Gravitational collapse with rotating thin shells and cosmic censorship

Jorge V. Rocha (Centra-IST, U.Lisboa)

- JVR and R. Santarelli, Phys. Rev. D89, 064065 (2014) [1402.4840 [gr-qc]]
- Ongoing work
Introduction: **Black holes and gravitational collapse**

- There is overwhelming observational evidence that black holes (BHs) exist.
  
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[non-spherical gravitational collapse]
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✧ Good understanding of rotating but stationary BHs.

✧ Poor control over highly dynamical scenarios.

In particular, little is known about gravitational collapse with rotation.

Why should we care?

1. realistic collapses should include rotation;
2. known ‘violations’ of the cosmic censorship conjecture (CCC) occur in non-rotating — thus non-generic — settings;
Introduction: Approaching the problem

- Advantage of non-rotating setups is their large amount of symmetry. Spherical symmetry reduces problem to 1+1 dims.
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- ∃ a larger class of (rotating) BH spacetimes that are stationary and whose metric depends on a single radial coordinate:

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  **Cohomogeneity-1 solutions**
  
  - Perturbative approach
    - Gravitational perturbations
    - [JVR, Santarelli, Delsate (2014)]
  - Exact approach
    - Darmois-Israel junction conditions
    - [Delsate, JVR, Santarelli (2014)]
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✦ ∃ a larger class of (rotating) BH spacetimes that are stationary and whose metric depends on a single radial coordinate:

- The price to pay for the convenience provided by cohomogeneity-1 spacetimes is the restriction to higher (odd) dimensions, \( D = 2N + 3 \) with \( N = 1, 2, 3, \ldots \)
Background: **Cohomogeneity-1 black holes**

- Myers-Perry(-AdS) BHs in $D=2N+3$ dims possess isometry group $\mathbb{R} \times U(1)^{N+1}$. 

\[
M^\pm = A^{2N+1} 4\pi^{N+1} \left( N + 1 \right) M^\pm, J^\pm = A^{2N+1} 4\pi \left( N + 1 \right) M^\pm a^\pm.
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- When all spin parameters are set equal, $a_i = a$, this symmetry is enhanced:

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and coordinates can be found that reflect this large amount of symmetry, such that the metric depends on just one (radial) coordinate.

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- Constant $t$ and $r$ sections are squashed $(2N+1)$-spheres.

- $S^{2N+1}$ can be written as an $S^1$ bundle over $CP^N$. 
Background: **Cohomogeneity-1 black holes**

- The metric for these cohomogeneity-1 BHs is

\[
ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -f(r)^2 dt^2 + g(r)^2 dr^2 + r^2 \tilde{g}_{ab}dx^a dx^b + h(r)^2 [d\psi + A_a dx^a - \Omega(r) dt]^2
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where

\[
g(r)^2 = \left( 1 + \frac{r^2}{\ell^2} - \frac{2M\Xi}{r^{2N}} + \frac{2Ma^2}{r^{2N+2}} \right)^{-1},
\]

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f(r) = \frac{r}{g(r)h(r)},
\]

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\( \hat{g}_{ab} \) denotes the Fubini-Study metric on \( CP^N \) and \( A_a dx^a \) is its Kahler potential.

For \( N=1 \): \( \hat{g}_{ab} dx^a dx^b = \frac{1}{4} \left( d\theta^2 + \sin^2 \theta \, d\phi^2 \right) \), \( A = \frac{1}{2} \cos \theta \, d\phi \).
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- n.b. These solutions accommodate a non-vanishing cosmological constant:

\[ R_{\mu\nu} = -(D-1)\ell^{-2}g_{\mu\nu} \]
Background: Thin shells in cohomogeneity-1 spacetimes

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- Take advantage of high degree of symmetry: consider shells that respect full set of spatial isometries. Focus on $N=1$, for simplicity.
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- Take advantage of high degree of symmetry: consider shells that respect full set of spatial isometries. Focus on $N=1$, for simplicity.

- n.b. For test particles, the dynamics on the $CP^1\cong S^2$ and on the $S^1$ separate:

\[
\{r, \theta, \phi\} \quad \text{and} \quad \{r, \psi\}
\]

[JVR, Santarelli, Delsate (2014)]
Rotating thin shells: **Junction conditions**

- Use junction conditions along a timelike hypersurface, \( t = T(\tau), r = R(\tau) \):

\[
\begin{align*}
g^{(+)}_{ij} &= g^{(-)}_{ij} = g_{ij}, \\
(k^{(+)}_{ij} - k^{(-)}_{ij}) - g_{ij}(k^{(+)} - k^{(-)}) &= -8\pi G S_{ij}
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induced metric

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shell's stress-energy tensor
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\text{extrinsic curvature} & \quad (k_{ij}^{(+)} - k_{ij}^{(-)}) - \mathcal{g}_{ij}(k^{(+)}) - k^{(-)} = -8\pi G S_{ij}
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This formalism has been applied to rotating spacetimes in \((2+1)\) dims. [Mann-Oh-Park (2009)]
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- For \(D>3\), we get one additional constraint from the 1st junction condition:

  \[
  M_+ a_+^2 = M_- a_-^2 \quad \Rightarrow \quad h_+(R) = h_-(R) \equiv h(R)
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  \[ M_+ a_+^2 = M_- a_-^2 \quad \text{and} \quad h_+(\mathcal{R}) = h_-(\mathcal{R}) \equiv h(\mathcal{R}) \]

- The 2nd junction condition requires the shell stress-energy tensor to take the form of an imperfect fluid:
  \[ S_{ij} = (\rho + P) u_i u_j + P g_{ij} + 2\varphi u_{(i} \xi_{j)} + \Delta P \mathcal{R}^2 \hat{g}_{ij} \]
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  energy density  pressure  intrinsic momentum / heat flow  pressure anisotropy
Rotating thin shells: Equation of state and shell equation of motion

- The stress-energy tensor components are dictated by the metric components:

\[
\rho = -\frac{(\beta_+ - \beta_-)(\mathcal{R}^2h)'}{8\pi \mathcal{R}^3}
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\[
P = \frac{h}{8\pi \mathcal{R}^3} \left[ \mathcal{R}^2(\beta_+ - \beta_-) \right]'
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\varphi = -\frac{(\mathcal{I}_+ - \mathcal{I}_-)(\mathcal{R}h)'}{4\pi^2 \mathcal{R}^4 h}
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\Delta P = \frac{(\beta_+ - \beta_-)}{8\pi} \left[ \frac{h}{\mathcal{R}} \right]'
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- These equations can be integrated, yielding the shell’s equation of motion:

\[
\dot{\mathcal{R}}^2 + V_{\text{eff}} (\mathcal{R}) = 0
\]
Rotating thin shells: Full collapse in asymptotically flat spacetime

- Take asymptotically flat limit, $\ell \to \infty$. 

---

**I. BACKGROUND**

Isometries:

$$R \mathcal{U} (1)$$

\[a_i = a_{(2)}\]

The metric is a solution of the Einstein equations with a negative cosmological constant,

$$R_{\mu \nu} = -\frac{1}{2} \mathcal{R} g_{\mu \nu}. \tag{4}$$

The largest real root, $r_+$, of $g_{rr}$ marks an event horizon which possesses the geometry of a homogeneously squashed $S^2$. The mass $M$ and angular momentum $J$ of the spacetime are given by

$$M = \mathcal{F} 2 N + 1 4 \pi G M \right( N + 1 2 + a^2 \right), \tag{5}$$

$$J = \mathcal{F} 2 N + 1 4 \pi G (N + 1) M a, \tag{6}$$

where $\mathcal{F} 2 N + 1$ is the area of the unit $(2N + 1)$-sphere.

A system of coordinates can be found such that the metric only depends on a single radial coordinate,

$$ds^2 = f(r)^2 dt^2 + g(r)^2 dr^2 + r^2 g_{ab} dx^a dx^b + h(r) \left[ d\delta + A^a dx^a \right]^2, \tag{7}$$

where

$$g(r)^2 = \sqrt{1 + r^2 \frac{1}{2} a^2}, f(r) = r g(r) h(r), \tag{8}$$

$$h(r)^2 = \frac{r^2}{2} \frac{1}{a^2}, \delta = 1, A^a = \frac{2 M a}{r^2} h(r)^2. \tag{9}$$

In the simplest case, $D = 5$, the base space is $\mathbb{CP}_1$, which is isomorphic to the sphere $S^2$, and $g_{ab} dx^a dx^b = \frac{1}{4} d\theta^2 + \sin^2 \theta d\phi^2$, $A^a = \frac{1}{2} \cos \theta d\phi$. \(\mathcal{F}^{11}\)
Rotating thin shells: Full collapse in asymptotically flat spacetime

- Take asymptotically flat limit, \( \ell \to \infty \).
- Collapse starting from rest at infinity imposes: \( w = 0 \) i.e., matter on the shell has EoS of dust
  \( m_0 = \Delta M \) i.e., the increment in mass of the spacetime is given precisely by the mass of the shell

[Delsate, JVR, Santarelli (2014)]
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Weak energy conditions (WEC) are satisfied

No fine tuning of parameters is necessary

[Images and equations as described in the document]
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[Diagram showing WEC and $V_{eff}$ vs $R(\tau)$]

- If initially one has a (sub-extremal) BH, then after the shell collapses there will be a larger horizon covering the singularity.

[Reference: Delsate, JVR, Santarelli (2014)]

Weak energy conditions (WEC) are satisfied

No fine tuning of parameters is necessary

CCC is preserved
Rotating thin shells: **Diverse scenarios**

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**Bounce**
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Rotating thin shells: **Diverse scenarios**

- **Full collapse**
  - $M_-=0.2$, $M_+=0.25$, $Ma^2=0.012$, $m_0=0.05$

- **Bounce**
  - $M_-=0.2$, $M_+=0.25$, $Ma^2=0.016$, $m_0=0.05$

- **Oscillatory**
  - $M_-=0.2$, $M_+=0.25$, $Ma^2=0.016$, $m_0=0.04$, $w=-0.15$
Rotating thin shells: Diverse scenarios

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Bounce

Oscillatory
Rotating thin shells: **Stationary shell around a BH in AdS**

- Confining nature of the potential (due to negative cosmological constant) + Centrifugal barrier (due to rotation)
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\[
m_0/\ell^2 = 0.324, \quad w = 0.285, \quad Ma^2/\ell^4 = 0.02, \quad R_*/\ell = 1.8
\]

[Delsate, JVR, Santarelli (2014)]
Test particles: Spinning up equal angular momenta AdS BHs

- Attempt to over-spin extremal AdS rotating BHs and test the CCC in higher (odd) dimensions with a cosmological constant, following Wald’s gedanken experiment.

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- Mass and angular momentum of cohomogeneity-1 BH spacetime:
  \[
  \mathcal{M} = \frac{\Omega_{2N+1}}{4\pi G} M \left( N + \frac{1}{2} + \frac{a^2}{2\ell^2} \right),
  \]
  \[
  \mathcal{J} = \frac{\Omega_{2N+1}}{4\pi G} (N + 1) M a,
  \]

- Dimensionless combinations:
  \[
  m \equiv \frac{M}{\ell^{2N}}, \quad j \equiv \frac{a}{\sqrt{M}} \ell^{N-1}
  \]

\[D = 5\]

![](chart.png)
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- Throw in \((N+1)\) point particles, one for each rotation plane, to preserve symmetry.
Test particles: Cosmic censorship in AdS (and hiD)

- For each geodesic particle (of given energy parameter $E$), determine critical value of the particles’ angular momentum $L$. separates bouncing trajectories from plunges
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- Consider the (non back-reacted) effect of the absorption of particles with maximal angular momentum by an already extremal BH:

  $$m_0 \rightarrow m_0 + \delta m, \quad j_0 \rightarrow j_0 + \delta j$$
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$$m_0 \rightarrow m_0 + \delta m, \quad j_0 \rightarrow j_0 + \delta j$$

No violation of the CCC in $D = 5, 7, 9, 11$.

Worst-case scenario generates a flow along the curve of extremal solutions.
Conclusions

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- Matching two rotating BH spacetimes across a thin shell is possible.
  - It requires matter on the shell to be an imperfect fluid.
  - Full collapse onto rotating, asymptotically flat, BH (satisfying energy conditions) respects the CCC.
  - Stationary solutions describing rotating shells of matter surrounding spinning BHs exist in AdS.
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Thank you.
Appendix: Effective potential for shell equation of motion

- For generic values of $N$, and a linear equation of state:

$$
\dot{R}^2 + V_{\text{eff}}(R) = 0
\quad \Rightarrow \quad
V_{\text{eff}}(R) = 1 + \frac{R^2}{\ell^2} + \frac{2Ma^2}{\ell^2 R^{2N}} + \frac{2Ma^2}{R^{2N+2}} - \frac{M_+ + M_-}{R^{2N}}
- \left( \frac{M_+ - M_-}{m_0} \right)^2 \left( \frac{R^{2N}}{m_0} \right)^{\frac{2N+1}{N}w} \left( 1 + \frac{2Ma^2}{R^{2N+2}} \right)^{w-1}
- \frac{1}{4} \left( \frac{m_0}{R^{2N}} \right)^{2+\frac{2N+1}{N}w} \left( 1 + \frac{2Ma^2}{R^{2N+2}} \right)^{1-w}.
$$

- For $N=1$ and large values of $R$:

$$
V_{\text{eff}} \approx 1 + \frac{R^2}{\ell^2} - \left( \frac{\Delta M}{m_0} \right)^2 \left( \frac{R^2}{m_0} \right)^{3w} - \frac{1}{4} \left( \frac{m_0}{R^2} \right)^{2+3w}
$$

- For $N=1$ and small values of $R$:

$$
V_{\text{eff}} \approx \frac{2Ma^2}{R^4} - \frac{M_+ + M_-}{R^2} - \frac{1}{4} \left( \frac{2Ma^2}{m_0^2} \right)^{1-w} \left( \frac{m_0}{R^2} \right)^{4+w} - \left( \frac{2Ma^2}{m_0^2} \right)^{w-1} \left( \frac{\Delta M}{m_0} \right)^2 \left( \frac{R^2}{m_0} \right)^{2+w}.
$$