q_2 + \frac{2R_2 - Y_1}{h\mathrm{P}_1}(d\mathrm{P}_0 + \alpha d\mathrm{P}_1) Q_0\right),
\end{aligned}$$

$$Y_1 = \frac{\kappa_2 h^2 \mathrm{P}_1(\kappa_2 + \kappa_1\sigma^2)(d\mathrm{P}_0 + 2\alpha d\mathrm{P}_1 + \alpha^2 d\mathrm{P}_2) - 2\sigma \mathrm{P}_1(\kappa_2 + \kappa_1\alpha^2)}{2\sigma h\left[(\kappa_2 + 2\kappa_1\alpha\sigma)d\mathrm{P}_0 + 2\kappa_1\sigma\alpha^2 d\mathrm{P}_1 - \kappa_2\alpha^2 d\mathrm{P}_2\right] - 2\alpha h \mathrm{P}_1(\kappa_2 + \kappa_1\sigma^2)}.$$

with $\alpha = \frac{1-\kappa_1 h^2 \mathrm{P}_0}{\kappa_1 h^2 \mathrm{P}_1}$. The local solution is extended numerically.

Wormholes – global solutions



Solutions for $\kappa_1 = 0.688$, $\kappa_2 = 0.312$, $b_k = b_k(c_3, c_4)$, $c_3 = 3$, $c_4 = -6$, for the neck radius $h = 2.2$. Here $\sigma = 0.444$ and $N = R'$.

Asymptotic behavior

For $R \to \infty$ solutions approach the AdS solution, $ds_f^2 = \lambda^2 ds_g^2$ where

$$ds_g^2 = -Q^2 dt^2 + \frac{dR^2}{N^2} + R^2 d\Omega^2$$

with

$$N^2 \to N_0^2 = 1 - \frac{\Lambda r^2}{3}$$

and $Q^2 \to const \times N_0^2$. One has for large R

$$N^2 = N_0^2 \times \left(1 + \frac{C}{R^3} + \frac{A}{R\sqrt{R}} \cos\left(\omega \ln(R) + \varphi\right)\right)$$

C-term is the Newtonian tail, the A-term is the effect of the massive mode – scalar polarization of the massive graviton. Oscillations: the massive graviton becomes a tachyon, with

$$m_{FP}^2 = \left(\frac{\kappa_2}{\lambda} + \kappa_1 \lambda\right)(b1 + 2b_2\lambda + b_3\lambda^2) < 0$$

Conclusions

- The ghost-free bigravity theory admits solutions for which the f-metric can be singular, but the g-metric describes globally regular wormholes.
- The wormholes interpolate between two AdS spaces.
- The wormhole throat is cosmologically large (could we live inside it ?)
- Fields approach the tachyon phase for $r \to \pm\infty$ but tachyons belong to unphysical sectors $\Rightarrow$ wormholes should be disregarded as unphysical solutions.
- However, they may have a holographic interpretation.