VII Black Holes Workshop

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Sharp bounds on the radius of relativistic charged spheres: Guilfoyle's stars saturate the Buchdahl-Andréasson bound

José P. S. Lemos

(collaboration: Vilson Zanchin)

CENTRA

IST - University of Lisbon

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Outline

1. *The Schwarzschild limit, the Buchdahl bound, and the no trapped surface bound*
2. *Limits and bounds for electric charged stars*

1. The Schwarzschild limit, the Buchdahl bound, and the no trapped surface bound

- **The Schwarzschild limit**

· Schwarzschild (SPAW 1916) in his second paper argued that within general relativity, a star of mass m and radius r_0, and with an incompressible equation of state

$$\rho = \text{constant}$$

can sustain itself against gravitational collapse as long as the central pressure $p_c < \infty$. For $p_c \to \infty$ then he showed that

$$r_0 = \frac{9}{4}m\,.$$

This is the Schwarzschild limit. It can also be put in the form

$$r_0 = \frac{9}{8}r_h\,,$$

where r_h (here $r_h = 2m$) is the gravitational radius of the star. The star is really compact. This limit was further discussed, e.g., Volkoff (PR 1939), using the Openheimer-Volkoff equation (PR 1939).

1. The Schwarzschild limit, the Buchdahl bound, and the no trapped surface bound

- **The Buchdahl bound**

Buchdahl (PR 1959) clinched the problem. He established in general relativity a model-independent bound on stellar structure whereby the total mass m and radius r_0 of any fluid star body is bounded.

Buchdahl's theorem: Suppose a static spherical system with a perfect fluid source where
(a) p and ρ are everywhere non-negative,
(b) ρ is non-increasing outward,
(c) the boundary $p=0$ at r_0 is joined to the vacuum Schwarzschild solution.
Then the fluid matter in the star is packed such that

$$r_0 \geq \frac{9}{4}m\,.$$

The Schwarzchild limit saturates the bound.

1. The Schwarzschild limit, the Buchdahl bound, and the no trapped surface bound

Other proofs by Andréasson (JDE 2008), Karageorgis, Stalker (CQG 2008) rely on $p+2p_T \leq \rho$, where p is the radial pressure and p_T the tangential pressure. Remarkably, the bound is also saturated by a thin shell $p=0$ and $2p_T \leq \rho$.

- **No trapped surface bound**

A trapped surface is a set of points defined as a closed surface on which outward-pointing light rays are actually converging (moving inwards). Trapped null surfaces are used in the definition of the apparent horizon which typically surrounds a black hole.

To exlude trapped surfaces in a matter static region one imposes the no trapped surface bound, Penrose (PRL 1965), given by

$$r_0 \geq 2m\,,$$

or, more generally,

$$r_0 \geq r_h\,.$$

2. Limits and bounds for electric charged stars

- **Solutions**

Bonnor (1950-1970, Nature, GRG),
de Felice et al (MNRAS 1995, CQG 1999),
Lemos, Zanchin, Zaslavskii (2003-2014 PRD),
Giuliani, Rothman (GRG 2008).
Assume $q \leq m$, so the gravitational radius of the star is $r_h = m + \sqrt{m^2 - q^2}$.

Are their similar analogues in the charged case to the Schwarzschild limit and the Buchdahl bound?

The ultimate goal is to have a configuration hovering at its own gravitational radius, i.e., to have a star at the trapped surface limit, $r_0 \geq r_h$. Has to find a different fluid, a non-perfect one. One form of matter that yields repulsion is electrically charged matter.

2. Limits and bounds for electric charged stars

- **The Buchdahl-Andréasson bound**

Andréasson (CMP 2009) set a sharp limit to electric charged stars.
Assumed still that $p+2p_T \leq \rho$, and that the boundary is matched to the exterior Reissner-Nordström solution.
Then the Buchdahl-Andréasson bound (CMP 2009) is

$$r_0 \geq \frac{9}{\left(1+\sqrt{1+3\,q^2/r_0^2}\right)^2}\, m\,,$$

where q is the total electric charge of the star. The Buchdahl bound is

$$r_0 \geq \frac{9}{4}\, m \quad \text{for} \quad q=0\,.$$

The other end of the bound is

$$r_0 \geq m \quad \text{for} \quad q=m\,.$$

The lower bound $r_0=m$ is a configuration at its own gravitational radius, i.e., a quasiblack hole.

2. Limits and bounds for electric charged stars

- **The Schwarzschild-electric limit**

What stars saturate the Buchdahl-Andréasson bound?

Andréasson (CMP 2009) acknowledged that thin electric shells that obey $2p_T = \rho$ (the radial pressure $p = 0$) saturate the bound.

And further asked: are there any (Schwarzschild-like) electric solutions with continuous matter that saturate the bound? Giuliani, Rothman solution (GRG 2008), $\rho =$ constant and some specific electric charge inside the sphere r, $q(r)$, does not saturate the bound when $p_c \to \infty$ as he verified.

We started to search for those solutions, Arbañil, Lemos, Zanchin (PRD 2014), Lemos, Lopes, Quinta, Zanchin (arXiv 2014), with the very stiff equations $\rho =$ constant and $\rho_e = \alpha\,\rho$ with $0 \leq \alpha \leq 1$. Send $p_c \to \infty$. Still: the bound is not saturated.

2. Limits and bounds for electric charged stars

Try the ansatz

$$ds^2 = -B(r)\,dt^2 + A(r)\,dr^2 + r^2\left(d\theta^2 + \sin^2\theta\,d\varphi^2\right),$$

and $\phi(r)$ spherically symmetric. Maxwell equation gives

$$\phi'(r) = q(r)\frac{\sqrt{B(r)A(r)}}{r^2},$$

where $q(r)$ is the electric charge inside the sphere r. Further put

$$\rho(r) + \frac{q^2(r)}{8\pi r^4} = \text{constant},$$

as Cooperstock, de la Cruz (GRG 1978) and Florides (JPA 1983) did. Solve Einstein equations to get electric charge stars, the Guilfoyle's solutions (GRG 1999), Lemos, Zanchin (PRD 2010).

Find that the Cooperstock-de la Cruz-Florides equation of state is indeed the electric charge equivalent to the Schwarzschild's interior $\rho(r) = \text{constant}$ solution.

It answers Andréasson's question. Such an equation of state yields electrically charged stars that saturate the Buchdahl-Andréasson bound exactly when $p_c \to \infty$, see Lemos, Zanchin (IP 2015).