RADIATION FROM A D-DIMENSIONAL COLLISION OF SHOCK WAVES: EXACT RESULTS

Flávio Coelho & Carlos Herdeiro & Marco O. P. Sampaio

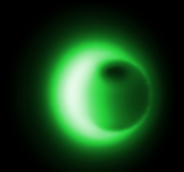
msampaio@ua.pt

Aveiro University & I3N

VII BLACK HOLES WORKSHOP

Aveiro University 18-19 December 2014

gravitation.web.ua.pt/bhw7

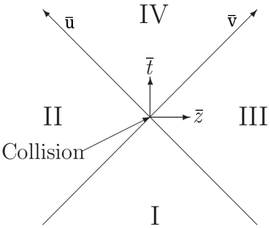


References: JHEP 07 (2011) 121
Phys. Rev. Lett. 108 (2012) 181102
Phys. Rev. D87 (2013) 084034
Int. J. Mod. Phys. A28 (2013) 1340019
arXiv:1410.0964 – accepted in JHEP

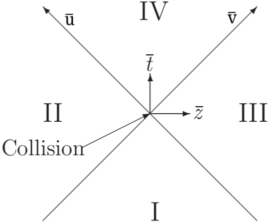
FCT Fundação para a Ciência e a Tecnologia
MINISTÉRIO DA EDUCAÇÃO E CIÊNCIA
Gr@v

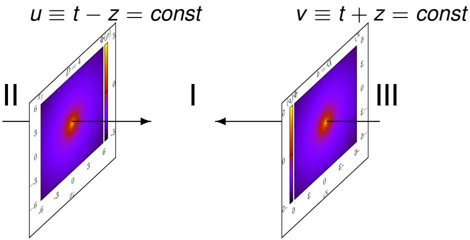
THE PROBLEM

- Gravitational collision of 2 point particles @ speed of light



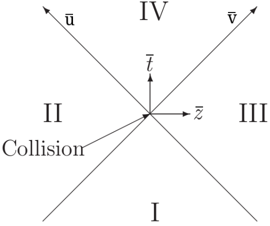
- Gravitational collision of 2 point particles @ speed of light

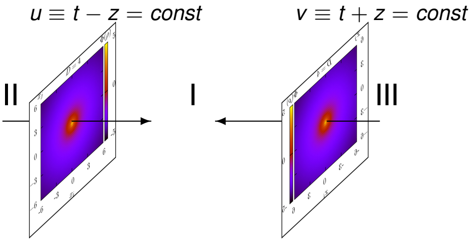




THE PROBLEM

- Gravitational collision of 2 point particles @ speed of light

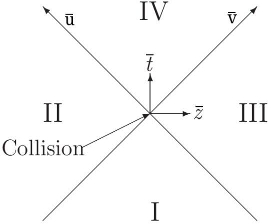




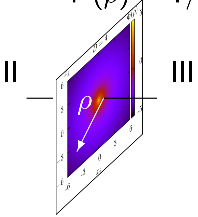
- Each solves Einstein's Eqs, **point source** $P^\mu = E\, n^\mu$

$$T^{\mu\nu} = E\delta(u)\delta^{(D-2)}(x^i)n^\mu n^\nu \quad , \quad n^\mu n_\mu = 0 \quad , \quad \kappa = \frac{8\pi G_D}{\Omega_{D-3}} E$$

- Gravitational collision of 2 point particles @ speed of light



$$\Phi'(\rho) \sim 1/\rho^{D-3}$$

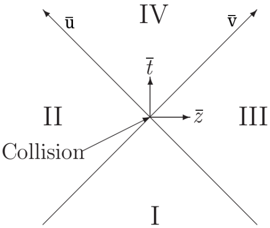


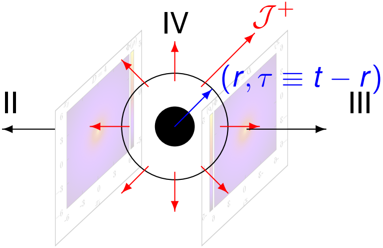
- Each solves Einstein's Eqs, **point source** $P^\mu = E\, n^\mu$

$$T^{\mu\nu} = E\delta(u)\delta^{(D-2)}(x^i)n^\mu n^\nu \quad , \quad n^\mu n_\mu = 0 \quad , \quad \kappa = \frac{8\pi G_D}{\Omega_{D-3}} E$$

THE PROBLEM

- Gravitational collision of 2 point particles @ speed of light





- Each solves Einstein's Eqs, **point source** $P^\mu = E\, n^\mu$

$$T^{\mu\nu} = E\delta(u)\delta^{(D-2)}(x^i)n^\mu n^\nu \quad , \quad n^\mu n_\mu = 0 \quad , \quad \kappa = \frac{8\pi G_D}{\Omega_{D-3}} E$$

- Flat space everywhere except in future of collision!

WHY?

Some Reasons to study this:

- *D*-dim **N**um. **GR**, BH collisions difficult @ large boost

  U. Sperhake, V. Cardoso, F. Pretorius, E. Berti, J. Gonzalez, arXiv:0806.1738
  M. Shibata, H. Okawa, T. Yamamoto, arXiv:0810.4735 + etc . . .

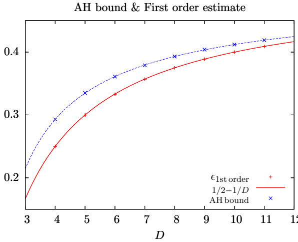
Some Reasons to study this:

- *D*-dim **N**um. **GR**, BH collisions difficult @ large boost

  U. Sperhake, V. Cardoso, F. Pretorius, E. Berti, J. Gonzalez, arXiv:0806.1738
  M. Shibata, H. Okawa, T. Yamamoto, arXiv:0810.4735 + etc ...

- **D is better than four!** PRL 108 (2012) 181102
  - In $D \geq 4$, $\epsilon^{(1)}_{\rm rad} = \frac{1}{2} - \frac{1}{D}$, $\epsilon^{(2)}_{\rm rad} = ?$
  - In $D = 4$, $\epsilon^{(1)}_{\rm rad} \simeq 0.25$, & $\epsilon^{(2)}_{\rm rad} \simeq 0.163$ (within Num. GR err)

    D'Eath and Payne, PRD Volume 46, Number 2, 658, 675 and 694
    see also East and Pretorius, arXiv:1210.0443



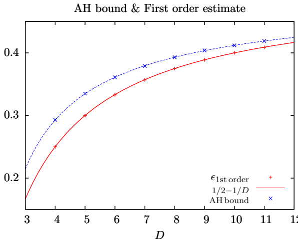
Some Reasons to study this:

- *D*-dim **N**um. **GR**, BH collisions difficult @ large boost

  U. Sperhake, V. Cardoso, F. Pretorius, E. Berti, J. Gonzalez, arXiv:0806.1738
  M. Shibata, H. Okawa, T. Yamamoto, arXiv:0810.4735 + etc . . .

- **D is better than four!** PRL 108 (2012) 181102
  - In $D \geq 4$, $\epsilon^{(1)}_{\rm rad} = \frac{1}{2} - \frac{1}{D}$, $\epsilon^{(2)}_{\rm rad} =?$
  - In $D = 4$, $\epsilon^{(1)}_{\rm rad} \simeq 0.25$, & $\epsilon^{(2)}_{\rm rad} \simeq 0.163$ (**within Num. GR err**)

    D'Eath and Payne, PRD Volume 46, Number 2, 658, 675 and 694
    see also East and Pretorius, arXiv:1210.0443



WHY?

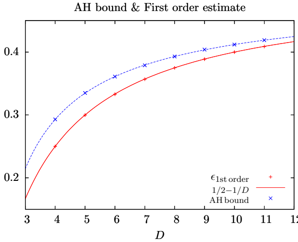
Some Reasons to study this:

- *D*-dim **N**um. **GR**, BH collisions difficult @ large boost

  U. Sperhake, V. Cardoso, F. Pretorius, E. Berti, J. Gonzalez, arXiv:0806.1738
  M. Shibata, H. Okawa, T. Yamamoto, arXiv:0810.4735 + etc ...

- **D is better than four!** PRL 108 (2012) 181102
  - In $D \geq 4$, $\epsilon^{(1)}_{\rm rad} = \frac{1}{2} - \frac{1}{D}$, $\epsilon^{(2)}_{\rm rad} =?$
  - In $D = 4$, $\epsilon^{(1)}_{\rm rad} \simeq 0.25$, & $\epsilon^{(2)}_{\rm rad} \simeq 0.163$ (**within Num. GR err**)

    D'Eath and Payne, PRD Volume 46, Number 2, 658, 675 and 694
    see also East and Pretorius, arXiv:1210.0443



- Black holes in ADD scenarios @ LHC, or not...

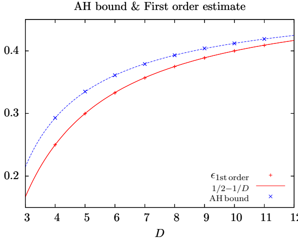
Some Reasons to study this:

- *D*-dim **N**um. **GR**, BH collisions difficult @ large boost

  U. Sperhake, V. Cardoso, F. Pretorius, E. Berti, J. Gonzalez, arXiv:0806.1738
  M. Shibata, H. Okawa, T. Yamamoto, arXiv:0810.4735 + etc ...

- **D is better than four!** PRL 108 (2012) 181102
  - In $D \geq 4$, $\epsilon_{\rm rad}^{(1)} = \frac{1}{2} - \frac{1}{D}$, $\epsilon_{\rm rad}^{(2)} = ?$
  - In $D = 4$, $\epsilon_{\rm rad}^{(1)} \simeq 0.25$, & $\epsilon_{\rm rad}^{(2)} \simeq 0.163$ (**within Num. GR err**)

    D'Eath and Payne, PRD Volume 46, Number 2, 658, 675 and 694
    see also East and Pretorius, arXiv:1210.0443



- Black holes in ADD scenarios @ LHC, or not...

Before this study outstanding questions were:

1. Is the formula exact? Is there a deeper meaning to it?
2. Angular expansion ⇔ perturbative expansion?
3. Can we find equally simple results at higher orders?

WHY?

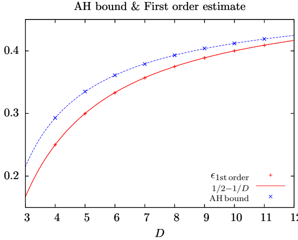
Some Reasons to study this:

- D-dim **N**um. **GR**, BH collisions difficult @ large boost

  U. Sperhake, V. Cardoso, F. Pretorius, E. Berti, J. Gonzalez, arXiv:0806.1738
  M. Shibata, H. Okawa, T. Yamamoto, arXiv:0810.4735 + etc ...

- **D is better than four!** PRL 108 (2012) 181102
  - In $D \geq 4$, $\epsilon^{(1)}_{\rm rad} = \frac{1}{2} - \frac{1}{D}$, $\epsilon^{(2)}_{\rm rad} = ?$
  - In $D = 4$, $\epsilon^{(1)}_{\rm rad} \simeq 0.25$, & $\epsilon^{(2)}_{\rm rad} \simeq 0.163$ **(within Num. GR err)**

    D'Eath and Payne, PRD Volume 46, Number 2, 658, 675 and 694
    see also East and Pretorius, arXiv:1210.0443



- Black holes in ADD scenarios @ LHC, or not...

Before this study outstanding questions were:

1. Is the formula exact? Is there a deeper meaning to it?
2. Angular expansion $\Leftrightarrow$ perturbative expansion?
3. Can we find equally simple results at higher orders?

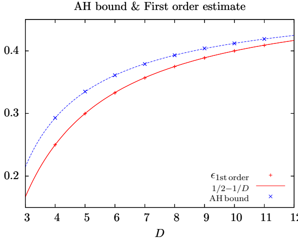
Some Reasons to study this:

- *D*-dim **N**um. **GR**, BH collisions difficult @ large boost

  U. Sperhake, V. Cardoso, F. Pretorius, E. Berti, J. Gonzalez, arXiv:0806.1738
  M. Shibata, H. Okawa, T. Yamamoto, arXiv:0810.4735 + etc . . .

- **D is better than four!** PRL 108 (2012) 181102
  - In $D \geq 4$, $\epsilon^{(1)}_{\text{rad}} = \frac{1}{2} - \frac{1}{D}$, $\epsilon^{(2)}_{\text{rad}} = ?$
  - In $D = 4$, $\epsilon^{(1)}_{\text{rad}} \simeq 0.25$, & $\epsilon^{(2)}_{\text{rad}} \simeq 0.163$ (**within Num. GR err**)

  D'Eath and Payne, PRD Volume 46, Number 2, 658, 675 and 694
  see also East and Pretorius, arXiv:1210.0443



- Black holes in ADD scenarios @ LHC, or not...

Before this study outstanding questions were:

1. Is the formula exact? Is there a deeper meaning to it?
2. Angular expansion $\Leftrightarrow$ perturbative expansion?
3. Can we find equally simple results at higher orders?

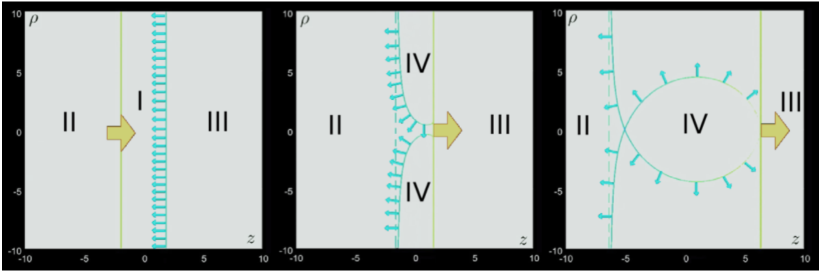
OUTLINE

OUTLINE

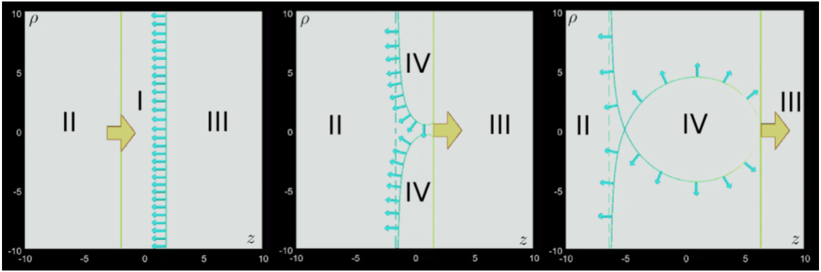
1. Exact results – The role of symmetry

2. The Penrose diagram

SHOCK WAVE PERTURBATION THEORY IN A FLASH!



SHOCK WAVE PERTURBATION THEORY IN A FLASH!

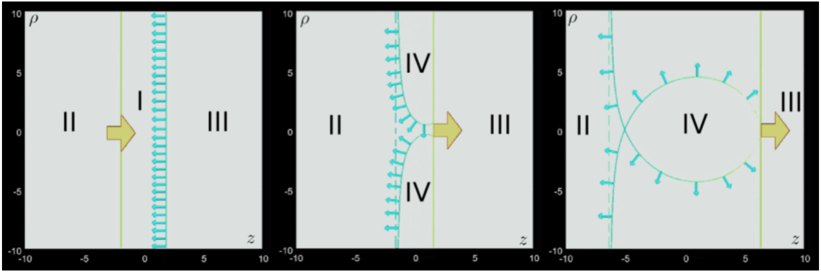


Basic facts:

- Scattering of test rays gives exact initial conditions $u = 0^+$

$$g_{\mu\nu}(v, x_i) = \eta_{\mu\nu} + \kappa h^{(1)}_{\mu\nu} + \kappa^2 h^{(2)}_{\mu\nu}$$

SHOCK WAVE PERTURBATION THEORY IN A FLASH!



Basic facts:

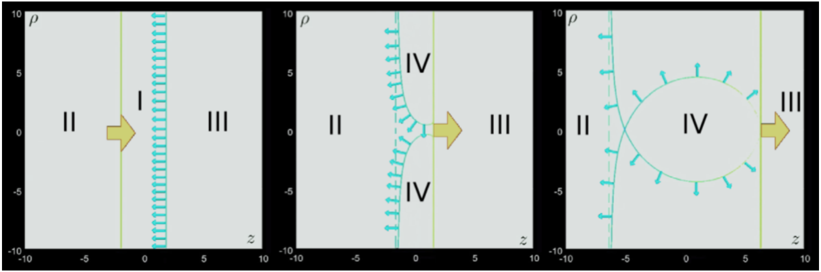
- Scattering of test rays gives exact initial conditions $u = 0^+$

$$g_{\mu\nu}(v, x_i) = \eta_{\mu\nu} + \kappa h^{(1)}_{\mu\nu} + \kappa^2 h^{(2)}_{\mu\nu}$$

- Assume perturbative ansatz in $u > 0$

$$g_{\mu\nu}(u > 0, v, x_i) = \eta_{\mu\nu} + \sum_{n=1}^{\infty} \kappa^n h^{(n)}_{\mu\nu}$$

SHOCK WAVE PERTURBATION THEORY IN A FLASH!



Basic facts:

- Scattering of test rays gives exact initial conditions $u = 0^+$

$$g_{\mu\nu}(v, x_i) = \eta_{\mu\nu} + \kappa h^{(1)}_{\mu\nu} + \kappa^2 h^{(2)}_{\mu\nu}$$

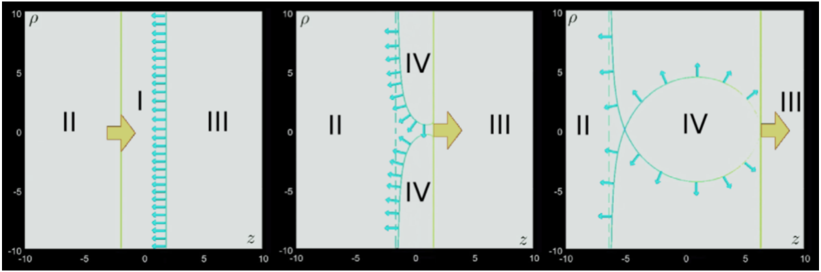
- Assume perturbative ansatz in $u > 0$

$$g_{\mu\nu}(u > 0, v, x_i) = \eta_{\mu\nu} + \sum_{n=1}^{\infty} \kappa^n h^{(n)}_{\mu\nu}$$

- de Donder gauge $\Rightarrow$ tower of wave Eqs with source

$$\Box h^{(n)N}_{\mu\nu} = T^{(n-1)}_{\mu\nu}\left[h^{(k<n)}_{\alpha\beta}\right]$$

SHOCK WAVE PERTURBATION THEORY IN A FLASH!

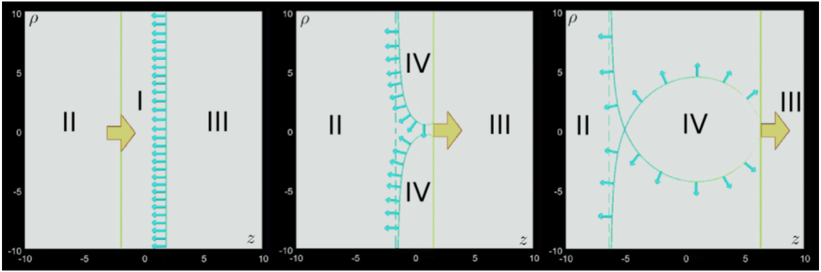


Basic facts:

- Integral solution Volume + Surface terms

$$h^{(n)N}_{\mu\nu}(y) = \int_{u'>0} d^D y' \, G(y, y') \left[T^{(n-1)}_{\mu\nu}(y') + 2\delta(u')\partial_{v'} h^{(n)N}_{\mu\nu}(y') \right]$$

SHOCK WAVE PERTURBATION THEORY IN A FLASH!



Basic facts:

- Integral solution Volume + Surface terms

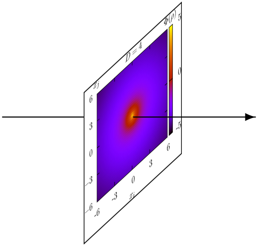
$$h^{(n)N}_{\mu\nu}(y) = \int_{u'>0} d^D y' \, G(y, y') \left[T^{(n-1)}_{\mu\nu}(y') + 2\delta(u')\partial_{v'} h^{(n)N}_{\mu\nu}(y') \right]$$

- Axial symmetry allows expansion $(A(u, v, \rho), B(u, v, \rho), \ldots)$:

$$h_{uu} \equiv A = A^{(1)} + A^{(2)} + \ldots \quad h_{ui} \equiv B\,\Gamma_i = (B^{(1)} + B^{(2)} + \ldots)\Gamma_i$$

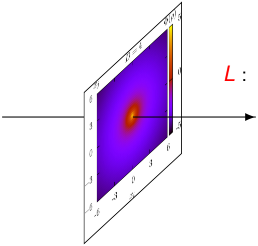
$$h_{ij} \equiv E\,\Delta_{ij} + H\,\delta_{ij} = (E^{(1)} + E^{(2)} + \ldots)\Delta_{ij} + (0 + H^{(2)} + \ldots)\delta_{ij}$$

Background shock conf. symm:

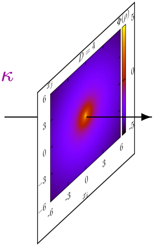


THE CL-SYMMETRY & SEPARATION OF VARIABLES

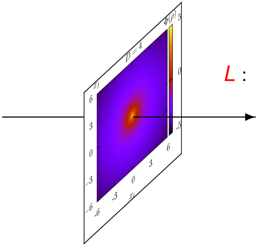
Background shock conf. symm: Boost (L)



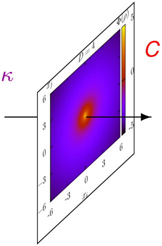
$L: \ \kappa \to e^{\beta}\kappa$



Background shock conf. symm: Boost (L) + Conformal scaling (C)

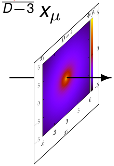


$L: \ \kappa \to e^{\beta}\kappa$

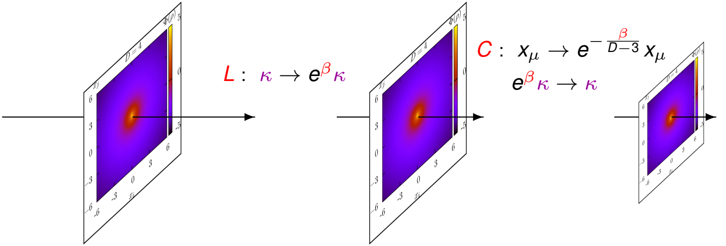


$C: \ x_\mu \to e^{-\frac{\beta}{D-3}} x_\mu$

$e^{\beta}\kappa \to \kappa$



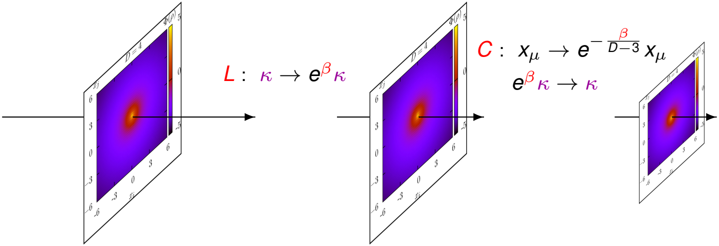
Background shock conf. symm: Boost (L) + Conformal scaling (C)



Up to conformal factor, metric perturbations invariant order by order

$$g_{\mu\nu}(X) \xrightarrow{CL} g_{\mu\nu}(X') = e^{\frac{2}{D-3}\beta}\left[\eta_{\mu\nu} + \sum_{k=1}^{\infty} e^{-2k\beta} h^{(k)}_{\mu\nu}(X')\right]$$

Background shock conf. symm: Boost (L) + Conformal scaling (C)



Up to conformal factor, metric perturbations invariant order by order

$$g_{\mu\nu}(X) \xrightarrow{CL} g_{\mu\nu}(X') = e^{\frac{2}{D-3}\beta}\left[\eta_{\mu\nu} + \sum_{k=1}^{\infty} e^{-2k\beta} h^{(k)}_{\mu\nu}(X')\right]$$

In adapted coordinates $p(u, v, \rho)$ $q(u, v, \rho)$, ρ separates!

$$h^{(k)}_{\mu\nu}(u, v, \rho, \phi_i) = \frac{f^{(k)}_{\mu\nu}(p, q, \phi_i)}{\rho^{(D-3)(2k+N_u-N_v)}}$$

ASYMPTOTIC ANGULAR FACTORISATION (AT ALL ORDERS!)

Waveform $\dot{\mathcal{E}}^{(k)}(r,\tau,\theta) \propto \frac{d}{d\tau}(E^{(k)} + H^{(k)})$ when $r \to +\infty$ is

$$\dot{\mathcal{E}}(\tau,\theta)$$

ASYMPTOTIC ANGULAR FACTORISATION (AT ALL ORDERS!)

Waveform $\dot{\mathcal{E}}^{(k)}(r,\tau,\theta) \propto \frac{d}{d\tau}(E^{(k)} + H^{(k)})$ when $r \to +\infty$ is

$$\dot{\mathcal{E}}(\tau,\theta)$$

The associated inelasticity is:

$$\epsilon \equiv \int_{-1}^{1} \frac{d\cos\theta}{2} \int d\tau \left(\sum_k \dot{\mathcal{E}}^{(k)}(\tau,\theta) \right)^2 .$$

ASYMPTOTIC ANGULAR FACTORISATION (AT ALL ORDERS!)

Waveform $\dot{\mathcal{E}}^{(k)}(r,\tau,\theta) \propto \frac{d}{d\tau}(E^{(k)} + H^{(k)})$ when $r \to +\infty$ is

$$\dot{\mathcal{E}}(\tau,\theta)$$

The associated inelasticity is:

$$\epsilon \equiv \int_{-1}^{1} \frac{d\cos\theta}{2} \int d\tau \left(\sum_k \dot{\mathcal{E}}^{(k)}(\tau,\theta) \right)^2 .$$

As a consequence of the CL-symmetry & asymptotics @ $\mathcal{J}^+$:

$$\dot{\mathcal{E}}^{(k)}(\tau,\theta) = \left(\frac{1}{1-\cos\theta} \right)^2 \left(\frac{1+\cos\theta}{1-\cos\theta} \right)^{k-1-\frac{1}{4}\frac{D-4}{D-3}} \dot{\mathcal{E}}^{(k)} \left(\bar{\tau}(\tau,\theta), \frac{\pi}{2} \right) .$$

ASYMPTOTIC ANGULAR FACTORISATION (AT ALL ORDERS!)

Waveform $\dot{\mathcal{E}}^{(k)}(r,\tau,\theta) \propto \frac{d}{d\tau}(E^{(k)} + H^{(k)})$ when $r \to +\infty$ is

$$\dot{\mathcal{E}}(\tau,\theta)$$

The associated inelasticity is:

$$\epsilon \equiv \int_{-1}^{1} \frac{d\cos\theta}{2} \int d\tau \left(\sum_k \dot{\mathcal{E}}^{(k)}(\tau,\theta) \right)^2 .$$

As a consequence of the CL-symmetry & asymptotics @ $\mathcal{J}^+$:

$$\dot{\mathcal{E}}^{(k)}(\tau,\theta) = \left(\frac{1}{1-\cos\theta}\right)^2 \left(\frac{1+\cos\theta}{1-\cos\theta}\right)^{k-1-\frac{1}{4}\frac{D-4}{D-3}} \dot{\mathcal{E}}^{(k)}\left(\bar{\tau}(\tau,\theta), \frac{\pi}{2}\right) .$$

Order of perturbation theory ⇔ order of angular expansion!

ASYMPTOTIC ANGULAR FACTORISATION (AT ALL ORDERS!)

Waveform $\dot{\mathcal{E}}^{(k)}(r,\tau,\theta) \propto \frac{d}{d\tau}(E^{(k)} + H^{(k)})$ when $r \to +\infty$ is

$$\dot{\mathcal{E}}(\tau,\theta)$$

The associated inelasticity is:

$$\epsilon \equiv \int_{-1}^{1} \frac{d\cos\theta}{2} \int d\tau \left(\sum_k \dot{\mathcal{E}}^{(k)}(\tau,\theta) \right)^2 .$$

As a consequence of the CL-symmetry & asymptotics @ $\mathcal{J}^+$:

$$\dot{\mathcal{E}}^{(k)}(\tau,\theta) = \left(\frac{1}{1-\cos\theta} \right)^2 \left(\frac{1+\cos\theta}{1-\cos\theta} \right)^{k-1-\frac{1}{4}\frac{D-4}{D-3}} \dot{\mathcal{E}}^{(k)}\left(\bar{\tau}(\tau,\theta), \frac{\pi}{2} \right) .$$

Order of perturbation theory $\Leftrightarrow$ order of angular expansion!

<u>Note</u>: This was claimed but not proved before

SURFACE INTEGRALS (@ $\mathcal{J}^+$) ARE BESSEL FUNCTIONS!!!

Magic happens (@ $\mathcal{J}^+$) when:

1. Fourier transform $\hat{\dot{F}}(\omega, \theta) = \int d\tau\, \dot{F}(\tau, \theta) e^{-i\omega\tau}$
2. Go to new frequency $\omega \to \Omega(\omega, \theta)$ & new wave $\hat{\mathcal{F}}(\Omega, \theta)$
3. Clever use of CL + back Fourier transform $\Omega \to t_{(\mathrm{new})}$ + horrific (but "straightforward") calculations

SURFACE INTEGRALS (@ $\mathscr{J}^+$) ARE BESSEL FUNCTIONS!!!

Magic happens (@ $\mathscr{J}^+$) when:

1. Fourier transform $\hat{\dot{F}}(\omega, \theta) = \int d\tau \, \dot{F}(\tau, \theta) e^{-i\omega\tau}$
2. Go to new frequency $\omega \to \Omega(\omega, \theta)$ & new wave $\hat{\mathcal{F}}(\Omega, \theta)$
3. Clever use of CL + back Fourier transform $\Omega \to t_{(\mathrm{new})}$ + horrific (but "straightforward") calculations

SURFACE INTEGRALS (@ $\mathscr{J}^+$) ARE BESSEL FUNCTIONS!!!

Magic happens (@ $\mathscr{J}^+$) when:

1. Fourier transform $\hat{\dot{F}}(\omega, \theta) = \int d\tau\, \dot{F}(\tau, \theta) e^{-i\omega\tau}$
2. Go to new frequency $\omega \to \Omega(\omega, \theta)$ & new wave $\hat{\mathcal{F}}(\Omega, \theta)$
3. Clever use of CL + back Fourier transform $\Omega \to t_{\text{(new)}}$ + horrific (but "straightforward") calculations

SURFACE INTEGRALS (@ $\mathscr{J}^+$) ARE BESSEL FUNCTIONS!!!

Magic happens (@ $\mathscr{J}^+$) when:

1. Fourier transform $\hat{\dot{F}}(\omega, \theta) = \int d\tau \, \dot{F}(\tau, \theta) e^{-i\omega\tau}$
2. Go to new frequency $\omega \to \Omega(\omega, \theta)$ & new wave $\hat{\mathcal{F}}(\Omega, \theta)$
3. Clever use of CL + back Fourier transform $\Omega \to t_{(\mathrm{new})}$ + horrific (but “straightforward”) calculations

SURFACE INTEGRALS (@ $\mathscr{J}^+$) ARE BESSEL FUNCTIONS!!!

Magic happens (@ $\mathscr{J}^+$) when:

1. Fourier transform $\hat{\dot{F}}(\omega,\theta) = \int d\tau\, \dot{F}(\tau,\theta) e^{-i\omega\tau}$
2. Go to new frequency $\omega \to \Omega(\omega,\theta)$ & new wave $\hat{\mathcal{F}}(\Omega,\theta)$
3. Clever use of CL + back Fourier transform $\Omega \to t_{(\mathrm{new})}$ + horrific (but "straightforward") calculations

Tadá!!!

$$\mathcal{F}(t) = \ldots \left[\frac{1}{\Phi'(R)}\frac{d}{dR}\right]^{k-1} \left(\Phi'(R)^{-1} R^{\frac{D-2}{2}} J_{\frac{D-4}{2}+m}(2R) f(R)\right)$$

- $R = \Phi^{-1}(t)$ and t new time in $D > 4$
- $\Phi(R)$ is the Gravitational shock profile
- $f(R)$ is the initial data for given metric perturbation

Magic happens (@ $\mathscr{J}^+$) when:

1. Fourier transform $\hat{\dot{F}}(\omega,\theta) = \int d\tau\, \dot{F}(\tau,\theta) e^{-i\omega\tau}$
2. Go to new frequency $\omega \to \Omega(\omega,\theta)$ & new wave $\hat{\mathcal{F}}(\Omega,\theta)$
3. Clever use of CL + back Fourier transform $\Omega \to t_{(\text{new})}$ + horrific (but “straightforward”) calculations

Tadá!!!

$$\mathcal{F}(t) = \ldots \left[\frac{1}{\Phi'(R)}\frac{d}{dR}\right]^{k-1} \left(\Phi'(R)^{-1} R^{\frac{D-2}{2}} J_{\frac{D-4}{2}+m}(2R) f(R)\right)$$

- $R = \Phi^{-1}(t)$ and t new time in $D > 4$
- $\Phi(R)$ is the Gravitational shock profile
- $f(R)$ is the initial data for given metric perturbation

SURFACE INTEGRALS (@ $\mathscr{J}^+$) ARE BESSEL FUNCTIONS!!!

Magic happens (@ $\mathscr{J}^+$) when:

1. Fourier transform $\hat{\dot{F}}(\omega, \theta) = \int d\tau \, \dot{F}(\tau, \theta) e^{-i\omega\tau}$
2. Go to new frequency $\omega \to \Omega(\omega, \theta)$ & new wave $\hat{\mathcal{F}}(\Omega, \theta)$
3. Clever use of CL + back Fourier transform $\Omega \to t_{(\mathrm{new})}$ + horrific (but "straightforward") calculations

Tadá!!!

$$\mathcal{F}(t) = \ldots \left[\frac{1}{\Phi'(R)} \frac{d}{dR} \right]^{k-1} \left(\Phi'(R)^{-1} R^{\frac{D-2}{2}} J_{\frac{D-4}{2}+m}(2R) f(R) \right)$$

- $R = \Phi^{-1}(t)$ and t new time in $D > 4$
- $\Phi(R)$ is the Gravitational shock profile
- $f(R)$ is the initial data for given metric perturbation

SURFACE INTEGRALS (@ $\mathscr{J}^+$) ARE BESSEL FUNCTIONS!!!

Magic happens (@ $\mathscr{J}^+$) when:

1. Fourier transform $\hat{\dot{F}}(\omega,\theta) = \int d\tau\, \dot{F}(\tau,\theta) e^{-i\omega\tau}$
2. Go to new frequency $\omega \to \Omega(\omega,\theta)$ & new wave $\hat{\mathcal{F}}(\Omega,\theta)$
3. Clever use of CL + back Fourier transform $\Omega \to t_{(\mathrm{new})}$ + horrific (but "straightforward") calculations

Tadá!!!

$$\mathcal{F}(t) = \ldots \left[\frac{1}{\Phi'(R)}\frac{d}{dR}\right]^{k-1} \left(\Phi'(R)^{-1} R^{\frac{D-2}{2}} J_{\frac{D-4}{2}+m}(2R) f(R)\right)$$

- $R = \Phi^{-1}(t)$ and t new time in $D > 4$
- $\Phi(R)$ is the Gravitational shock profile
- $f(R)$ is the initial data for given metric perturbation

Surf. integral contributions to the inelasticity’s angular series $\epsilon(\theta) = \sum_{N=1}^{\infty} \epsilon^{(N)}(\theta)$

N	Term	contribution to $\epsilon^{(N)}(\frac{\pi}{2})$
1	$\mathcal{F}_{E^{(1)}}\mathcal{F}_{E^{(1)}}$	$8\left(\frac{1}{2}-\frac{1}{D}\right)$
2	$2\mathcal{F}_{E^{(1)}}\mathcal{F}_{E^{(2)}}$	$-32\left(\frac{1}{2}-\frac{1}{D}\right)\frac{D-4}{D+2}$
	$2\mathcal{F}_{E^{(1)}}\mathcal{F}_{H^{(2)}}$	$-32\left(\frac{1}{2}-\frac{1}{D}\right)\frac{D-3}{D-4}$
3	$\mathcal{F}_{E^{(2)}}\mathcal{F}_{E^{(2)}}$	$64\left(\frac{1}{2}-\frac{1}{D}\right)\frac{(D-4)^2}{(D+2)(D+4)}$
	$2\mathcal{F}_{E^{(2)}}\mathcal{F}_{H^{(2)}}$	$64\left(\frac{1}{2}-\frac{1}{D}\right)\frac{(D-3)}{(D+2)}$
	$\mathcal{F}_{H^{(2)}}\mathcal{F}_{H^{(2)}}$	$64\left(\frac{1}{2}-\frac{1}{D}\right)\frac{(D-3)^2}{(D-4)(D-8)}$

- Checked with numerics with relative error of less than 10^{-4}
- Note divergent cases which agree with non-integrable tails

Surf. integral contributions to the inelasticity’s angular series $\epsilon(\theta) = \sum_{N=1}^{\infty} \epsilon^{(N)}(\theta)$

N	Term	contribution to $\epsilon^{(N)}(\frac{\pi}{2})$
1	$\mathcal{F}_{E^{(1)}}\mathcal{F}_{E^{(1)}}$	$8\left(\frac{1}{2}-\frac{1}{D}\right)$
2	$2\mathcal{F}_{E^{(1)}}\mathcal{F}_{E^{(2)}}$	$-32\left(\frac{1}{2}-\frac{1}{D}\right)\frac{D-4}{D+2}$
	$2\mathcal{F}_{E^{(1)}}\mathcal{F}_{H^{(2)}}$	$-32\left(\frac{1}{2}-\frac{1}{D}\right)\frac{D-3}{D-4}$
3	$\mathcal{F}_{E^{(2)}}\mathcal{F}_{E^{(2)}}$	$64\left(\frac{1}{2}-\frac{1}{D}\right)\frac{(D-4)^2}{(D+2)(D+4)}$
	$2\mathcal{F}_{E^{(2)}}\mathcal{F}_{H^{(2)}}$	$64\left(\frac{1}{2}-\frac{1}{D}\right)\frac{(D-3)}{(D+2)}$
	$\mathcal{F}_{H^{(2)}}\mathcal{F}_{H^{(2)}}$	$64\left(\frac{1}{2}-\frac{1}{D}\right)\frac{(D-3)^2}{(D-4)(D-8)}$

- Checked with numerics with relative error of less than 10^{-4}
- Note divergent cases which agree with non-integrable tails

Surf. integral contributions to the inelasticity's angular series $\epsilon(\theta) = \sum_{N=1}^{\infty} \epsilon^{(N)}(\theta)$

N	Term	contribution to $\epsilon^{(N)}(\frac{\pi}{2})$
1	$\mathcal{F}_{E^{(1)}}\mathcal{F}_{E^{(1)}}$	$8\left(\frac{1}{2}-\frac{1}{D}\right)$
2	$2\mathcal{F}_{E^{(1)}}\mathcal{F}_{E^{(2)}}$	$-32\left(\frac{1}{2}-\frac{1}{D}\right)\frac{D-4}{D+2}$
	$2\mathcal{F}_{E^{(1)}}\mathcal{F}_{H^{(2)}}$	$-32\left(\frac{1}{2}-\frac{1}{D}\right)\frac{D-3}{D-4}$
3	$\mathcal{F}_{E^{(2)}}\mathcal{F}_{E^{(2)}}$	$64\left(\frac{1}{2}-\frac{1}{D}\right)\frac{(D-4)^2}{(D+2)(D+4)}$
	$2\mathcal{F}_{E^{(2)}}\mathcal{F}_{H^{(2)}}$	$64\left(\frac{1}{2}-\frac{1}{D}\right)\frac{(D-3)}{(D+2)}$
	$\mathcal{F}_{H^{(2)}}\mathcal{F}_{H^{(2)}}$	$64\left(\frac{1}{2}-\frac{1}{D}\right)\frac{(D-3)^2}{(D-4)(D-8)}$

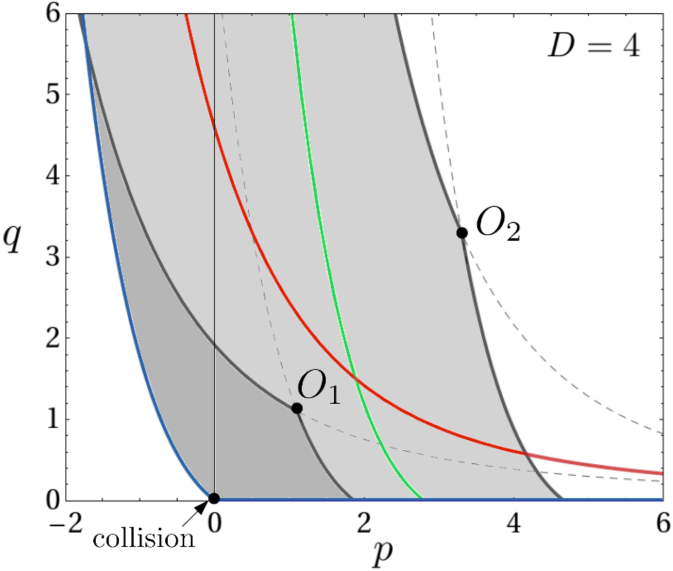
- Checked with numerics with relative error of less than 10^{-4}
- Note divergent cases which agree with non-integrable tails

OUTLINE

1. Exact results – The role of symmetry
2. The Penrose diagram

(p, q) [OR (P, Q)] NOT INTUITIVE!

REGION IV – FUTURE OF THE COLLISION



THE 2D WAVE OPERATOR AND ITS CHARACTERISTICS

- 2D wave operator at k-th order of rank m

$$\rho^2\Box_m^{(k)} \to \dots \frac{\partial^2}{\partial Q^2} + \dots \frac{\partial^2}{\partial P^2} + \dots \frac{\partial^2}{\partial Q \partial P} \dots$$

- Characteristic coordinates $\eta(P, Q)$ and $\xi(P, Q)$

$$\rho^2\Box_m^{(k)} \to \dots \frac{\partial^2}{\partial\eta\partial\xi} + 0 \times \frac{\partial^2}{\partial\eta^2} + 0 \times \frac{\partial^2}{\partial\eta^2} + \dots$$

- Define compactified versions

$$\hat{\eta} \equiv \frac{\eta}{\sqrt{1+\eta^2}} \quad , \quad \hat{\xi} \equiv \frac{\xi}{\sqrt{1+\xi^2}}$$

THE 2D WAVE OPERATOR AND ITS CHARACTERISTICS

- 2D wave operator at k-th order of rank m

$$\rho^2\square_m^{(k)} \to \dots \frac{\partial^2}{\partial Q^2} + \dots \frac{\partial^2}{\partial P^2} + \dots \frac{\partial^2}{\partial Q \partial P} \dots$$

- Characteristic coordinates $\eta(P, Q)$ and $\xi(P, Q)$

$$\rho^2\square_m^{(k)} \to \dots \frac{\partial^2}{\partial \eta \partial \xi} + 0 \times \frac{\partial^2}{\partial \eta^2} + 0 \times \frac{\partial^2}{\partial \eta^2} + \dots$$

- Define compactified versions

$$\hat{\eta} \equiv \frac{\eta}{\sqrt{1+\eta^2}} \quad , \quad \hat{\xi} \equiv \frac{\xi}{\sqrt{1+\xi^2}}$$

THE 2D WAVE OPERATOR AND ITS CHARACTERISTICS

- 2D wave operator at k-th order of rank m

$$\rho^2 \Box_m^{(k)} \to \ldots \frac{\partial^2}{\partial Q^2} + \ldots \frac{\partial^2}{\partial P^2} + \ldots \frac{\partial^2}{\partial Q \partial P} \ldots$$

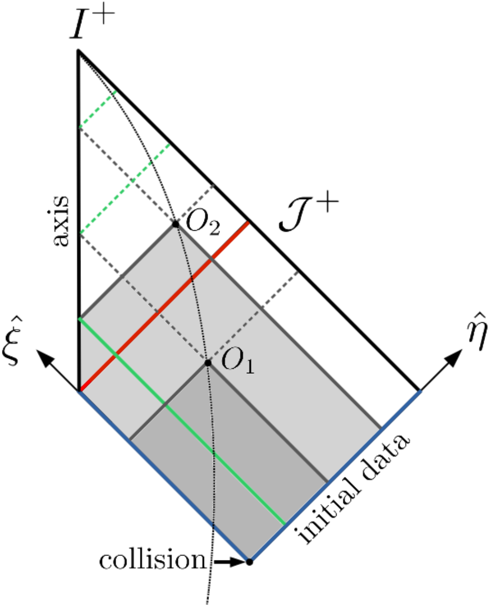
- Characteristic coordinates $\eta(P, Q)$ and $\xi(P, Q)$

$$\rho^2 \Box_m^{(k)} \to \ldots \frac{\partial^2}{\partial \eta \partial \xi} + 0 \times \frac{\partial^2}{\partial \eta^2} + 0 \times \frac{\partial^2}{\partial \eta^2} + \ldots$$

- Define compactified versions

$$\hat{\eta} \equiv \frac{\eta}{\sqrt{1+\eta^2}} \quad , \quad \hat{\xi} \equiv \frac{\xi}{\sqrt{1+\xi^2}}$$

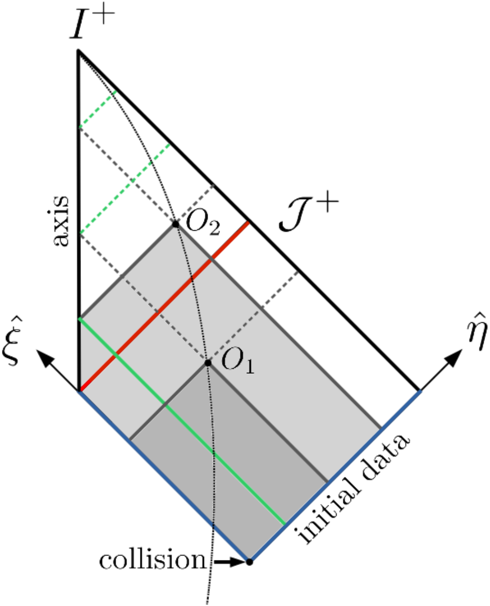
THE CONFORMAL DIAGRAM – AKA "THE MINE FIELD"



Many things clearer:

- Light rays @ 45º
- Past light cone of O_i
- Source singularity from rays that cross the axis
- (Retarded) Green function singularity from axis crossing
- Special coordinate $\bar{\tau} \Leftrightarrow \lim_{\eta\to+\infty} \xi$

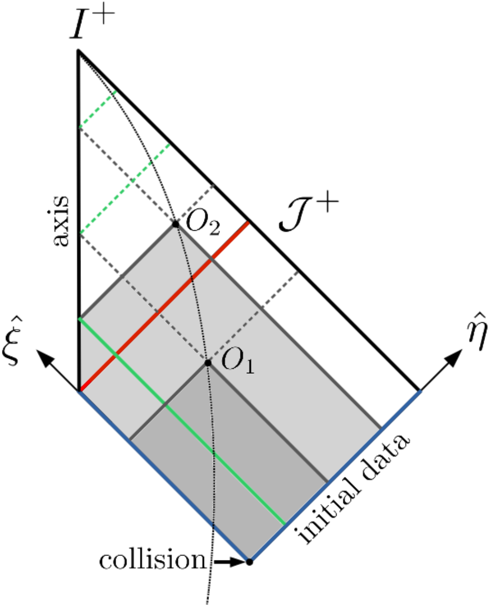
THE CONFORMAL DIAGRAM – AKA "THE MINE FIELD"



Many things clearer:

- Light rays @ 45º
- Past light cone of O_i
- Source singularity from rays that cross the axis
- (Retarded) Green function singularity from axis crossing
- Special coordinate $\bar{\tau} \Leftrightarrow \lim_{\eta\to+\infty} \xi$

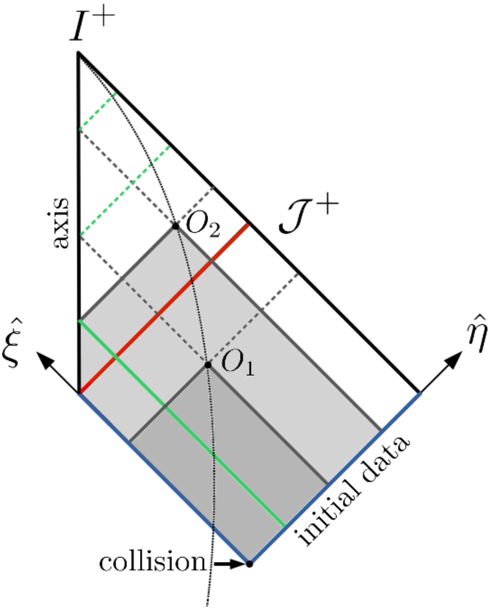
THE CONFORMAL DIAGRAM – AKA "THE MINE FIELD"



Many things clearer:

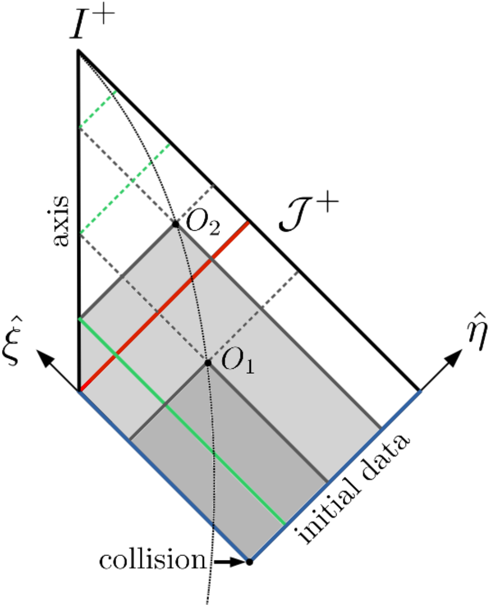
- Light rays @ 45º
- Past light cone of O_i
- Source singularity from rays that cross the axis
- (Retarded) Green function singularity from axis crossing
- Special coordinate $\bar{\tau} \Leftrightarrow \lim_{\eta \to +\infty} \xi$

THE CONFORMAL DIAGRAM – AKA "THE MINE FIELD"



Many things clearer:

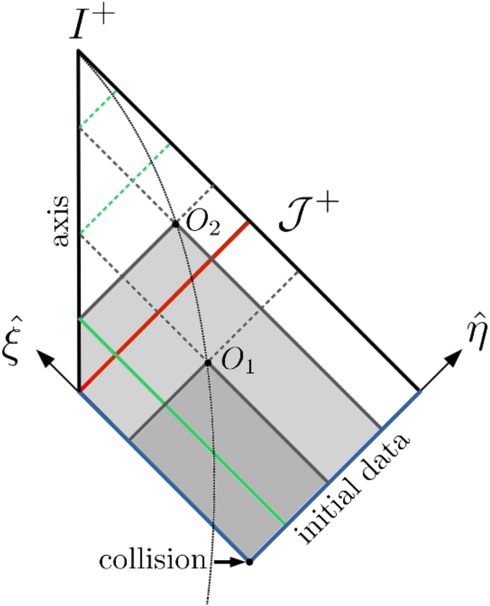
- Light rays @ 45º
- Past light cone of O_i
- Source singularity from rays that cross the axis
- (Retarded) Green function singularity from axis crossing
- Special coordinate $\bar{\tau} \Leftrightarrow \lim_{\eta\to+\infty} \xi$



Many things clearer:

- Light rays @ 45º
- Past light cone of O_i
- Source singularity from rays that cross the axis
- (Retarded) Green function singularity from axis crossing
- Special coordinate $\bar{\tau} \Leftrightarrow \lim_{\eta \to +\infty} \xi$

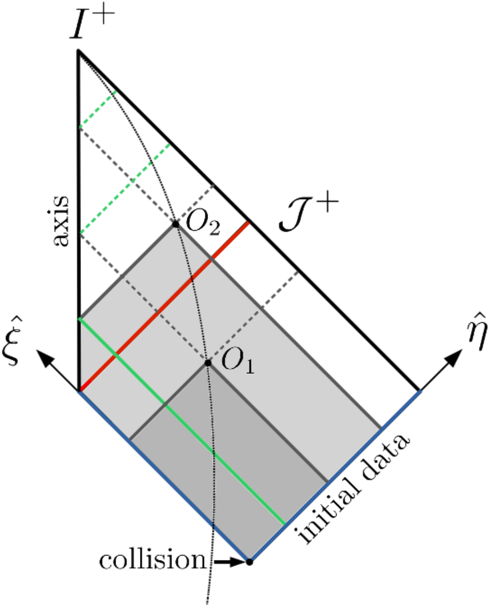
THE CONFORMAL DIAGRAM – AKA "THE MINE FIELD"



Many things clearer:

- Light rays @ 45º
- Past light cone of O_i
- Source singularity from rays that cross the axis
- (Retarded) Green function singularity from axis crossing
- Special coordinate $\bar{\tau} \Leftrightarrow \lim_{\eta \to +\infty} \xi$

THE CONFORMAL DIAGRAM – AKA "THE MINE FIELD"



Many things clearer:

- Light rays @ 45°
- Past light cone of O_i
- Source singularity from rays that cross the axis
- (Retarded) Green function singularity from axis crossing
- Special coordinate $\bar{\tau} \Leftrightarrow \lim_{\eta \to +\infty} \xi$

Challenges:

- Control numerical errors of higher order integrals
- All boundaries in the diagram have coordinate singularities!

Where we got:

- Proved correspondence between pertubative expansion and axis expansion
- Showed all first order results are exact (Bessel functions @ $\mathscr{J}^+$) by working in Fourier space
- Found characteristic coordinates for all *D* and an extremely useful conformal diagram

In progress: Control errors @ higher orders in the new promising compactified version of the problem

CONCLUSIONS

Where we got:

- Proved correspondence between pertubative expansion and axis expansion
- Showed all first order results are exact (Bessel functions @ $\mathscr{I}^+$) by working in Fourier space
- Found characteristic coordinates for all D and an extremely useful conformal diagram

In progress: Control errors @ higher orders in the new promising compactified version of the problem

THANK YOU!