Kerr-Newman scalar clouds

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Contents

Introduction

- "Black holes have no hair."
- Kerr black holes: Superradiance and superradiant instability:
  (i) $\mathcal{I}(\omega) < 0$, for $\mathcal{R}(\omega) > m\Omega_H$;
  (ii) $\mathcal{I}(\omega) > 0$, for $\mathcal{R}(\omega) < m\Omega_H$;
  (iii) $\mathcal{I}(\omega) = 0$, for $\omega = m\Omega_H$.

Scalar clouds

The line element for the Kerr-Newman black hole is

$$\begin{aligned} ds^2 = & - \frac{\Delta}{\rho^2}(dt - a\sin^2\theta d\phi)^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 \\ & + \frac{\sin^2\theta}{\rho^2}[(r^2+a^2)d\phi - adt]^2, \end{aligned} \tag{1}$$

with

$$\rho^2 \equiv r^2 + a^2\cos^2\theta, \qquad \Delta \equiv r^2 - 2Mr + a^2 + Q^2, \tag{2}$$

where M and Q are the ADM mass and charge of the BH, respectively, and the ADM angular momentum is given by $J = aM$. The background electromagnetic 4-potential is $A_\alpha = (-rQ/\rho^2, 0, 0, aQr\sin^2\theta/\rho^2)$.

The Klein-Gordon equation for a massive charged particle is given by

$$(\nabla^\alpha - iqA^\alpha)(\nabla_\alpha - iqA_\alpha)\Psi - \mu^2\Psi = 0, \tag{3}$$

where μ is the mass of the scalar field and q is its charge.

$$\Delta\frac{d}{dr}\left(\Delta\frac{dR_{lm}}{dr}\right)+\left[H^2 + (2ma\omega - K_{lm} - \mu^2(r^2+a^2))\Delta\right]R_{lm} = 0, \tag{4}$$

where $H \equiv (r^2+a^2)\omega - am - qQr$.

We find that the asymptotic behaviour of the radial solution is

$$R_{lm}(r) \approx \begin{cases} e^{-i(\omega-\omega_c)r_*}, & \text{for } r \to r_+, \\ \frac{e^{-\sqrt{\mu^2-\omega^2}r}}{r}, & \text{for } r \to \infty, \end{cases} \tag{5}$$

where we defined the critical frequency ω_c, given by

$$\omega_c \equiv m\Omega_H + q\Phi_H = \frac{ma}{r_+^2 + a^2} + \frac{qQr_+}{r_+^2 + a^2} \ . \tag{6}$$

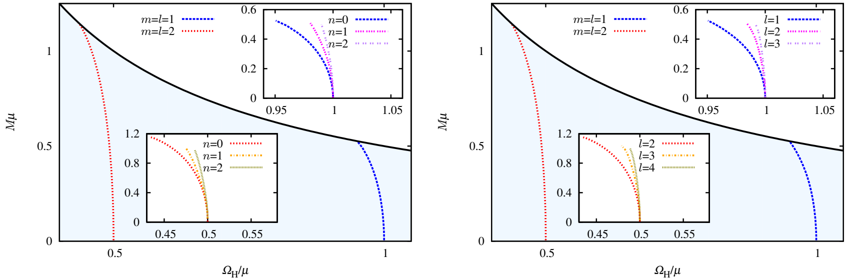

Figure : Mass vs. horizon angular velocity parameter space of Kerr BHs. The insets of the left figure compare the nodeless solutions ($n = 0$) with the solutions with $n = 1, 2$. The insets of the right figure compare the solutions for $m = l$ with the solutions with $m < l$, all with $n = 0$.

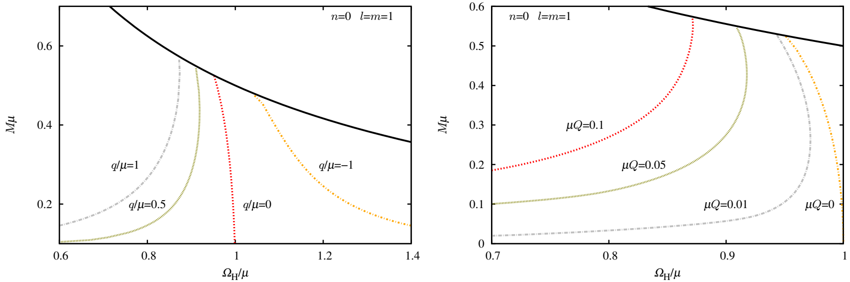

Figure : Existence lines for charged scalar bound states in the Kerr-Newman background, for $n = 0$ and $l = m = 1$, for different values of the field charge and fixed background charge $\mu Q = 0.1$ (left) and for different values of the background charge and fixed field charge $q/\mu = 1$ (right).

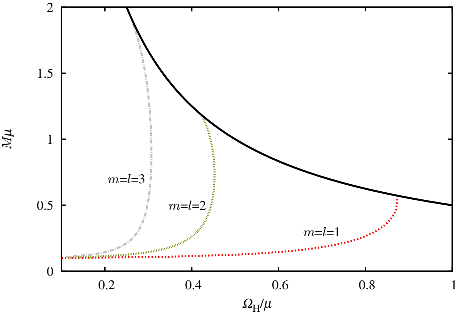

Figure : Existence lines for charged scalar bound states in the Kerr-Newman background, for $\mu Q = 0.1$, $q/\mu = 1$, $n = 0$ and $m = l = 1, 2$ and 3.

The position of the cloud r_{MAX} is the value of r where the function $4\pi r^2|R_{lm}|^2$ attains its maximum.

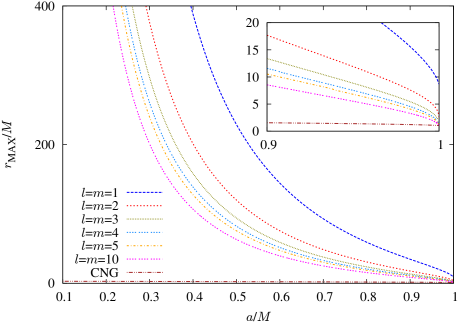

Figure : “Position” of the clouds, r_{MAX}/M, and of the co-rotating circular null geodesic (CNG), as a function of a/M for clouds with $n = 0$ and $l = m = 1, 2, 3, 4, 5, 10$ in the Kerr background.

Analytical expressions

- Hod (2013) found a formula for the near extremal Kerr black hole;
- Detweiler (1980) studied superradiance for the Kerr black hole for small values of the mass coupling, $M\mu \ll 1$;
- Furuhashi and Nambu (2004) studied superradiance for the Kerr-Newman black hole in the limit $M\mu \ll 1$ and $Qq \ll 1$. They also showed that in order to have bound states we must have $M\mu \gtrsim Qq$.

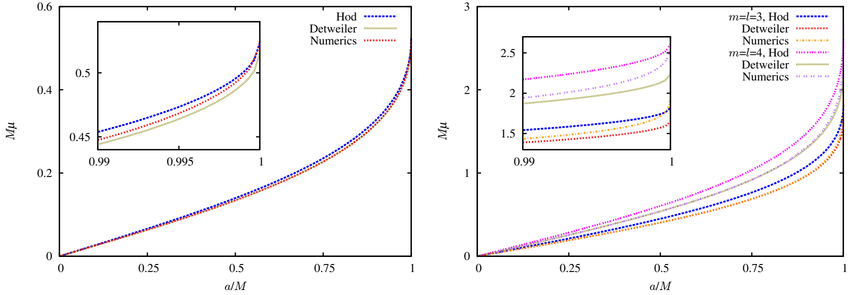

Figure : Comparison between our numerical solution for the nodeless clouds with $m, l = 1$ (left) and $m, l = 2$ and 3 (right) and the analytical results by Hod and Detweiler, in a mass vs. angular momentum parameter for Kerr BHs.

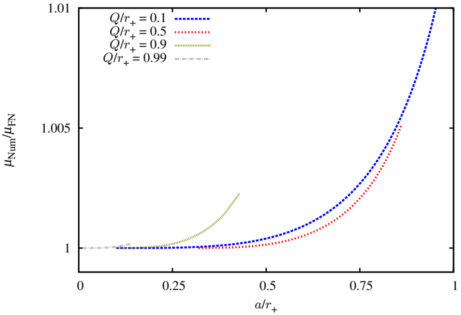

Figure : Comparison between our numerical solutions and the analytical formula by Furuhashi and Nambu, for clouds with $n = 0$, $m = l = 1$ and $qr_+ = 0.1$.

Final remarks

- The radial position of the clouds corroborates a generalization of the ‘no-short hair’(Hod, 2014).
- The existence of clouds are a sufficient condition for the existence of hairy solutions, but not a necessary one.
- Clouds are dynamical attractors.
- Clouds in the laboratory: Acoustic clouds.

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