Signatures of accretion on gravitational and electromagnetic waves from black holes.

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Plan

- Our goal is to describe the gravitational waves generated by the motion of disks of matter in the background of a black hole using perturbation theory.
- For simplicity we focused on the emission of gravitational waves when a black hole is perturbed by a surrounding pressure-less fluid matter
- Specifically, we work with the curvature perturbations within the null tetrad formulations developed by Newman and Penrose.

Formalism

In order to obtain the perturbed equations, one starts from the Bianchi identities and the definition of the Riemann tensor (Chandrasekhar '83)

$$R_{\mu\nu\lambda\tau\,;\sigma} + R_{\mu\nu\sigma\lambda\,;\tau} + R_{\mu\nu\tau\sigma\,;\lambda} = 0\ , \qquad R_{\sigma\mu\nu\lambda}\, Z_a{}^{\sigma} = Z_{a\,\mu;\nu\lambda} - Z_{a\,\mu;\lambda\nu}, \quad .$$

These equations are projected on a null tetrad and become equations for the Weyl scalars and spinor coefficients. The perturbed expression of these equations is computed for the case of vacuum type D spacetimes to obtain a master equation for the perturbed scalar

$$\Psi_4 \quad = \quad -C_{\mu\nu\lambda\tau}\, k^{\mu}\, m^{*\nu}\, k^{\lambda}\, m^{*\tau},$$

including the source terms given by Einstein's equations.

We derived the perturbation equation for the Ψ_4 Weyl scalar for a static spherically symmetric space time.

$$ds^2 = -\left(\alpha^2 - \gamma^2\,\beta^2\right)\,dt^2 + 2\,\gamma^2\,\beta\,dt\,dr\, + \gamma^2\,dr^2 + r^2\,d\Omega^2,$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta\,d\varphi^2$ is the solid angle element, and the lapse, α, the radial component of the shift vector, β.

We obtained the so called Teukolsky equation for vacuum type D spacetimes (Teukolsky '73), and then focused on the Schwarzschild spacetime written in Kerr-Schild coordinates.

$$ds^2 = -\left(1 - \frac{2M}{r}\right)\, dt^2 + \frac{4M}{r}\, dt\, dr + \left(1 + \frac{2M}{r}\right)\, dr^2 + r^2\, d\Omega^2 \quad .$$

and the tetrad

$$l^\mu = \frac{1}{2}\,\left(1 + \frac{2M}{r}, 1 - \frac{2M}{r}, 0, 0\right)\;, \quad k^\mu = (1, -1, 0, 0)\;,$$

$$m^\mu = \frac{1}{\sqrt{2}\, r}\,(0, 0, 1, i\,\csc\theta)\;,$$

The resulting perturbation equation is an inhomogeneous wave equation.

Matter content

We consider the matter source to be described by a pressure-less fluid

$$T_{\mu\nu} = \rho\, u_\mu\, u_\nu \ ,$$

where ρ is the rest mass density and u_μ is the four velocity of the dust. Furthermore, we consider that the fluid is in-falling radially in the black hole and the four velocity has only temporal and radial components,

$$u^\mu = \left(u^0, u^1, 0, 0\right) \ .$$

The evolution of the fluid is described by the continuity equation for the current vector, $J^\mu = \rho\, u^\mu$, and the conservation equation for the stress energy tensor:

$$J^\mu{}_{;\mu} = 0 \quad \text{and} \quad T^{\mu\nu}{}_{;\mu} = 0 \ .$$

With the radial in-falling matter, plus a decomposition of the density in terms of the spherical harmonics with zero weight

$$\rho = \sum_{lm} \rho_{l,m}(t,r)\, {Y_0}^{l,m}(\theta,\phi)\ .$$

it is possible to separate the angular and radial part of the sources. Furthermore, expanding the perturbation of Ψ_4 as

$$\Psi_4 = \sum_{lm} R^G_{l,m}(t,r)\, {Y_{-2}}^{l,m}(\theta,\phi)\ ,$$

we can get a radial-time equation.

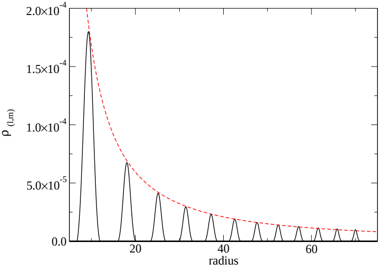
The resulting second order equation can be reduced to a first order system.

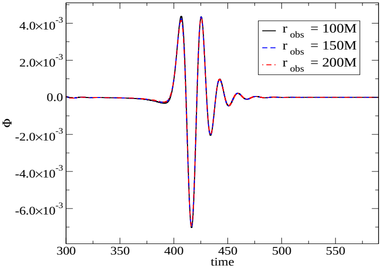
We solved numerically this system using the method of lines with a total variation diminishing Runge Kutta integrator with a four order spatial stencil.

Results

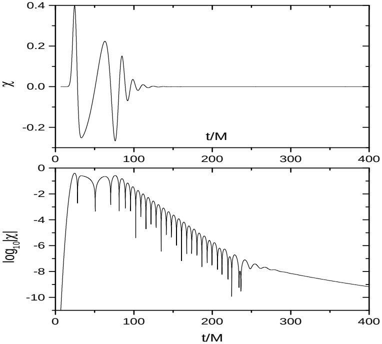
We parametrize the shells of matter using a Gaussian pulse

$$\rho_{l,m}(r, t=0) = A_0\, e^{-(r-r_0)^2/\sigma^2},$$

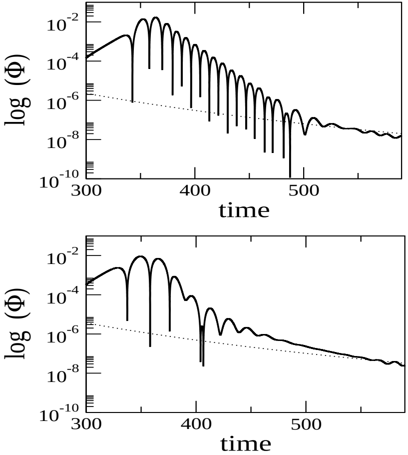




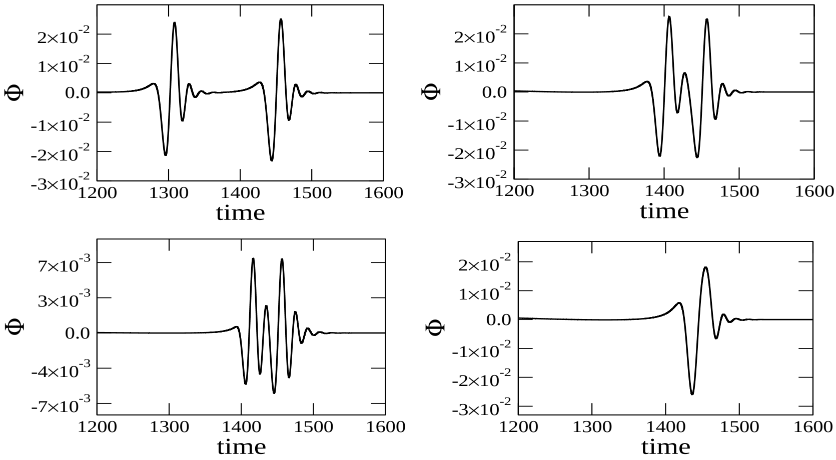
The three phases of a gravitational signal: Initial burst, quasinormal ringing and tail (Kokkotas and Schmidt '99)



We analyze the signal with respect the compactness of the shells,
$\sigma = 1/2\,M$, M, $3/2\,M$, and $5/2\,M$,



Gravitational response due to two consecutive pulses of fluid



Then we consider a charged fluid falling into the black hole.

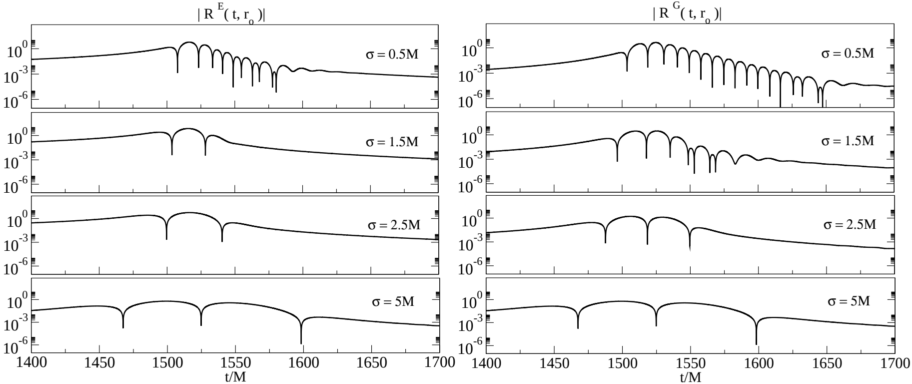
$$J^{\mu}_{el} = q\rho u^{\mu},$$

where q is the charge to mass ratio e/m. We solve the Maxwell equations ${F^{\mu\nu}}_{;\nu} = 4\,\pi\,J^{\mu}_{el}$ for the scalar ϕ_2

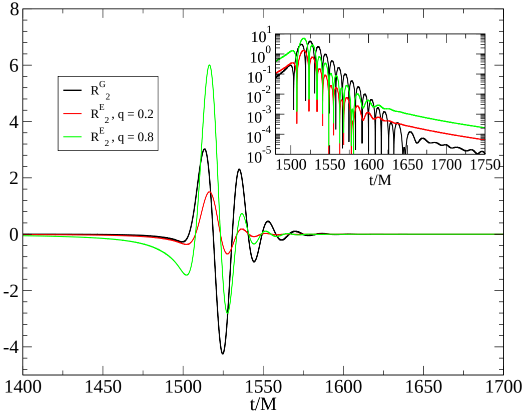
$$\phi_2 \equiv F_{\mu\nu} m^{*\mu} k^{\nu} \ .$$

coupled with the perturbation equation.

The compactness of the shell is reflected in both electromagnetic and gravitational signals



The electric and gravitational quasi-normal modes are present if the shells are compact enough. We obtained the frequencies using a fourier transform and the results from the evolution match the results in the frequency domain (Kokkotas and Schmidt '99).



We estimate the energy radiated as

$$\frac{d}{dt}E_{GW} = \lim_{r\to\infty} \frac{1}{16\pi} \sum_{\ell,m} |\int_{-\infty}^{t} dt' R_{\ell}^{G}(t')|^2 \ . \quad \frac{d}{dt}E_{EM} = \lim_{r\to\infty} \frac{1}{4\pi} \sum_{\ell,m} |R_{\ell}^{E}(t)|^2 \ .$$

We found a quadratic dependence between the electromagnetic and gravitational energy emitted, of the form $E_{EM}/E_{GW} = a\, q^2$, with $a = 12.417, 11.128, 10.928$ for $\ell = 2, 3, 4$ respectively.

Conclusions

- The compactness of the shells affects the gravitational and electromagnetic emission.
- Pressure-less matter induces electric and gravitational quasi-normal modes on both signals. However there is no direct mixing between frequencies.
- The electromagnetic energy emitted after the falling of matter is related with the energy carried by the gravitational waves via q^2.