Bouncing shells in Anti de Sitter space

Alexandre Serantes

Universidade de Santiago de Compostela

alexandre.serantes@rai.usc.es

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Overview

Motivation

- Gravitational collapse process on $AdS_{d+2} \sim$ thermalization of large N, large λ $CFT_d + 1$
- Look for model that allows explicit and straightforward calculations: thin shell spacetimes
- Usually considered in Poincare-AdS, mostly Vaidya spacetimes [Abajo-Arrastia, Aparicio, Lopez; Valasubramaian, Bernamonti, de Boer, Copland, Craps, Keski-Vakkuri, Muller, Schafer, Shigemori, Staessens; Allais, Tonni; Keranen, Nishimura, Stricker, Taanila, Vuorinen and many more...]
- What happens if the field theory lives on a compact space?

What happens in global AdS?

Motivation

- Global AdS-Vaidya describes direct black hole formation
- More general matter models lead to different behavior
  Example: massless scalar field
  - Localized scalar pulse can bounce back-and-forth between $r = 0$ and $r = \infty$ several times before collapsing [Bizon, Rostworowski]
  - There even exist exactly periodic solutions that never collapse [Maliborski, Rostworowski]
- Finite size systems may exhibit instrinsic quantum periodicity: bouncing $\sim$ revivaling $+$ relaxation?

The question I will try to answer...

Can we find exactly periodic solutions in other matter models?

The problem

- Simplifying assumption (i): *thin shell approximation*
- Simplifying assumption (ii): $SO(d)$ symmetric thin shell
- Simplifying assumption (iii): perfect fluid shell

$$\begin{aligned} S_{ab} &= \sigma u_a u_b + p\left(h_{ab} + u_a u_b\right) \\ p &= \frac{\alpha}{d}\sigma \end{aligned}$$

- Schwarzschild coordinates $(t_\pm, r, \theta_1 ... \theta_d)$ outside the shell
- Comoving coordinates $(\tau, \theta_1 ... \theta_d)$ on the shell worldvolume
- Line element

$$\begin{aligned} &ds_\pm^2 = -f_\pm(r)dt_\pm^2 + f_\pm(r)^{-1}dr^2 + r^2 d\Omega_d^2 \\ &dh^2 = -d\tau^2 + r(\tau)^2 d\Omega_d^2 \end{aligned}$$

$$\begin{aligned} &f_-(r) = 1 + r^2 \\ &f_+(r) = 1 + r^2 - \frac{m}{r^{d-1}} \end{aligned}$$

How do we find the shell trajectory $r(\tau)$? Israel juction conditions

$$
\begin{aligned}
[h_{ab}] &= 0 \\
[K_{ab}] &= -8\pi G S_{ab}
\end{aligned}
$$

Shell equation of motion

$$\dot{r}(\tau)^2 + V(r(\tau)) = 0$$

$$V(r) = 1 + r^2 - \frac{1}{2} m r^{1-d} - \frac{m^2}{4M^2} r^{2\alpha} - \frac{1}{4} M^2 r^{-2(d-1+\alpha)}$$

- Shell rest energy $E = \sigma A = M A^{-d\alpha}$

Looking for oscillating trajectories

- Asymptotics of V: $V \to -\infty$ as $r \to 0$; $V \to \infty$ as $r \to \infty$
- Oscillating shell trajectories are confined in an intermediate radial region bounded by two zeros of the potental, such that $V < 0$ between them

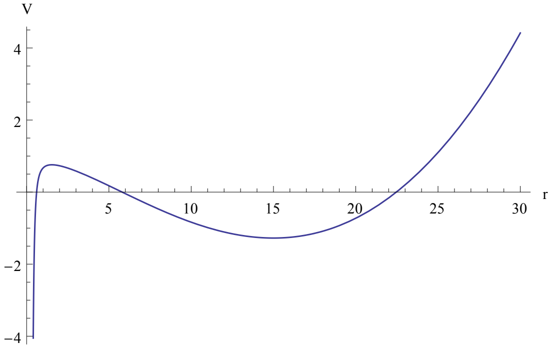

Figure: Example of potential well for $d = 3$, $\alpha = 0.99$, $m = 0.5$, $M = 0.2421$

- Fix d, α and m, oscillating shells iff $M \in [M_l, M_u]$
  - $M = M_u$, local minimum of V touching the $V = 0$ axis (static shell)
  - $M = M_l$, local maximum of V touching the $V = 0$ axis (shell infalls below)

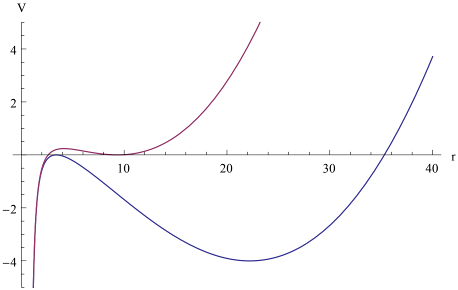

Figure: Potential for $d = 3$, $\alpha = 0.99$, $m = 10$, $M_u = 4.863$ -red- and $M_l = 4.823$ -blue-

- We have to solve the system of equations

$$V(d,\alpha,m,M,r) = \partial_r V(d,\alpha,m,M,r) = 0$$

- The allowed M-region for oscillating solutions has an implicit analytic expression

$$\begin{aligned} r &= r(d,\alpha,m,M) \\ m &= \frac{4r^{d-1}(\alpha-(1-\alpha)r^2)(d-1+\alpha+(d+\alpha)r^2)}{(d-1+2\alpha)\sqrt{1+r^2}} \\ M &= \frac{2r^{d-1+\alpha}(\alpha-(1-\alpha)r^2)}{(d-1+2\alpha)\sqrt{1+r^2}} \end{aligned}$$

- As stated, $M(d,\alpha,m)$ has to branches

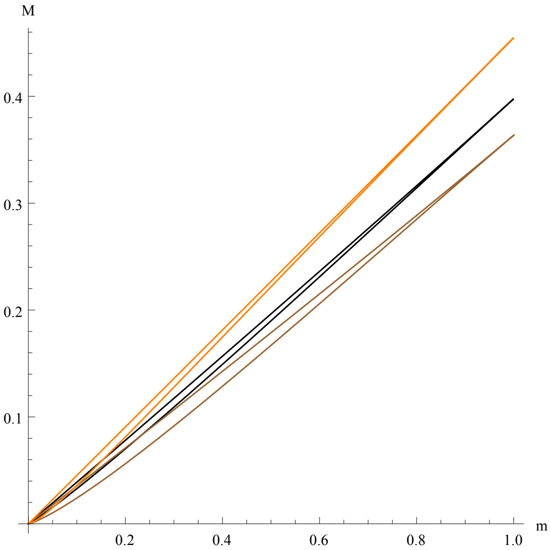

Figure: $M(d, \alpha, m)$ for $\alpha = 0.2, 0.5, 0.95$ in AdS_5 -black, brown and orange curves respectively-. All quantities are measured in units of $m_{max}(d, \alpha)$

Some properties for AdS_{d+2}, $d > 1$

- An oscillating shell never crosses its Schwarzschild radius r_h
- There is always a maximum allowed mass $m_{max}(d, \alpha)$ above which there are no oscillating solutions
- $m_{max}(d, \alpha)$ diverges as $\alpha \to 1$

$$m_{max}(d, \alpha) \sim (1 - \alpha)^{\frac{1-d}{2}} + \dots$$

We can have oscillating solutions of arbitrary mass as long as the shell matter is sufficiently near conformality

- $m_{max}(d, \alpha)$ goes to zero as $\alpha \to 0$

$$m_{max}(d, \alpha) \sim \alpha^{\frac{d+3}{2}}$$

There are no pressureless oscillating solutions

AdS_3 is special

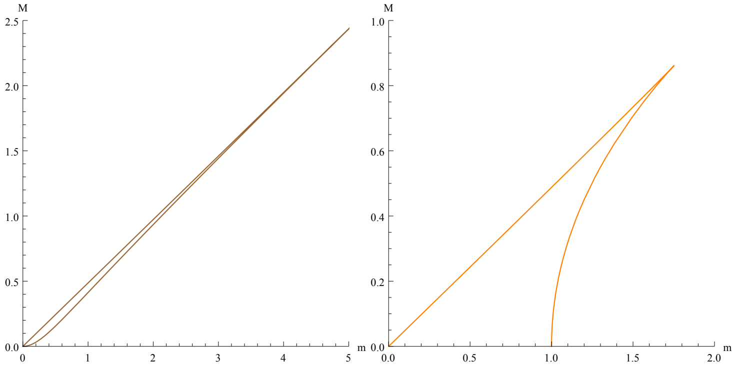

Figure: For $\alpha = 0.99$, $M(2, 0.99, m)$ -left, brown- versus $M(1, 0.99, m)$ -right, orange-

- There are two different regimes separated by $m = 1$
- $m > 1$
  - There are still two M branches; for $M < M_l(\alpha, m)$ there are no oscillating solutions
  - There is still a maximal mass $m_{max}(\alpha)$ that an oscillating solution can have

$$m_{max}(\alpha) = \frac{2}{1 + \sqrt{1 - \alpha^2}}$$

    Again, *there are no oscillating pressureless solutions*

    Now, m does not diverge as $\alpha \to 1$: *there is an absolute upper bound on the mass of an oscillating shell*
- $m < 1$
  - There is just one M branch; for $M < M_u(\alpha, m)$ there are always oscillating solutions
  - These solutions are protected against forming naked singularities

No naked singularities in AdS_3

- The potential barrier that keeps the shell away from $r = 0$ approaches the origin, but it does not vanish at any finite m

$$r_{barrier} \sim \left(\frac{M}{\sqrt{m}}\right)^{\frac{1}{\alpha}} \qquad V_{barrier} \sim 1 - m$$

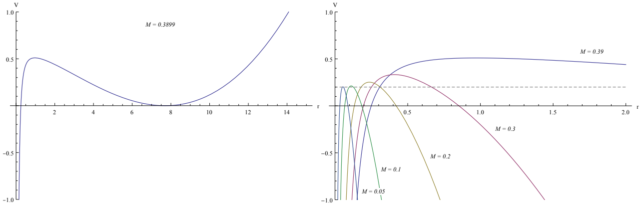

Figure: Left: for $\alpha = 0.99$ and $m = 0.8$, potential V at $M = 0.3899$, which corresponds to a static shell located at $r_u = 7.699$. Right: evolution of the potential barrier for the bouncing shell at different M

Open questions

Several simplifying assumptions... is the model too simple?

- Do they still exist for other equations of state $p = p(\sigma)$?
- Are these solutions stable?
  - Perturbatively: non spherical deformations and/or coupling to external fields...
    These may lead to radiation emission by the shell, energy loss and eventual collapse to form a black hole
  - Non-perturbatively: quantum tunneling
    This effect should be relevant on a time scale parametrically larger than the one associated to perturbative effects

Is the collapse time t_c that the shell needs to form a black hole larger than the period t_r of the shell?

Thanks for your time!