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Traversable Wormholes in Distorted Gravity

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Introduction

- A wormhole can be represented by two asymptotically flat regions joined by a bridge.
- One very simple and at the same time fundamental example of wormhole is represented by the Schwarzschild solution of the Einstein's field equations.
- One of the prerogatives of a wormhole is its ability to connect two distant points in space-time. In this amazing perspective, it is immediate to recognize the possibility of traveling crossing wormholes as a short-cut in space and time.
- A Schwarzschild wormhole does not possess this property.

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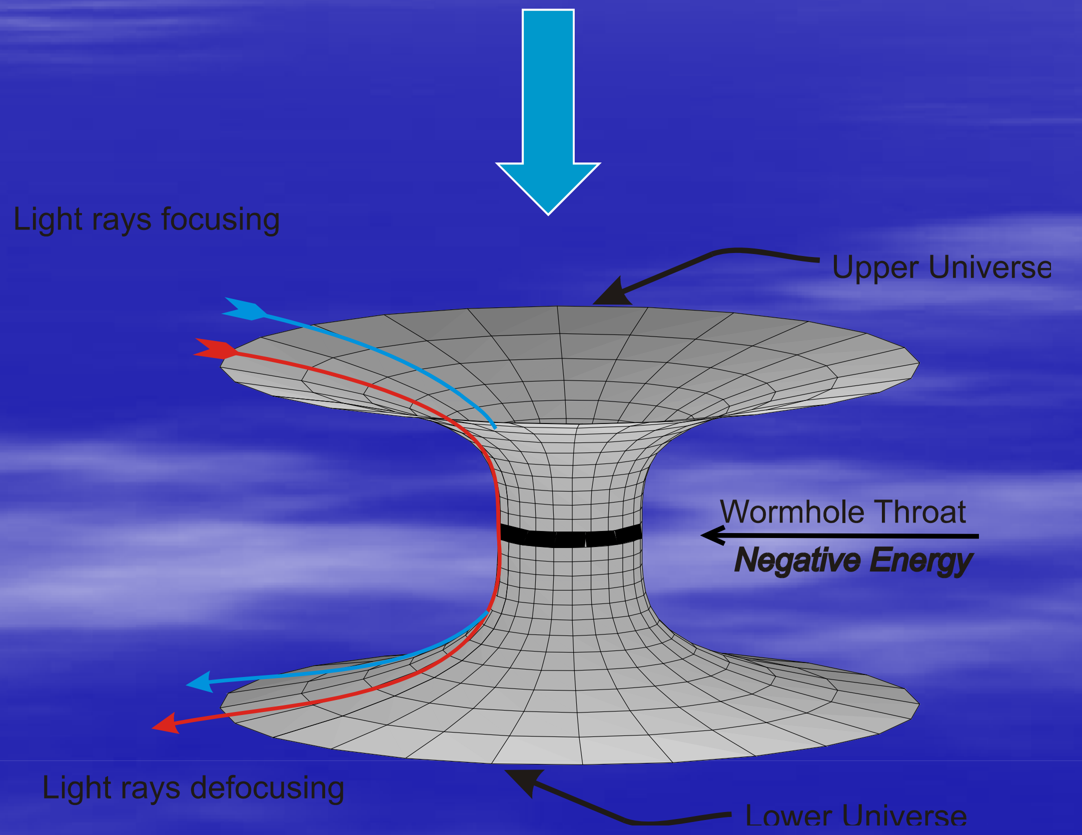
Traversable wormholes

Introduction

Traversable wormholes



The traversable wormhole metric

M. S. Morris and K. S. Thorne, Am. J. Phys. 56, 395 (1988).

$$ds^2 = -\exp\left(-2\phi(r)\right)dt^2 + \frac{dr^2}{1-\frac{b(r)}{r}} + r^2 d\theta^2 + r^2\sin^2\theta d\varphi^2$$

Condition

- $b(r)$ is the shape function
- $\phi\ (r)$ is the redshift function

$$r \in \left[r_0, +\infty\right)$$

$$b_\pm(r_0) = r_0$$

$$b_\pm(r) < r$$

Proper radial distance

$$l(r) = \pm\int_{r_0}^{r}\frac{dr'}{\sqrt{1-b_\pm(r')/r'}}$$

$$\lim_{r\to\infty} b_\pm(r) = b_\pm$$ Appropriate asymptotic

$$\lim_{r\to\infty} \phi_\pm(r) = \phi_\pm$$ limits

Einstein Field Equations

$$G_{\mu\nu} = \kappa T_{\mu\nu} \qquad \kappa = 8\pi G$$

Orthonormal frame

$$b'(r) = 8\pi G \rho r^2$$

$$\phi'(r) = \frac{b + 8\pi G p_r r^3}{2r^2\left(1 - b(r)/r\right)}$$

$$\tau(r) = -p_r$$

General setting for self sustained traversable wormholes

Instead of $G_{\mu\nu} = \kappa T_{\mu\nu}$ consider $G_{\mu\nu} = \kappa \left\langle T_{\mu\nu} \right\rangle^{ren}$

where $\left\langle T_{\mu\nu} \right\rangle^{ren}$ renormalized expectation value of the stress-energy tensor operator of the quantized field

If the matter field source is absent $\left\langle T_{\mu\nu} \right\rangle^{ren} = -\frac{1}{\kappa} \left\langle \Delta G_{\mu\nu} \left(\tilde{g}_{\alpha\beta}, h_{\alpha\beta} \right) \right\rangle^{ren}$

$$G_{\mu\nu} \left(g_{\alpha\beta} \right) = G_{\mu\nu} \left(\tilde{g}_{\alpha\beta} \right) + \Delta G_{\mu\nu} \left(\tilde{g}_{\alpha\beta}, h_{\alpha\beta} \right) \qquad g_{\alpha\beta} = \tilde{g}_{\alpha\beta} + h_{\alpha\beta}$$

$\Delta G_{\mu\nu} \left(\tilde{g}_{\alpha\beta}, h_{\alpha\beta} \right)$ is a perturbation series in terms of $g_{\alpha\beta}$

The Einstein tensor $G_{\mu\nu}$ can be divided into a part which is unperturbed related to the background geometry and a part related to quantum fluctuations like the metric $g_{\alpha\beta}$

General setting for self sustained traversable wormholes

Usually $\left\langle \Delta G_{\mu\nu}\left(\tilde{g}_{\alpha\beta}, h_{\alpha\beta}\right)\right\rangle$ is divergent!!!

We need a regularization/renormalization scheme to handle with divergences

Distorting Gravity avoids such a scheme

Proposal

$\searrow$

Gravity's Rainbow

Gravity's Rainbow

Doubly Special Relativity

G. Amelino-Camelia, Int.J.Mod.Phys. D 11, 35 (2002); gr-qc/001205.
G. Amelino-Camelia, Phys.Lett. B 510, 255 (2001); hep-th/0012238.

$$E^2 g_1^2\left(E/E_P\right) - p^2 g_2^2\left(E/E_P\right) = m^2$$

$$\lim_{E/E_P \to 0} g_1\left(E/E_P\right) = \lim_{E/E_P \to 0} g_2\left(E/E_P\right) = 1$$

Curved Space Proposal → *Gravity's Rainbow*
[J. Magueijo and L. Smolin, Class. Quant. Grav. 21, 1725 (2004) arXiv:gr-qc/0305055].

$$G_{\mu\nu} = 8\pi G\left(E\right) T_{\mu\nu}\left(E\right) + g_{\mu\nu}\Lambda\left(E\right)$$

$$G\left(E\right) \to G\left(0\right) \quad \text{when} \quad E \ll E_P$$

$$\Lambda\left(E\right) \to G\left(0\right) \quad \text{when} \quad E \ll E_P$$

Gravity's Rainbow

Doubly Special Relativity

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Curved Space Proposal → *Gravity's Rainbow*
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$$ds^2 = -\left(1 - \frac{2MG(0)}{r}\right)\frac{d\tilde{t}^2}{g_1^2\left(E/E_P\right)} + \frac{d\tilde{r}^2}{\left(1 - \frac{2MG(0)}{r}\right)g_2^2\left(E/E_P\right)} + \frac{\tilde{r}^2}{g_2^2\left(E/E_P\right)}\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

Gravity's Rainbow

$$ds^2 = -\exp\left(-2\Lambda(r)\right)\frac{dt^2}{g_1^2(E/E_P)} + \frac{dr^2}{\left(1-\frac{b(r)}{r}\right)g_2^2(E/E_P)} + \frac{r^2}{g_2^2(E/E_P)}d\theta^2 + \frac{r^2}{g_2^2(E/E_P)}\sin^2\theta d\varphi^2$$

- $b(r)$ is the shape function
- $\Lambda(r)$ is the redshift function

$$b(r_0) = r_0 \quad r \in [r_0, +\infty)$$

$$\frac{b'(r)}{8\pi G r^2} = \rho$$

Self Sustained Equation

$$\frac{b'(r)}{2Gr^2 g_2(E/E_P)} = \frac{2}{3\pi^2}\sum_{i=1}^{2}\int_{E^*}^{+\infty} E_i \frac{g_1(E/E_P)}{g_2^2(E/E_P)}\frac{d}{dE_i}\sqrt{\left(\frac{E_i^2}{g_2^2(E/E_P)} - m_i^2(r)\right)^3}dE_i$$

[R.G. and F.S.N. Lobo, arXiv:1102.3803 [gr-qc]] Phys.Rev. D85 (2012) 024043

Effective Einstein's Field Equations

$$\frac{b'(r)}{2Gr^2 g_2(E/E_P)} = \frac{2}{3\pi^2}\sum_{i=1}^{2}\int_{E^*}^{+\infty} E_i \frac{g_1(E/E_P)}{g_2^2(E/E_P)} \frac{d}{dE_i}\sqrt{\left(\frac{E_i^2}{g_2^2(E/E_P)} - m_i^2(r)\right)^3}\, dE_i$$

The only solution is in the trans-Planckian region

$$g_1(E/E_P) = \exp\left(-\alpha\frac{E^2}{E_P^2}\right)\left(1+\beta\frac{E}{E_P}\right)$$

$$\mathrm{b}(r) = \mathrm{r}_0$$

$$g_2(E/E_P) = 1$$

$$\mathrm{b}(r) = \mathrm{r}_0^2/r$$

We can also fix the geometry

We find $\alpha \simeq 1/4$

$$r_0 = 1.46/E_P$$

$$\begin{cases} m_1^2(r) = \dfrac{6}{r^2}\left(1-\dfrac{b(r)}{r}\right) - \dfrac{3}{2r^2}\left[b'(r) - \dfrac{b(r)}{r}\right] \\ m_2^2(r) = \dfrac{6}{r^2}\left(1-\dfrac{b(r)}{r}\right) - \dfrac{3}{2r^2}\left[\dfrac{b'(r)}{3} + \dfrac{b(r)}{r}\right] \end{cases}$$

Lichnerowicz Potentials

Topology Change

R.G. and F.S.N. Lobo, arXiv:1303.5566 [gr-qc] Eur.Phys.J. C74 (2014) 2884

Recursive way (n) is the order of approximation

$$\frac{\left(b'(r)\right)^{(n)}}{2Gr^2 g_2\left(E/E_P\right)}=\frac{2}{3\pi^2}\sum_{i=1}^{2}\int_{E^*}^{+\infty}E_i\,\frac{g_1(E/E_P)}{g_2^2(E/E_P)}\frac{d}{dE_i}\sqrt{\left(\frac{E_i^2}{g_2^2(E/E_P)}-\left(m_i^2(r)\right)^{(n-1)}\right)^3}\,dE_i$$

Specific example III: $g_2\left(E/E_P\right)=1+E/E_P$ and $g_1\left(E/E_P\right)=g\left(E/E_P\right)\left(1+E/E_P\right)^6$

We fix on the r.h.s. $b(r)=0\rightarrow Minkowski$

asymptotically flat solution:

$$b(r)=r_0+\frac{3\sqrt{6}}{\pi^2 E_P e^{\alpha}\alpha}\left\{1-\exp\left[\left(1-\frac{\sqrt{6}}{E_P r}\right)\alpha\right]\right\}$$

Gravity's Rainbow Applications

◆ Cosmological Constant computation

R.G. and G.Mandanici, Phys. Rev. D 83, 084021 (2011), arXiv:1102.3803 [gr-qc]

◆ Cosmological Constant computation+f(R)

R.G., **JCAP 1306 (2013) 017** arXiv:1210.7760

◆ Particle Propagation

R.G. and G. Mandanici Phys.Rev. D85 (2012) 023507 e-Print: arXiv:1109.6563 [gr-qc]

◆ Naked Singularity and Charge Creation

R.G. and B. Majumder, N**ucl.Phys. B884 (2014) 125** e-Print: **arXiv:1311.1747** **[gr-qc]**
R.G. and B. Majumder, N**ucl.Phys. B883 (2014) 598** e-Print: **arXiv:1305.3390** **[gr-qc]**

◆ Inflation

R.G. and M.Sakellariadou, **Phys.Rev. D90 (2014) 4, 043521** arXiV:1212.4987